G52MAL
Machines and Their Languages
Lecture 1
Administrative Details and Introduction

Henrik Nilsson

University of Nottingham

Finding People and Information

- Henrik Nilsson
  Room A08
- Moodle
- Main module web page:
  `www.cs.nott.ac.uk/~nhn/G52MAL`

Aims of the Course

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- To familiarize you with key Computer Science ***concepts*** in central areas like
  - Automata Theory
  - Formal Languages
  - Models of Computation
  - Complexity Theory

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  - Complexity Theory
- To equip you with ***tools*** with wide applicability in the fields of CS and IT.

Organization (1)

- ***Lectures:***
  - Two 1 h lectures per week.
  - Detailed but somewhat tentative schedule available on the module web page.

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  - Detailed but somewhat tentative schedule available on the module web page.
- ***Coursework:***
  - 4 Bi-weekly problem sets.
  - Made available via the module web page.
  - Best 3 counts.
  - Deadlines: 5/2, 19/2, 4/3, 16/3 (Wed.!).

Organization (2)

- ***Assessment:***
  - Coursework, 25 %
  - 2 hour written examination, 75 %

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  - 2 hour written examination, 75 %
- However, ***resits*** are by 100 % written examination (standard School policy)

Literature (1)

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- Dr. Thorsten Altenkirch's and my G52MAL lecture notes.
  (Available via the G52MAL module page.)

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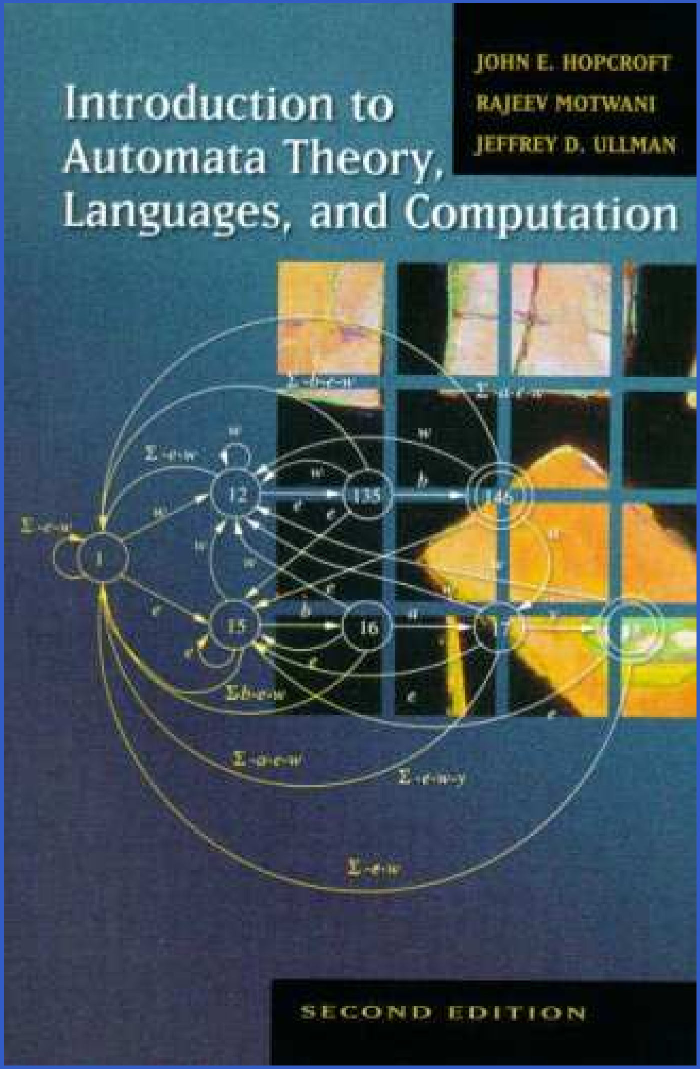
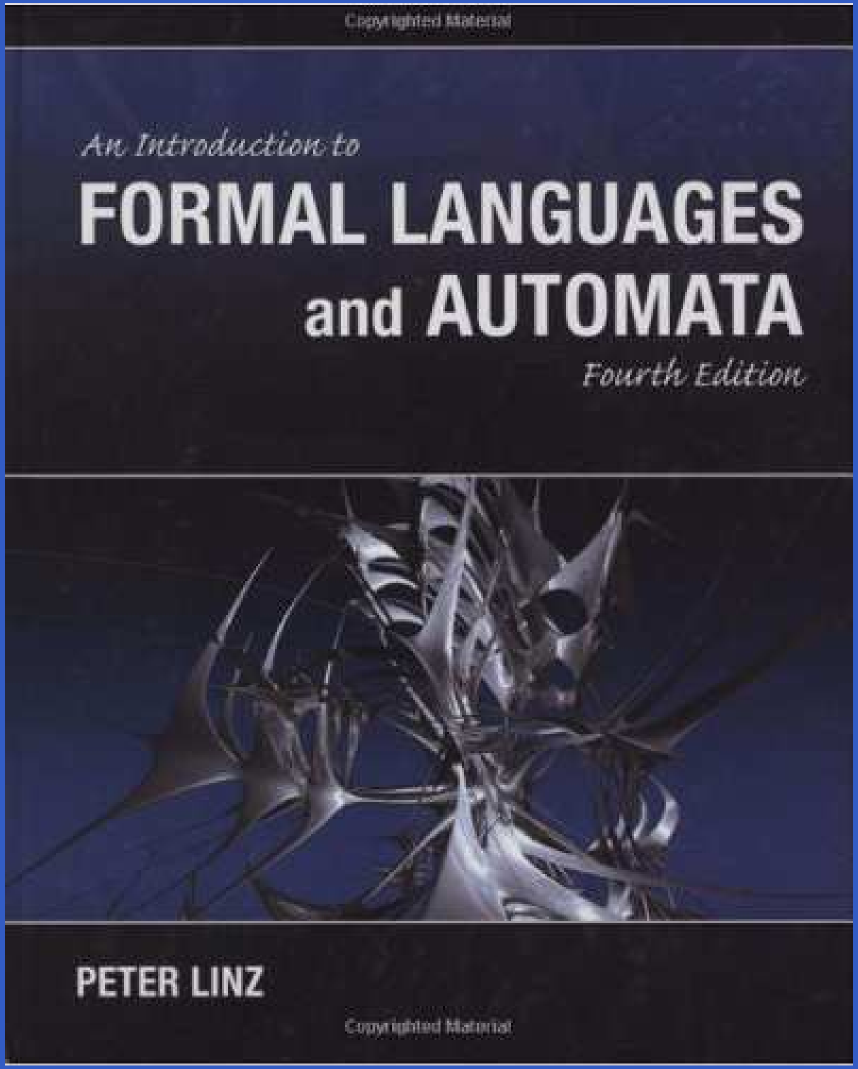
- Your own notes from the lectures!

