

G52MAL

Machines and Their Languages

Lecture 13

Pushdown Automata (PDA)

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 - Through **automata**: a mechanical procedure for deciding whether or not a word belongs to the language.
 - Through **regular expressions**: a direct description of the words in a language.
- We have learned that not all languages are regular. In particular, the class of **context-free languages** includes the regular languages but is strictly larger.



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QUESTION: Can the context-free languages also be described through some form of automaton?

In other words: is there a mechanical procedure for determining language membership for the context-free languages, and if so, what is the “simplest” such machine?

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 - Suffices for describing the context-free languages (CFL).
 - Is not enough to describe languages more general than the CFLs (as we will see).