G51MAL
Machines and Their Languages
Lecture 1
Administrative Details and Introduction

Henrik Nilsson

University of Nottingham, UK

Finding People and Information (1)

- Henrik Nilsson
  Room B47, Computer Science Building
  e-mail: `nhn@cs.nott.ac.uk`
  tel: 0115 846 6506
- Teaching Assistants:
  - Ondrej Rypacek
    e-mail: `oxr@cs.nott.ac.uk`
  - Wouter Swierstra (head TA)
    e-mail: `wss@cs.nott.ac.uk`

Finding People and Information (2)

- Main module web page:
  `www.cs.nott.ac.uk/~nhn/G51MAL`
- Coursework/Tutorial web page:
  `www.cs.nott.ac.uk/~wss/teaching/mal`

Contacting Me

- I will be available immediately after each lecture for course-related matters.
- Make an appointment if necessary.
- Please keep e-mail traffic to a minimum.

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- ***Assessment:*** 2 hour exam in May/June, 100% of the mark.

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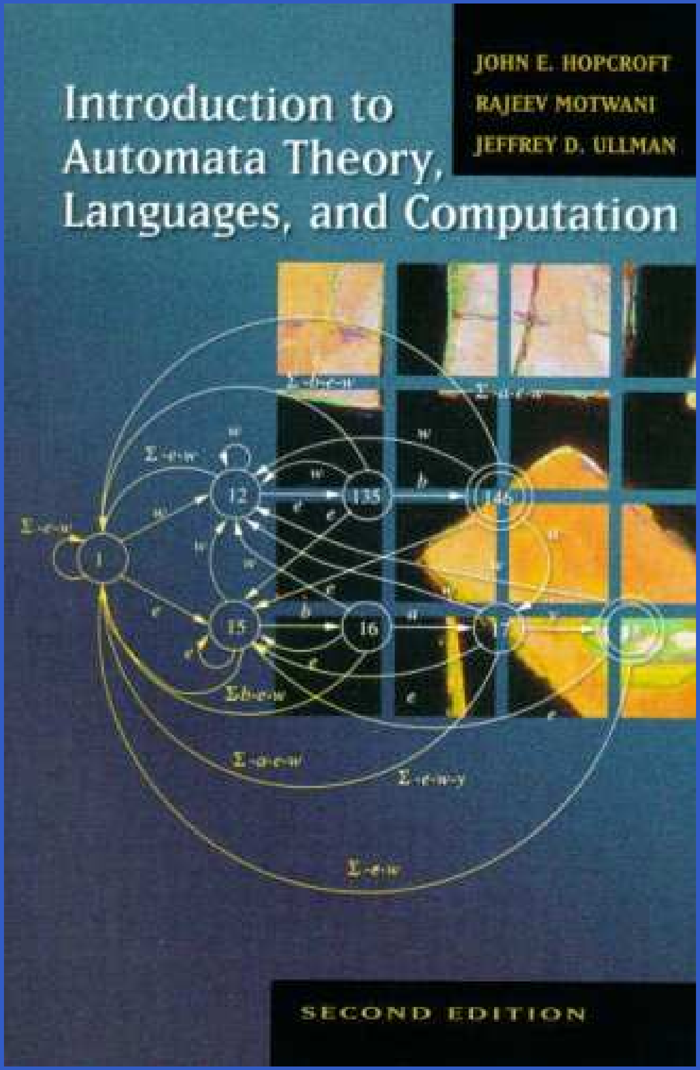
- Supplementary material, e.g. slides, sample program code.

Literature (2)