Literature (3)





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3. The relation between 1 and 2.
4. Applications: Scanning and Parsing

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- Finding words and patterns in large bodies of text, e.g. in web pages.
- Verification of systems with finite number of states, e.g. communication protocols.

Why Study All This? (2)

As a concrete example, a job opening from some time ago:

> The Strats team at Standard Chartered is hiring a developer for a 1 year contracting role in London.
> The role is to develop and extend our parsing and validation library for FpML, using the FpML Haskell library to parse and build financial product data into our internal Haskell data types.

https://donsbot.wordpress.com/2015/01/28/

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- What can a computer do ***at all***?
  Decidability
- What can a computer do ***efficiently***?
  Time and space ***Complexity***

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- If you succeed, your salary will be doubled. But if you fail, you'd have to look for a new job.
- Should you accept?

Example: The Halting Problem (1)

Consider the following program. Does it terminate for all values of $n \geq 1$?

```
while (n > 1) {
    if even(n) {
        n = n / 2;
    } else {
        n = n * 3 + 1;
    }
}
```

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. . . but so far, ***no proof***! (See e.g. Wikipedia.)

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> ***It is impossible to write a program that decides if another, arbitrary, program terminates (halts) or not.***

What might be surprising is that it ***is*** possible to ***prove*** such a result. This was first done by the British mathematician ***Alan Turing*** using Turing Machines.

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- Instrumental in the success of British code breaking efforts during WWII.

Alan Turing (2)

