COMP3012/G53CMP: Lecture 3

Syntactic Analysis: Bottom-Up Parsing

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This Lecture

- Parsing strategies: top-down and bottom-up.
- Shift-Reduce parsing theory.
- LR(0) parsing.
- LR(0), LR(k), and LALR(k) grammars

Parsing Strategies

There are two basic strategies for parsing:

- ***Top-down parsing***:
  - Attempts to construct the parse tree ***from the root downward***.
  - Traces out a ***leftmost derivation***.
  - E.g. Recursive-Descent Parsing (see G52LAC).
- ***Bottom-up parsing***:
  - Attempts to construct the parse tree ***from the leaves working up toward the root***.
  - Traces out a ***rightmost derivation in reverse***.

Top-Down: Leftmost Derivation

Consider the grammar:

$$S \to aABe \qquad A \to bcA \mid c \qquad B \to d$$

Call sequence for predictive parser on $abccde$:

`parseS`	S	$\underset{lm}{\Rightarrow}$	$aABe$
read `a`			
`parseA`		$\underset{lm}{\Rightarrow}$	$abcABe$
read `b`			
read `c`			
`parseA`		$\underset{lm}{\Rightarrow}$	$abccBe$
read `c`			
`parseB`		$\underset{lm}{\Rightarrow}$	$abccde$
read `d`			
read `e`			

Shift-Reduce Parsing

Shift-reduce parsing is a general style of bottom-up syntax analysis:

- Works from the leaves toward the root of the parse tree.
- Has two basic actions:
  - ***Shift*** (read) next terminal symbol.
  - ***Reduce*** a sequence of read terminals and previously reduced nonterminals corresponding to the RHS of a production to LHS nonterminal of that production.

Bottom-Up: Rightmost Der. in Reverse

Consider (again) the grammar:

$$S \to aABe \qquad A \to bcA \mid c \qquad B \to d$$

Reduction steps for the sentence $abccde$ to S

$abccde$	(reduce by $A \to c$)
$abcAde$	(reduce by $A \to bcA$)
$aAde$	(reduce by $B \to d$)
$aABe$	(reduce by $S \to aABe$)
S	

Trace out rightmost derivation in reverse:

$$S \underset{rm}{\Rightarrow} aABe \underset{rm}{\Rightarrow} aAde \underset{rm}{\Rightarrow} abcAde \underset{rm}{\Rightarrow} abccde$$

How can we know when and what to reduce???

Shift-Reduce Parsing: Idea

How can we know when and what to reduce???

Idea:

- Construct a DFA where each state is labelled by "all possibilities" given the input and reductions thus far. (Similar to how an NFA is turned into a DFA.)
- Whenever reduction is possible, if there is only one possible reduction, then it is always clear what to do.

Will make this more precise in the following.

LL, LR, and LALR parsing (1)

Three important classes of parsing methods:

- ***LL(k)***:
  - input scanned **L**eft to right
  - **L**eftmost derivation
  - k symbols of lookahead
- ***LR(k)***:
  - input scanned **L**eft to right
  - **R**ightmost derivation in reverse
  - k symbols of lookahead
- ***LALR(k)***: **L**ook**A**head ***LR***, simplified LR parsing

LL, LR, and LALR parsing (2)

By extension, the classes of grammars these methods can handle are also classified as LL(k), LR(k), and LALR(k).

Why study LR and LALR parsing?

- These methods handle a wide class of grammars of practical significance.
- In particular, handles left- and right-recursive grammars (but left rec. needs less stack).
- LALR is a good compromise between expressiveness and space cost of implementation.
- Consequently, many parser generator tools based on LALR.
- We will mainly study LR(0) parsing because it is the simplest, yet uses the same fundamental principles as LR(1) and LALR(1).

Shift-Reduce Parsing Theory (1)

Some terminology:

- An ***item*** for a CFG is a production with a dot anywhere in the RHS.

For example, the items for the grammar

$$S \to aAc \qquad A \to Ab \mid \epsilon$$

are

$$\begin{array}{lllllll} S & \to & \cdot\, aAc & \quad & A & \to & \cdot\, Ab \\ S & \to & a \cdot Ac & & A & \to & A \cdot b \\ S & \to & aA \cdot c & & A & \to & Ab\, \cdot \\ S & \to & aAc\, \cdot & & A & \to & \cdot \end{array}$$

Shift-Reduce Parsing Theory (2)

- Recap: Given a CFG $G = (N, T, P, S)$, a string $\phi \in (N \cup T)^*$ is a ***sentential form*** for G iff $S \overset{*}{\underset{G}{\Rightarrow}} \phi$.
- A ***right-sentential form*** is a sentential form that can be derived by a rightmost derivation.
- A ***handle*** of a right-sentential form ϕ is a substring α of ϕ such that $S \overset{*}{\underset{rm}{\Rightarrow}} \delta A w \underset{rm}{\Rightarrow} \delta \alpha w$ and $\delta \alpha w = \phi$, where $\alpha, \delta, \phi \in (N \cup T)^*$, and $w \in T^*$.

