G51MAL
Machines and Their Languages
Lecture 1
Administrative Details and Introduction

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Finding People and Information (1)

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Finding People and Information (2)

- Main module web page:
  `www.cs.nott.ac.uk/~nhn/G51MAL`
- Coursework/Tutorial web page:
  `www.cs.nott.ac.uk/~wss/teaching/mal`

Contacting Me

- I will be available immediately after each lecture for course-related matters.
- Make an appointment if necessary.
- Please keep e-mail traffic to a minimum.

Aims of the Course

- To familiarize you with key Computer Science concepts in central areas like
  - Automata Theory
  - Formal Languages
  - Models of Computation
  - Complexity Theory
- To equip you with tools with wide applicability in the fields of CS and IT, e.g. for
  - Complier Construction
  - Text Processing
  - XML

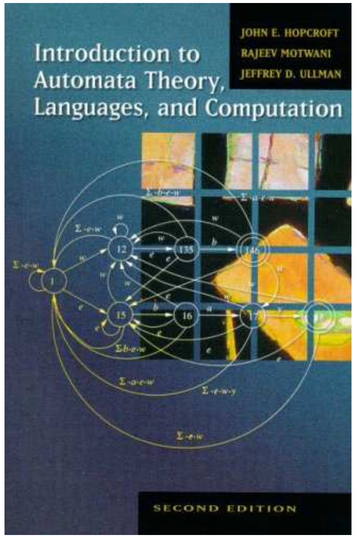
Organization

- ***Lectures:*** Two per week.
- ***Tutorials:*** Weekly in small ($\approx$ 15 students) groups.
  You are expected to participate regularly!
- ***Coursework:*** Weekly ***compulsory*** exercises. Marked and then discussed during tutorials.
- ***Assessment:*** 2 hour exam in May/June, 100% of the mark.

Literature

- Main reference: Hopcroft, Motwani, & Ullman. *Introduction to Automata Theory, Languages, and Computation, 2nd edition*, Addison Wesley, 2001.
- Dr. Thorsten Altenkirch's G51MAL updated lecture notes.
- Your own notes from the lectures!
- Possibly a new version of the lecture notes later.
- Supplementary material, e.g. slides, sample program code.

Literature (2)

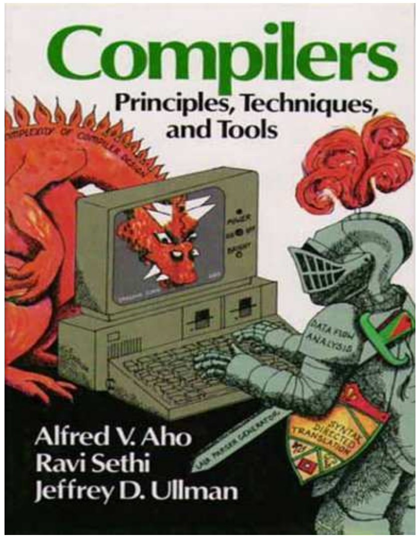


Literature (3)

If you are curious about an important application area you might want to check out:

> Alfred V Aho, Ravi Sethi, Jeffrey D. Ullman. *Compilers — Principles, Techniques, and Tools*, Addison-Wesley, 1986.
> (The “Dragon Book”.)

Literature (4)



Content

1. Mathematical models of computation, such as:
   - Finite automata
   - Pushdown automata
   - Turing machines
2. How to specify formal languages?
   - Regular expressions
   - Context free grammars
   - Context sensitive grammars
3. The relation between 1 and 2.

Why Study Automata Theory?

Finite automata are a useful model for important kinds of hardware and software:

- Software for designing and checking digital circuits.
- Lexical analyzer of compilers.
- Finding words and patterns in large bodies of text, e.g. in web pages.
- Verification of systems with finite number of states, e.g. communication protocols.

Why Study Automata Theory? (2)

The study of Finite Automata and Formal Languages are intimately connected. Methods for specifying formal languages are very important in many areas of CS, e.g.:

- ***Context Free Grammars*** are very useful when designing software that processes data with recursive structure, like the parser in a compiler.
- ***Regular Expressions*** are very useful for specifying lexical aspects of programming languages and search patterns.