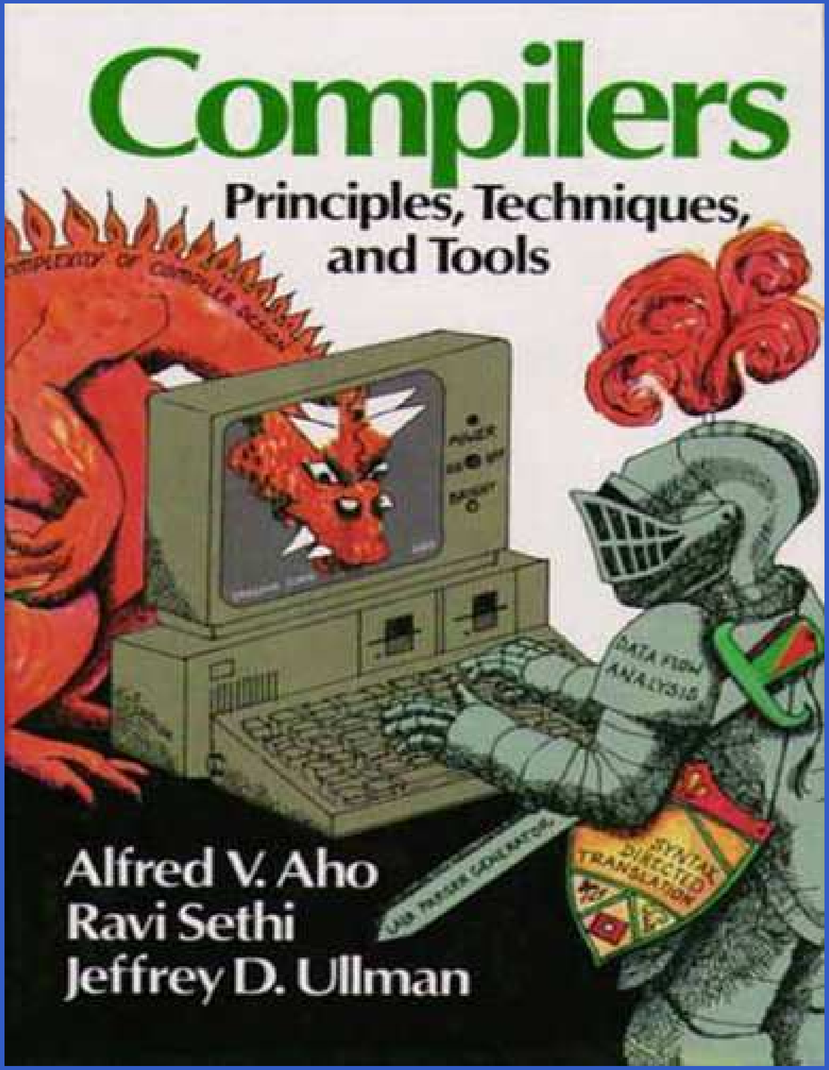


Literature (3)

If you are curious about an important application area you might want to check out:

> Alfred V Aho, Ravi Sethi, Jeffrey D. Ullman. *Compilers — Principles, Techniques, and Tools*, Addison-Wesley, 1986.
> (The “Dragon Book”.)

Literature (4)



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3. The relation between 1 and 2.

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- Lexical analyzer of compilers.
- Finding words and patterns in large bodies of text, e.g. in web pages.
- Verification of systems with finite number of states, e.g. communication protocols.

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- ***Context Free Grammars*** are very useful when designing software that processes data with recursive structure, like the parser in a compiler.
- ***Regular Expressions*** are very useful for specifying lexical aspects of programming languages and search patterns.

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- What can a computer do at all? (Decidability)
- What can a computer do efficiently? (Intractability)

Example: Regular Expressions (1)

Suppose you need to locate a piece of text in a directory containing a large number of files of various kinds. You recall only that the text mentions the year 1900-something.

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The following UNIX-command will do the trick:

```
grep "19[0-9][0-9]" *.txt
```

Example: Regular Expressions (2)

The result is a list of names of files containing text matching the pattern, together with the matching text lines:

```
history.txt:  In 1933 it became
notes.txt:  later on, around 1995,
```

Example: The Halting Problem (1)

Consider the following program. Does it terminate for all values of n ≥ 1?

```
while (n > 1) {
    if even(n) {
        n = n / 2;
    } else {
        n = n * 3 + 1;
    }
}
```

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Say we start with n $= 7$, for example:

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In fact, for all numbers that have been tried (***a lot!***), it does terminate . . .

. . . but no one has ever been able to ***prove*** that it always terminates!

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Then the following important decidability result should perhaps not come as a total surprise:

> It is impossible to write a program that decides if another, arbitrary, program terminates (halts) or not.

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> It is impossible to write a program that decides if another, arbitrary, program terminates (halts) or not.

What might be surprising is that it is possible to prove such a result. This was first done by the British mathematician ***Alan Turing***.

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- Thorsten recommends Andrew Hodges biography *Alan Turing: the Enigma*.