Shift-Reduce Parsing Theory (3)

For example, consider the grammar:

$$S \to aABe \qquad A \to bcA \mid c \qquad B \to d$$

The following is a rightmost derivation:

$$S \underset{rm}{\Rightarrow} aABe \underset{rm}{\Rightarrow} aAde \underset{rm}{\Rightarrow} abcAde$$

$aABe$, $aAde$ and $abcAde$ are right-sentential forms. Handle for each? $aABe$, d, and bcA

For an unambiguous grammar, the rightmost derivation is unique. Thus we can talk about ***"the handle"*** rather than merely "a handle".

Shift-Reduce Parsing Theory (4)

- A ***viable prefix*** of a right-sentential form ϕ is any prefix γ of ϕ ending no farther right than the right end of the handle of ϕ.
- An item $A \to \alpha \cdot \beta$ is ***valid*** for a viable prefix γ if there is a rightmost derivation

  $$S \overset{*}{\underset{rm}{\Rightarrow}} \delta A w \underset{rm}{\Rightarrow} \delta \alpha \beta w$$

  and $\delta\alpha = \gamma$.
- An item is ***complete*** if the the dot is the rightmost symbol in the item.

Shift-Reduce Parsing Theory (5)

Consider the grammar

$$S \to aABe \qquad A \to bcA \mid c \qquad B \to d$$

and the rightmost derivation

$$S \underset{rm}{\Rightarrow} aABe \underset{rm}{\Rightarrow} aAde \underset{rm}{\Rightarrow} abcAde$$

The right-sentential form $abcAde$ has handle bcA.

Viable prefixes? ϵ, a, ab, abc, $abcA$.

Shift-Reduce Parsing Theory (6)

Last derivation step $aAde \underset{rm}{\Rightarrow} abcAde$ by production $A \to bcA$, meaning the handle is bcA.

Valid item for each non-ϵ viable prefix of $abcAde$ considering this particular derivation only?

Viable prefix	Valid item
a	$A \to \cdot\, bcA$
ab	$A \to b \cdot cA$
abc	$A \to bc \cdot A$
$abcA$	$A \to bcA\, \cdot$

Any ***complete*** valid item?

Shift-Reduce Parsing Theory (7)

Knowing the valid item***s*** for a viable prefix allows a rightmost derivation in reverse to be found:

- If $A \to \alpha\, \cdot$ is a ***complete*** valid item for a viable prefix $\gamma = \delta\alpha$ of a right-sentential form γw ($w \in T^*$), then it ***appears*** that $A \to \alpha$ can be used at the last step, and that the previous right-sentential form is $\delta A w$.
- If this indeed ***always is the case*** for a CFG G, then for any $x \in L(G)$, since x ***is*** a right-sentential from, previous right-sentential forms can be determined until S is reached, giving a right-most derivation of x.

Shift-Reduce Parsing Theory (8)

Of course, if $A \to \alpha\, \cdot$ is a complete valid item for a viable prefix $\gamma = \delta\alpha$, in general, we only know it ***may be possible*** to use $A \to \alpha$ to derive γw from $\delta A w$. For example:

- $A \to \alpha\, \cdot$ may be valid because of a ***different*** rightmost derivation $S \overset{*}{\underset{rm}{\Rightarrow}} \delta A w' \underset{rm}{\Rightarrow} \phi w'$.
- There could be ***two or more complete items*** valid for γ.
- There could be a handle of γw that ***includes symbols of*** w.

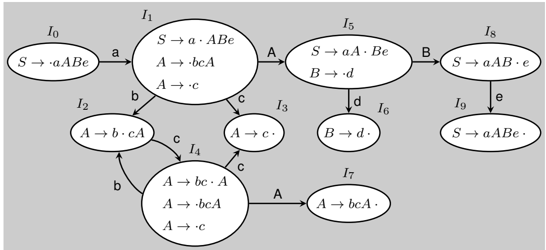
LR(0) Parsing (1)

- A CFG for which knowing a complete valid item is enough to determine the previous right-sentential form is called ***LR(0)*** grammar.
- The set of viable prefixes for ***any*** CFG is ***regular***! (Somewhat unexpected: the language of a CFG is obviously not regular in general.)
- Thus, an efficient parser can be developed for an LR(0) CFG based on a DFA for recognising viable prefixes and their valid items.
- The states of the DFA are ***sets*** of items valid for a recognised viable prefix.

LR(0) Parsing (2)

A DFA recognising viable prefixes for the CFG

$$S \to aABe \qquad A \to bcA \mid c \qquad B \to d$$



LR(0) Parsing (3)

Drawing conventions for "LR DFAs":

- For the purpose of recognizing the set of viable prefixes, all drawn states are considered accepting.
- Error transitions and error states are not drawn.

LR(0) Parsing (4)

How to construct such a DFA is beyond the scope of this course. See e.g. Aho, Sethi, Ullman (1986) for details. However, some observations:

- Recall that the viable prefixes for the right-sentential form $abcAde$ are ϵ, a, ab, abc, $abcA$. They are indeed all recognised by the DFA (all states are considered accepting).
- Recall that the item $A \to bc \cdot A$ is valid for the viable prefix abc. The corresponding DFA state indeed contains that item. (Along with ***more items*** in this case!)

