G52MAL
Machines and Their Languages
Lecture 9
Introduction to Context-free Grammars

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Non-regular Languages (1)

We have established that the following language is not regular:

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Others? What about B: the language of "balanced parentheses"? E.g.

$$\begin{aligned} ()() &\in B \\ ((()())()) &\in B \\ )(&\notin B \\ (() &\notin B \end{aligned}$$

Non-regular Languages (2)

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Use Pumping Lemma for regular languages to prove formally. ***Exercise!***

Non-regular Languages (3)

But of course, “balanced parentheses” is a key feature of many important classes of languages; e.g.:

- Arithmetic expressions: `(`, `)`
- Matching keywords in programming languages: `begin`, `end`, `repeat`, `until`
- Markup languages; e.g. HTML: `<p>`, `</p>`, `<a href="...">`, `</a>`

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Q: Can such languages be described formally? How?

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A: Through ***Context-free Grammars*** (CFG).

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Let us consider the language ***Grammatically Correct Sentences of Extremely Simplified English*** (GCSESE)

GCSESE (1)

Nonterminals		Terminals
S:	Sentence	boy
NP:	Noun Phrase	girl
VP:	Verb Phrase	little
N:	Noun	big
V:	Verb	walks
		runs
		slowly
		fast

GCSESE (2)

Productions for GCSESE:

$$
\begin{array}{rcl}
S & \rightarrow & NP\ VP \\
NP & \rightarrow & Adj\ NP \\
NP & \rightarrow & N \\
N & \rightarrow & \text{boy} \\
N & \rightarrow & \text{girl} \\
Adj & \rightarrow & \text{little} \\
Adj & \rightarrow & \text{big}
\end{array}
\qquad
\begin{array}{rcl}
VP & \rightarrow & V\ Adv \\
VP & \rightarrow & V \\
V & \rightarrow & \text{walks} \\
V & \rightarrow & \text{runs} \\
Adv & \rightarrow & \text{slowly} \\
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\end{array}
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$$

Note: The terminals constitute the ***alphabet*** of the language being defined.