Alan Turing (2)

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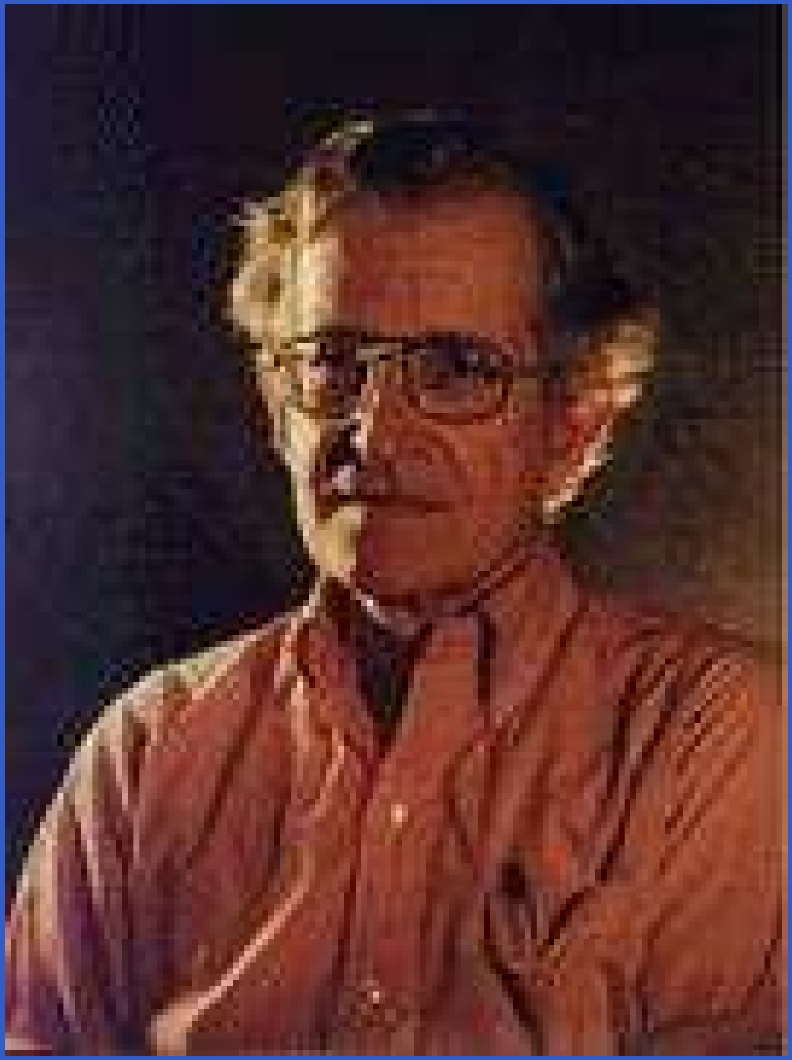
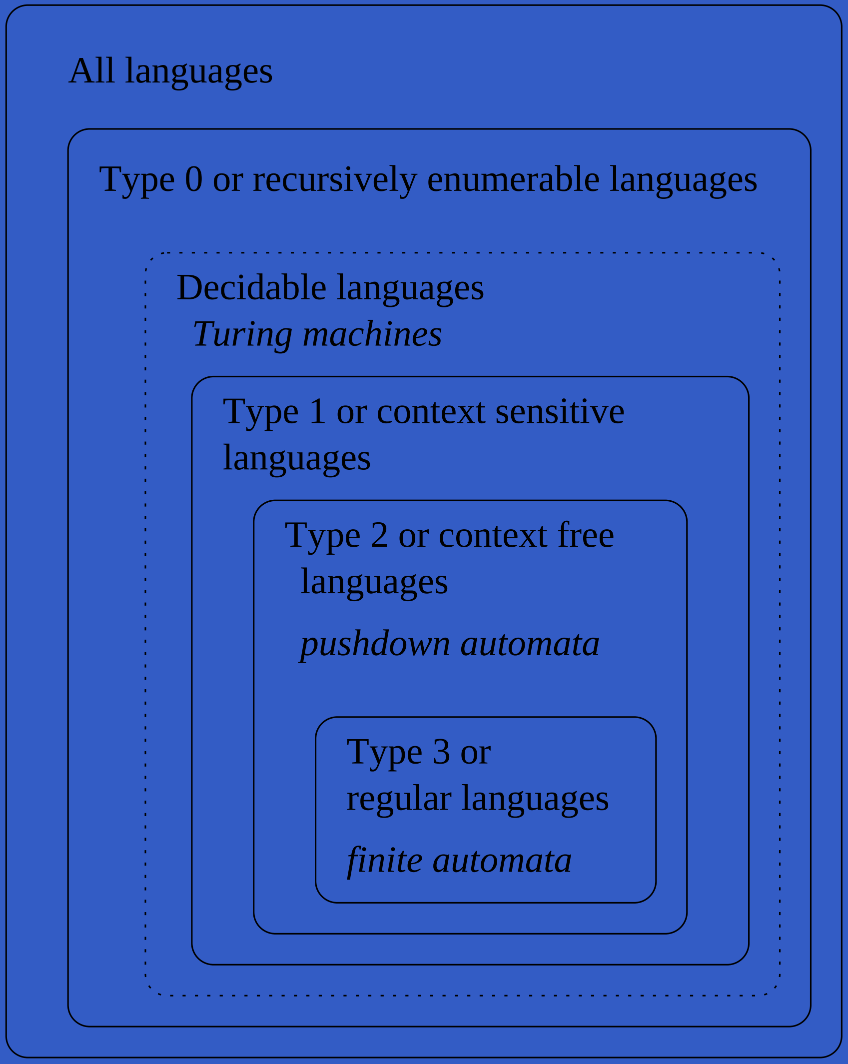
- American linguist who introduced ***Context Free Grammars*** in an attempt to describe natural languages formally.
- Also introduced the ***Chomsky Hierarchy*** which classifies grammars and languages and their descriptive power.
- Chomsky is also widely known for his controversial political views and his criticism of the foreign policy of U.S. governments.

