School of Computer Science, University of Nottingham

COMP2012/G52LAC Languages and Computation, Spring 2019

Dr. Venanzio Capretta

Solutions to Coursework Problems, Set 3

11 April 2019

1. This is the definition of the transition function for the Turing Machine:

$$
\begin{array}{lll}
\delta(q_0,a) = (q_1,X,R), & \delta(q_0,b) = (q_5,b,R), & \\
\delta(q_1,a) = (q_1,a,R), & \delta(q_1,b) = (q_2,Y,R), & \delta(q_1,Z) = (q_4,Z,L), \\
\delta(q_2,b) = (q_2,b,R), & \delta(q_2,Z) = (q_2,X,R), & \delta(q_2,c) = (q_3,Z,L), \\
\delta(q_3,b) = (q_3,b,L), & \delta(q_3,Z) = (q_3,Z,L), & \delta(q_3,Y) = (q_1,Y,R), \\
\delta(q_4,a) = (q_4,a,L), & \delta(q_4,Y) = (q_4,b,L), & \delta(q_4,X) = (q_0,X,R), \\
\delta(q_5,b) = (q_5,b,R), & \delta(q_5,Z) = (q_5,Z,R), & \delta(q_5,\text{␣}) = (q_6,\text{␣},L), \\
\delta(q_5,x) = \texttt{stop} \quad \text{in all other cases} & &
\end{array}
$$

2. This is the full computation trace of the Turing Machine on the word $aabcc$:

$$
\begin{array}{rl}
(\epsilon,q_0,aabcc) & \vdash (X,q_1,abcc) \vdash (Xa,q_1,bcc) \vdash (XaY,q_2,cc) \vdash (Xa,q_3,YZc) \\
& \vdash (XaY,q_1,Zc) \vdash (Xa,q_4,YZc) \vdash (X,q_4,abZc) \vdash (\epsilon,q_4,XabZc) \\
& \vdash (X,q_0,abZc) \vdash (XX,q_1,bZc) \vdash (XXY,q_2,Zc) \vdash (XXYZ,q_2,c) \\
& \vdash (XXY,q_3,ZZ) \vdash (XX,q_3,YZZ) \vdash (XXY,q_1,ZZ) \vdash (XX,q_4,YZZ) \\
& \vdash (X,q_4,XbZZ) \vdash (XX,q_0,bZZ) \vdash (XXb,q_5,ZZ) \vdash (XXbZ,q_5,Z) \\
& \vdash (XXbZZ,q_5,\epsilon) \vdash (XXbZZ,q_6,Z)
\end{array}
$$

   The final state q_6 is accepting, so the word $aabcc$ is accepted.

   For the other words we have:

   - $(\epsilon,q_0,aabbcc) \vdash^* (XXYbZZ,q_2,\epsilon)$, the word is rejected
   - $(\epsilon,q_0,aaabbcccccc) \vdash^* (XXXbbZZZZZ,q_6,Z)$, the word is accepted
   - $(\epsilon,q_0,abc) \vdash^* (Xb,q_6,Z)$, the word is accepted
   - $(\epsilon,q_0,ababc) \vdash^* (XY,q_2,abc)$, the word is rejected

3. $$L_M = b^+ \cup \{a^i b^j (cb^*)^{i\cdot j} \mid i>0, j>0\}$$

4. $$
\begin{array}{l}
\texttt{fib}_{\texttt{step}} = \lambda p.\langle \mathsf{snd}\,p, \texttt{plus}\,(\mathsf{fst}\,p)\,(\mathsf{snd}\,p)\rangle \\
\texttt{fib}_{\texttt{aux}} = \lambda n.n\;\texttt{fib}_{\texttt{step}}\;\langle \overline{0}, \overline{1}\rangle \\
\texttt{fib} = \lambda n.\mathsf{fst}\,(\texttt{fib}_{\texttt{aux}}\,n)
\end{array}
$$