The University of Nottingham

SCHOOL OF COMPUTER SCIENCE

A LEVEL 2 MODULE, SPRING SEMESTER 2017–2018

LANGUAGES AND COMPUTATION

Time allowed TWO hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced.

Answer ALL THREE questions

No calculators are permitted in this examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject-specific translation directories are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn your examination paper over until instructed to do so

Question 1

The following questions are multiple choice. There is at least one correct answer, but there may be several. To get all the marks you have to list all correct answers and none of the incorrect ones. 1 mistake results in 3 marks, 2 mistakes result in 1 mark, 3 or more mistakes result in zero marks.

(a) Which of the following statements are correct?

(i) An alphabet is a finite sequence of distinct symbols.

(ii) A language is the set of all possible words over a given alphabet.

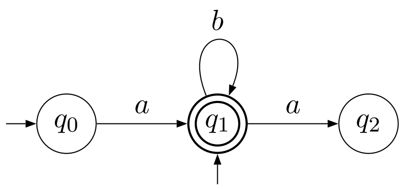
(iii) A language is always an infinite set of words.

(iv) A regular language is always infinite.

(v) An infinite language can be regular.

(5)

(b) Consider the following nondeterministic finite automaton (NFA) A over $\Sigma = \{a, b\}$:



Which of the following statements about A and the equivalent deterministic finite automaton (DFA) obtained through the subset construction are correct? Consider the entire DFA, even if some states are not reachable.

(i) $\{q_0, q_1\}$ is the initial state of the equivalent DFA.

(ii) Each of $\{q_1\}$, $\{q_2\}$, $\{q_1, q_2\}$ is an accepting state in the equivalent DFA

(iii) The transition function of the equivalent DFA has a transition from the state $\{q_1, q_2\}$ on the symbol b to the state $\{q_1\}$.

(iv) The transition function of the equivalent DFA has a transition from the state $\{q_2\}$ on the symbol a to the state $\emptyset$.

(v) The state $\emptyset$ is a *dead state* of the resulting DFA: no accepting state can be reached from it.

(5)

(c) Consider the following Context-Free Grammar (CFG) G:

$$\begin{array}{lcl} S & \to & XX \mid Y \\ X & \to & aXc \mid aYc \\ Y & \to & Yb \mid \epsilon \end{array}$$

where S, X, Y are nonterminal symbols, S is the start symbol, and a, b, c are terminal symbols.

Which of the following statements about G are correct?

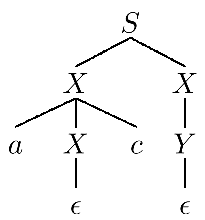
(i) $S \Rightarrow XX \Rightarrow aYcX \Rightarrow acX \Rightarrow acaYc \Rightarrow acac$ is a *left-most* derivation in the grammar G.

(ii) $S \Rightarrow XX \Rightarrow XaYc \Rightarrow Xac \Rightarrow aYcac \Rightarrow acac$ is a *right-most* derivation in the grammar G.

(iii) G is ambiguous.

(iv) $aXcbbb$ is a sentential form for grammar G.

(v) The following is a derivation tree in the grammar G:



(5)

(d) Which of the following statements about Complexity Theory is certainly true?

(i) The Satisfiability Problem SAT is NP-complete.

(ii) The Halting Problem is reducible to SAT.

(iii) Every NP-complete problem is reducible to SAT in polynomial time.

(iv) SAT is reducible to every NP-complete problem in polynomial time.

(v) Every NP-complete problem is solvable in polynomial time by a deterministic Turing machine.

(5)

(e) Which of the following statements about the λ-calculus are true?

(i) The Church numeral $\overline{3}$ is $\lambda f.\lambda x.((f\,x)\,x)\,x$.

(ii) The pair of two terms $\langle a, b\rangle$ is represented by $\lambda x.x\,a\,b$.

(iii) Every Turing-computable function can be represented by a λ-term.

(iv) Some λ-terms do not have a normal form.

(v) The problem of determining whether a λ-term has a normal form is decidable.

(5)

Question 2
Consider the following Context-Free Grammar (CFG):

$$\begin{aligned} S &\rightarrow AS \mid AB \mid AA \\ A &\rightarrow aA \mid \epsilon \\ B &\rightarrow BCDb \mid e \\ C &\rightarrow Dc \mid c \\ D &\rightarrow Cd \mid d \end{aligned}$$

S, A, B, C, and D are nonterminals, a, b, c, d, and e are terminals, and S is the start symbol. The set of nullable nonterminals is $N_\epsilon = \{S, A\}$.