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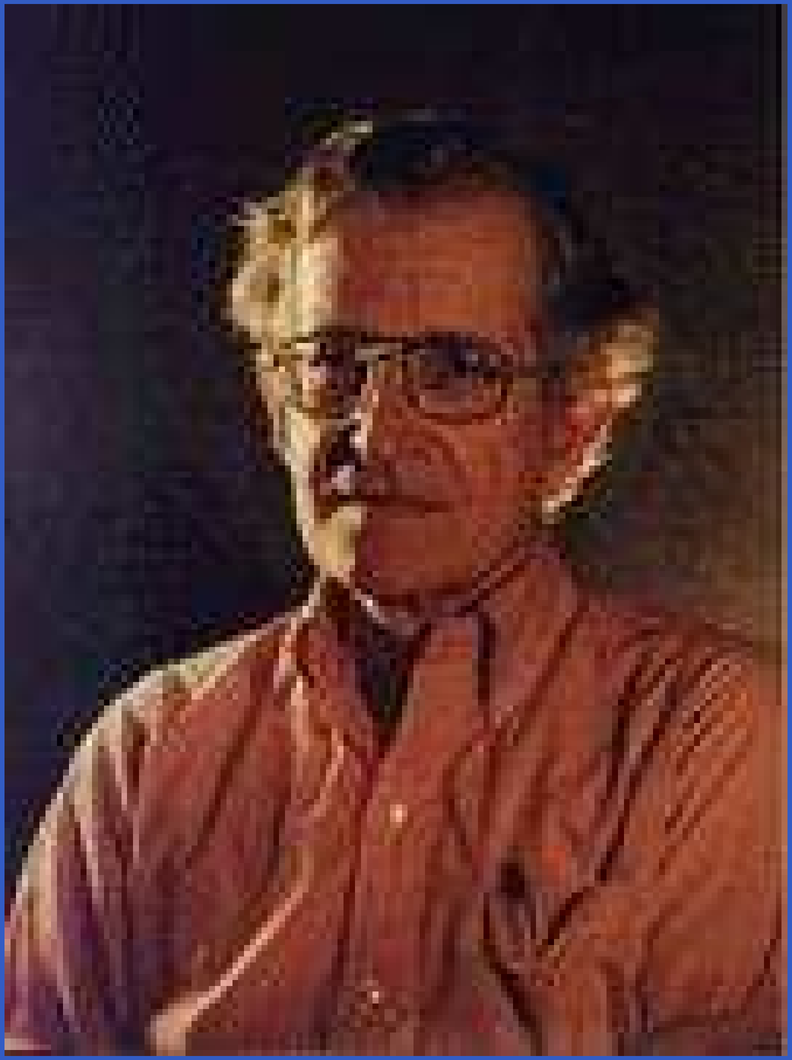
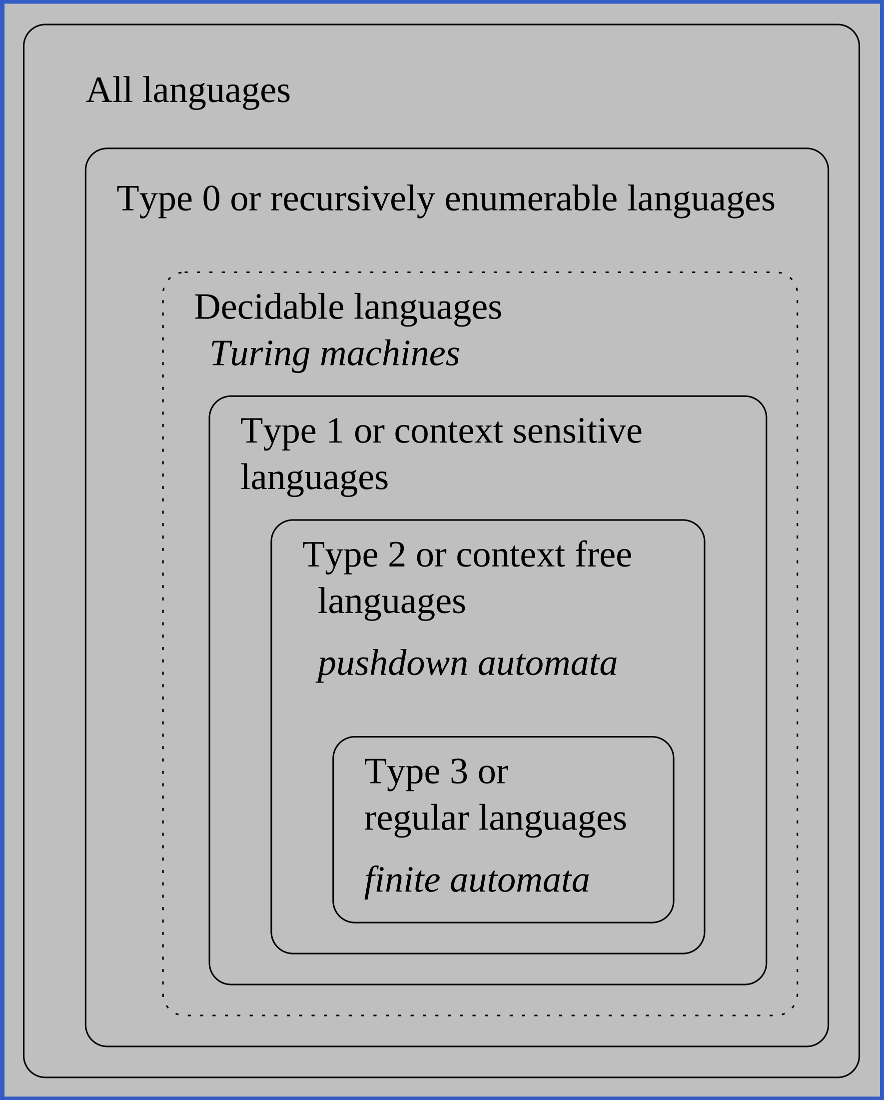
- American linguist who introduced ***Context Free Grammars*** in an attempt to describe natural languages formally.
- Also introduced the ***Chomsky Hierarchy*** which classifies grammars and languages and their descriptive power.