Why Study Automata Theory? (3)

Automata are essential for the study of the limits of computation. Two issues:

- What can a computer do at all? (Decidability)
- What can a computer do efficiently? (Intractability)

Example: Regular Expressions (1)

Suppose you need to locate a piece of text in a directory containing a large number of files of various kinds. You recall only that the text mentions the year 1900-something.

The following UNIX-command will do the trick:

```
grep "19[0-9][0-9]" *.txt
```


Example: Regular Expressions (2)

The result is a list of names of files containing text matching the pattern, together with the matching text lines:

```
history.txt:  In 1933 it became
notes.txt:  later on, around 1995,
```

Example: The Halting Problem (1)

Consider the following program. Does it terminate for all values of n ≥ 1?

```
while (n > 1) {
    if even(n) {
        n = n / 2;
    } else {
        n = n * 3 + 1;
    }
}
```


Example: The Halting Problem (2)

Not as easy to answer as it might first seem.

Say we start with n $= 7$, for example:

7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1

In fact, for all numbers that have been tried (***a lot!***), it does terminate . . .

. . . but no one has ever been able to ***prove*** that it always terminates!

Example: The Halting Problem (3)

Then the following important decidability result should perhaps not come as a total surprise:

> It is impossible to write a program that decides if another, arbitrary, program terminates (halts) or not.

What might be surprising is that it is possible to prove such a result. This was first done by the British mathematician ***Alan Turing***.

Alan Turing (1)

Alan Turing (1912–1954):

- Introduced an abstract model of computation, ***Turing Machines***, to give a precice definition of what problems that can be solved by a computer.
- Instrumental in the success of British code breaking efforts during WWII.
- Thorsten recommends Andrew Hodges biography *Alan Turing: the Enigma*.

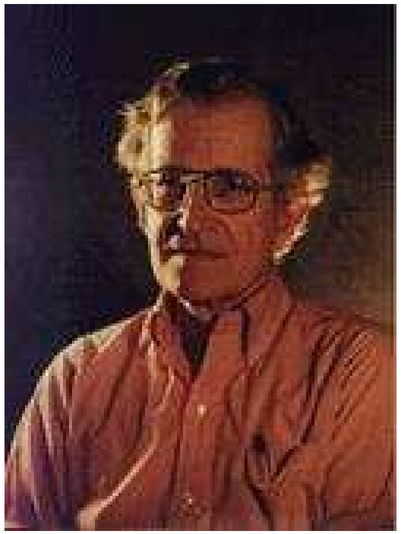
Alan Turing (2)

Noam Chomsky (1)

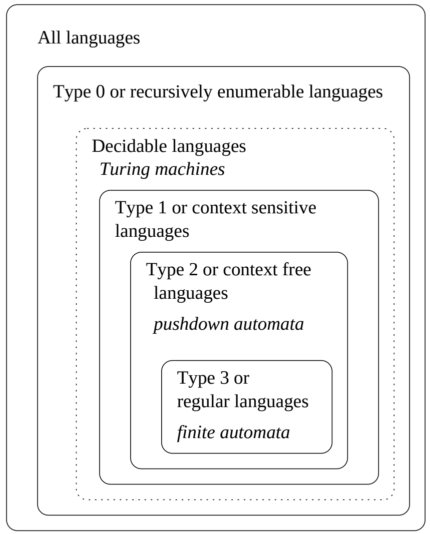
Noam Chomsky (1928–):

- American linguist who introduced ***Context Free Grammars*** in an attempt to describe natural languages formally.
- Also introduced the ***Chomsky Hierarchy*** which classifies grammars and languages and their descriptive power.
- Chomsky is also widely known for his controversial political views and his criticism of the foreign policy of U.S. governments.

Noam Chomsky (2)



The Chomsky Hierarchy



Languages

The terms *language* and *word* are used in a strict technical sense in this course:

- A ***language*** is a set of words.
- A ***word*** is a sequence (or string) of symbols.

ϵ denotes the ***empty word***, the sequence of zero symbols.

Symbols and Alphabets

What is a symbol, then?

Anything, but it has to come from an ***alphabet*** Σ which is a ***finite*** set.

A common (and important) instance is $\Sigma = \{0, 1\}$.

ϵ, the empty word, is ***never*** an symbol of an alphabet.