(a) Systematically compute the *first sets* for all nonterminals, i.e., $\mathrm{first}(S)$, $\mathrm{first}(A)$, $\mathrm{first}(B)$, $\mathrm{first}(C)$, and $\mathrm{first}(D)$, by setting up and solving the equations according to the definitions of first sets for nonterminals and strings of grammar symbols, looking for the *smallest* solutions. Recall that an equation of the form $X = X \cup Y$, in the absence of other constraints on X, simplifies to $X = Y$ when we are looking for the smallest solution. Show your calculations.

To get you started, the equation for $\mathrm{first}(A)$, before simplification, resulting from the productions for A ($A \rightarrow aA \mid \epsilon$), is:

$$\mathrm{first}(A) = \mathrm{first}(aA) \cup \mathrm{first}(\epsilon)$$

(8)

(b) Set up the subset constraint system that defines the *follow sets* for all nonterminals; i.e., $\mathrm{follow}(S)$, $\mathrm{follow}(A)$, $\mathrm{follow}(B)$, $\mathrm{follow}(C)$, and $\mathrm{follow}(D)$. Simplify where possible using the law

$$X \subseteq Z \wedge Y \subseteq Z \iff X \cup Y \subseteq Z$$

and by removing trivially satisfied constraints such as $\emptyset \subseteq X$ and $X \subseteq X$.

To get you started, the constraints on $\mathrm{follow}(S)$, before simplification, resulting from S being the start symbol and the one production where S occurs in the RHS ($S \rightarrow AS$), are:

$$\begin{aligned} \{\$\} &\subseteq \mathrm{follow}(S) \\ \mathrm{first}(\epsilon) &\subseteq \mathrm{follow}(S) \\ \mathrm{follow}(S) &\subseteq \mathrm{follow}(S) \end{aligned}$$

(12)

(c) Solve the subset constraint system for the follow sets from the previous question by finding the *smallest* sets satisfying the constraints. (5)

Question 3

(a) Write the λ-terms that represent the following values:

- The Church numerals $\overline{0}$ and $\overline{2}$;
- The exponential function exp such that $(\text{exp}\ \overline{n}\ \overline{m}) = \overline{n^m}$;
- The Boolean values true and false;
- The *'not and'* Boolean operator nand, such that

$$\begin{array}{ll} \text{nand true true} \rightsquigarrow^* \text{false}, & \text{nand false true} \rightsquigarrow^* \text{true}, \\ \text{nand true false} \rightsquigarrow^* \text{true}, & \text{nand false false} \rightsquigarrow^* \text{true}. \end{array}$$

(8)

(b) For every pairs of natural numbers $n > 0$ and i such that $1 \leq i \leq n$, consider the following λ-term:

$$\pi_i^n = \lambda x_1.\lambda x_2.\ldots\lambda x_n.x_i$$

where $x_1, \ldots, x_n$ are distinct variables. For example

$$\pi_1^1 = \lambda x_1.x_1, \qquad \pi_1^3 = \lambda x_1.\lambda x_2.\lambda x_3.x_1, \qquad \pi_3^4 = \lambda x_1.\lambda x_2.\lambda x_3.\lambda x_4.x_3.$$

If a, b and c are any terms, what values do the following terms reduce to?

$$\begin{array}{ll} \pi_1^2\, a\, b \rightsquigarrow^* ? & \pi_2^3\, a\, b\, c \rightsquigarrow^* ? \\ \pi_1^3\, a\, b\, c \rightsquigarrow^* ? & \pi_3^3\, a\, b\, c \rightsquigarrow^* ? \end{array}$$

Now consider the following two functions shift and slide:

$$\text{shift} = \lambda f.\text{true}\, f, \qquad \text{slide} = \lambda f.\lambda u.\lambda v.f\, u.$$

What values do the following terms reduce to (show the steps in the reductions)?

$$\text{shift}\, \pi_1^2 \rightsquigarrow^* ? \qquad \text{slide}\, \pi_1^2 \rightsquigarrow^* ?$$

In general, when we apply shift and slide to a term of the form π_i^m, we obtain a term in the same form. Determine what the indexes of the result are:

$$\text{shift}\, \pi_i^m \rightsquigarrow^* \pi_?^? \qquad \text{slide}\, \pi_i^m \rightsquigarrow^* \pi_?^?$$

[Hint: Remember that the names of the variables are arbitrary; you can rename them freely as long as you keep them distinct.] (10)

(c) In the context of complexity theory, give a definition of the class of NP-complete problems.

Name two examples of NP-complete problems, with brief informal definitions.

Explain what the open question P=NP means and why it is important in computer science. (7)