Noam Chomsky (2)



The Chomsky Hierarchy



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ϵ denotes the ***empty word***, the sequence of zero symbols.

Symbols and Alphabets

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A common (and important) instance is $\Sigma = \{0, 1\}$.

ϵ, the empty word, is ***never*** an symbol of an alphabet.

Alphabet, Word, and Language

alphabet	$\Sigma = \{a, b\}$
words over Σ	$\epsilon, a, b, aa, ab, ba, bb,$
	$aaa, aab, aba, abb, baa, bab, \ldots$
languages	$\emptyset, \{\epsilon\}, \{a\}, \{b\}, \{a, aa\},$
	$\{\epsilon, a, aa, aaa\},$
	$\{a^n \mid n \geq 0\},$
	$\{a^n b^n \mid n \geq 0, n \text{ even}\}$

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Note the distinction between ϵ, $\emptyset$, and $\{\epsilon\}$!

All Words over an Alphabet (1)

Given an alphabet Σ we define the set Σ^* as set of words (or sequences) over Σ:

- The empty word $\epsilon \in \Sigma^*$.
- given a symbol $x \in \Sigma$ and a word $w \in \Sigma^*$, $xw \in \Sigma^*$.
- These are all elements in Σ^*.

This is called an ***inductive definition***.

All Words over an Alphabet (2)

Example: Given $\Sigma = \{0, 1\}$, some elements of Σ^* are

- ϵ (the empty word)
- 0, 1
- 00, 10, 01, 11
- 000, 100, 010, 110, 010, 110, 011, 111
- $\ldots$

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Note: although there are infinitely many words in Σ^*, each word has a ***finite*** length!

Concatenation of Words (1)

An important operation on Σ^* is ***concatenation***:

> given $w, v \in \Sigma^*$, their concatenation $wv \in \Sigma^*$.

For example, concatenation of ab and ba yields $abba$.

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This operation can be defined by primitive recursion:

$$\begin{aligned} \epsilon v &= v \\ (xw)v &= x(wv) \end{aligned}$$

Concatenation of Words (2)

Concatenation is associative and has unit ϵ:

$$u(vw) = (uv)w$$

$$\epsilon u = u = u\epsilon$$

where u, v, w are words.

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- $L \in \mathcal{P}(\Sigma^*)$.

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- The set of words that contain the same number of 0s and 1s is a language over $\Sigma = \{0, 1\}$.

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- The set of correct Java programs. This is a language over the set of UNICODE characters.

Examples of Languages (3)

- The set of programs that, if executed successfully on a Windows machine, prints the text “Hello World!” in a window. This is a language over $\Sigma = \{0, 1\}$.

Concatenation of Languages (1)

Concatenation of words is extended to languages by:

$$MN = \{uv \,|\, u \in M \wedge v \in N\}$$

Example:

$$\begin{aligned} M &= \{\epsilon, a, aa\} \\ N &= \{b, c\} \\ MN &= \{uv \,|\, u \in \{\epsilon, a, aa\} \wedge v \in \{b, c\}\} \\ &= \{\epsilon b, \epsilon c, ab, ac, aab, aac\} \\ &= \{b, c, ab, ac, aab, aac\} \end{aligned}$$

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$$L\{\epsilon\} = L = \{\epsilon\}L$$

Concatenation of Languages (3)

- Concatenation distributes through set union:

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But note e.g. $L(M \cap N) \neq LM \cap LN$!
For example, with $L = \{\epsilon, a\}$, $M = \{\epsilon\}$, $N = \{a\}$, we have

$$
\begin{aligned}
L(M \cap N) = L\emptyset = \emptyset \\
LM \cap LN = \{\epsilon, a\} \cap \{a, aa\} = \{a\}
\end{aligned}
$$