Alphabet, Word, and Language

alphabet	$\Sigma = \{a, b\}$
words over Σ	$\epsilon, a, b, aa, ab, ba, bb,$
	$aaa, aab, aba, abb, baa, bab, \ldots$
languages	$\emptyset, \{\epsilon\}, \{a\}, \{b\}, \{a, aa\},$
	$\{\epsilon, a, aa, aaa\},$
	$\{a^n \| n \geq 0\},$
	$\{a^n b^n \| n \geq 0, n \, \mathrm{even}\}$

Note the distinction between ϵ, $\emptyset$, and $\{\epsilon\}$!

All Words over an Alphabet (1)

Given an alphabet Σ we define the set Σ^* as set of words (or sequences) over Σ:

- The empty word $\epsilon \in \Sigma^*$.
- given a symbol $x \in \Sigma$ and a word $w \in \Sigma^*$, $xw \in \Sigma^*$.
- These are all elements in Σ^*.

This is called an ***inductive definition***.

All Words over an Alphabet (2)

Example: Given $\Sigma = \{0, 1\}$, some elements of Σ^* are

- ϵ (the empty word)
- 0, 1
- 00, 10, 01, 11
- 000, 100, 010, 110, 010, 110, 011, 111
- $\ldots$

We are just applying the inductive definition.

Note: although there are infinitely many words in Σ^*, each word has a ***finite*** length!

Concatenation of Words (1)

An important operation on Σ^* is ***concatenation***:

given $w, v \in \Sigma^*$, their concatenation $wv \in \Sigma^*$.

For example, concatenation of ab and ba yields $abba$.

This operation can be defined by primitive recursion:

$$\begin{aligned} \epsilon v &= v \\ (xw)v &= x(wv) \end{aligned}$$

Concatenation of Words (2)

Concatenation is associative and has unit ϵ:

$$u(vw) = (uv)w$$
$$\epsilon u = u = u\epsilon$$

where u, v, w are words.

Languages Revisited

The notion of a language L of a set of words over an alphabet Σ can now be made precise:

- $L \subseteq \Sigma^*$, or equivalently
- $L \in \mathcal{P}(\Sigma^*)$.

Examples of Languages (1)

Some examples of languages:

- The set $\{0010, 00000000, \epsilon\}$ is a language over $\Sigma = \{0, 1\}$.
  This is an example of a ***finite*** language.
- The set of words with odd length over $\Sigma = \{1\}$.
- The set of words that contain the same number of 0s and 1s is a language over $\Sigma = \{0, 1\}$.

Examples of Languages (2)

- The set of words which contain the same number of 0s and 1s modulo 2 (i.e. both are even or odd) is a language over $\Sigma = \{0, 1\}$.
- The set of palindromes using the English alphabet, e.g. words which read the same forwards and backwards like abba. This is a language over $\{\mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}\}$.
- The set of correct Java programs. This is a language over the set of UNICODE characters.

Examples of Languages (3)

- The set of programs that, if executed successfully on a Windows machine, prints the text “Hello World!” in a window. This is a language over $\Sigma = \{0, 1\}$.

Concatenation of Languages (1)

Concatenation of words is extended to languages by:

$$MN = \{uv \,|\, u \in M \wedge v \in N\}$$

Example:

$$\begin{aligned} M &= \{\epsilon, a, aa\} \\ N &= \{b, c\} \\ MN &= \{uv \,|\, u \in \{\epsilon, a, aa\} \wedge v \in \{b, c\}\} \\ &= \{\epsilon b, \epsilon c, ab, ac, aab, aac\} \\ &= \{b, c, ab, ac, aab, aac\} \end{aligned}$$

Concatenation of Languages (2)

- Concatenation of languages is associative:

$$L(MN) = (LM)N$$

- Concatenation of languages has zero $\emptyset$:

$$L\emptyset = \emptyset = \emptyset L$$

- Concatenation of languages has unit $\{\epsilon\}$:

$$L\{\epsilon\} = L = \{\epsilon\}L$$

Concatenation of Languages (3)

- Concatenation distributes through set union:

$$\begin{aligned} L(M \cup N) &= LM \cup LN \\ (L \cup M)N &= LN \cup MN \end{aligned}$$

But note e.g. $L(M \cap N) \neq LM \cap LN$!
For example, with $L = \{\epsilon, a\}$, $M = \{\epsilon\}$, $N = \{a\}$, we have

$$L(M \cap N) = L\emptyset = \emptyset$$
$$LM \cap LN = \{\epsilon, a\} \cap \{a, aa\} = \{a\}$$