Noam Chomsky (2)



The Chomsky Hierarchy



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The term ***string*** is often used interchangeably with the term ***word***.

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ϵ, the empty word, is ***never*** a symbol of an alphabet.

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alphabet	$\Sigma = \{a, b\}$
words	?

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Note the distinction between ϵ, $\emptyset$, and $\{\epsilon\}$!

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- List some words over your alphabet?
- What might an interesting language over your alphabet be? Does your language include ***all*** possible words over your alphabet?

All Words Over an Alphabet (1)

Given an alphabet Σ we define the set Σ^* as set of words (or sequences) over Σ:

- The empty word $\epsilon \in \Sigma^*$.
- given a symbol $x \in \Sigma$ and a word $w \in \Sigma^*$, $xw \in \Sigma^*$.
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This is called an ***inductive definition***.

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Is Σ^* always infinite? Always non-empty?

All Words over an Alphabet (2)

Example: Given $\Sigma = \{0, 1\}$, some elements of Σ^* are

- ϵ (the empty word)
- $0, 1$
- $00, 10, 01, 11$
- $000, 100, 010, 110, 001, 101, 011, 111$
- $\ldots$

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We are just applying the inductive definition.

Note: although there are infinitely many words in Σ^* (when $\Sigma \neq \emptyset$), each word has a ***finite*** length!

Concatenation of Words (1)

An important operation on Σ^* is ***concatenation***:

> given $w, v \in \Sigma^*$, their concatenation $wv \in \Sigma^*$.

For example, concatenation of ab and ba yields $abba$.

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This operation can be defined by primitive recursion:

$$\begin{aligned} \epsilon v &= v \\ (xw)v &= x(wv) \end{aligned}$$

Concatenation of Words (2)

Concatenation is associative and has unit ϵ:

$$u(vw) = (uv)w$$
$$\epsilon u = u = u\epsilon$$

where u, v, w are words.

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- $L \subseteq \Sigma^*$, or equivalently
- $L \in \mathcal{P}(\Sigma^*)$.

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  This is an example of a ***finite*** language.
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- The set of words that contain the same number of 0s and 1s is a language over $\Sigma = \{0, 1\}$. (Finite or infinite?)

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- The set of correct Java programs. This is a language over the set of UNICODE characters.
- The set of programs that, if executed successfully on a Windows machine, prints the text “Hello World!” in a window. This is a language over $\Sigma = \{0, 1\}$.

Concatenation of Languages (1)

Concatenation of words is extended to languages by:

$$MN = \{uv \mid u \in M \wedge v \in N\}$$

Example:

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Concatenation of Languages (2)

- Concatenation of languages is associative:

$$L(MN) = (LM)N$$

- Concatenation of languages has zero $\emptyset$:

$$L\emptyset = \emptyset = \emptyset L$$

- Concatenation of languages has unit $\{\epsilon\}$:

$$L\{\epsilon\} = L = \{\epsilon\}L$$

Concatenation of Languages (3)

- Concatenation distributes through set union:

$$
\begin{aligned}
L(M \cup N) &= LM \cup LN \\
(L \cup M)N &= LN \cup MN
\end{aligned}
$$

Concatenation of Languages (3)

- Concatenation distributes through set union:

$$\begin{aligned} L(M \cup N) &= LM \cup LN \\ (L \cup M)N &= LN \cup MN \end{aligned}$$

But not through intersection! $L(M \cap N) \neq LM \cap LN$

Counterexample: $L = \{\epsilon, a\}$, $M = \{\epsilon\}$, $N = \{a\}$:

$$L(M \cap N) = L\emptyset = \emptyset$$
$$LM \cap LN = \{\epsilon, a\} \cap \{a, aa\} = \{a\}$$

Concatenation of Languages (4)

- Exponent notation is used to denote iterated concatenation:
  - $L^1 = L$
  - $L^2 = LL$
  - $L^3 = LLL$
  - $\ldots$
- By definition: $L^0 = \{\epsilon\}$ (for ***any*** language, incl. $\emptyset$)
- $$L^* = \bigcup_{n=0}^{\infty} L^n$$

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We need:

- A ***finite*** (and preferably concise) formal ***description*** of L.

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Various approaches to achieve this will be key a theme throughout the module.