LR(0) Parsing (5)

- Recall that item $A \to bcA \cdot$ is a ***complete*** valid item for the viable prefix $abcA$. The corresponding DFA state indeed contains that item (and ***only*** that item).

LR(0) Parsing (6)

Given a DFA recognising viable prefixes, an LR(0) parser can be constructed as follows:

- In a state ***without complete items***: ***Shift***
  - Read next terminal symbol and push it onto an internal parse stack.
  - Move to new state by following the edge labelled by the read terminal.

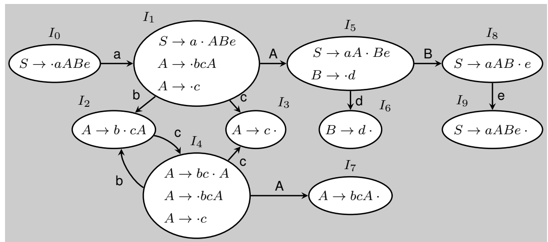
LR(0) Parsing (7)

- In a state with a ***single complete item***: ***Reduce***
  - The top of the parse stack contains the ***handle*** of the current right-sentential form (since we have recognised a viable prefix for which a single ***complete*** item is valid).
  - The handle is just the ***RHS*** of the valid item.
  - Reduce to the previous right-sentential form by ***replacing the handle*** on the parse stack with the ***LHS*** of the valid item.
  - ***Move*** to the state indicated by the new viable prefix on the parse stack.

LR(0) Parsing (8)

- If a state contains both complete and incomplete items, or if a state contains more than one complete item, then the grammar ***is not LR(0)***.

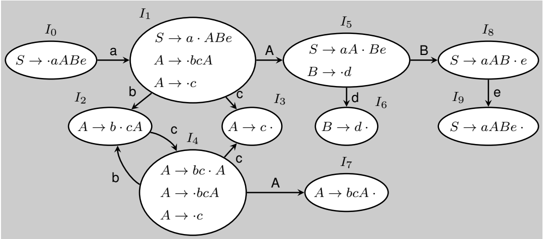
LR(0) Parsing (9)



Note: γw is the current right-sentential form.

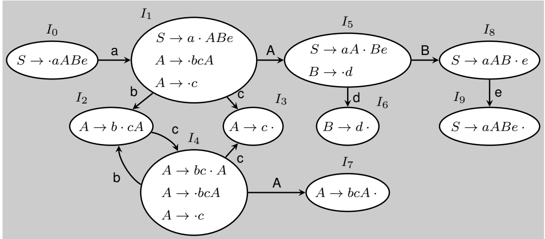
State	Stack (γ)	Input (w)	Move
I_0	ϵ	$abccde$	Shift
I_1	a	$bccde$	Shift

LR(0) Parsing (10)



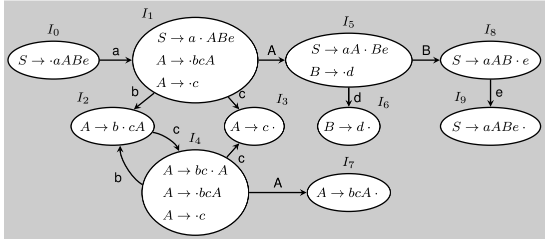
State	Stack (γ)	Input (w)	Move
I_2	ab	$ccde$	Shift
I_4	abc	cde	Shift
I_3	$abcc$	de	Reduce by $A \to c$

LR(0) Parsing (11)



State	Stack (γ)	Input (w)	Move
I_7	$abcA$	de	Reduce by $A \to bcA$
I_5	aA	de	Shift
I_6	aAd	e	Reduce by $B \to d$

LR(0) Parsing (12)



State	Stack (γ)	Input (w)	Move
I_8	aAB	e	Shift
I_9	$aABe$	ϵ	Reduce by $S \to aABe$
	S	ϵ	Done

LR(0) Parsing (13)

Complete sequence (γw is right-sentential form):

State	Stack (γ)	Input (w)	Move
I_0	ϵ	$abccde$	Shift
I_1	a	$bccde$	Shift
I_2	ab	$ccde$	Shift
I_4	abc	cde	Shift
I_3	$abcc$	de	Reduce by $A \to c$
I_7	$abcA$	de	Reduce by $A \to bcA$
I_5	aA	de	Shift
I_6	aAd	e	Reduce by $B \to d$
I_8	aAB	e	Shift
I_9	$aABe$	ϵ	Reduce by $S \to aABe$
	S	ϵ	Done

Cf: $S \underset{rm}{\Rightarrow} aABe \underset{rm}{\Rightarrow} aAde \underset{rm}{\Rightarrow} abcAde \underset{rm}{\Rightarrow} abccde$

LR(0) Parsing (14)

Even more clear that the parser carries out the rightmost derivation in reverse if we look at the right-sentential forms γw of the reduction steps only:

$$\begin{aligned} abccde &\underset{rm}{\Leftarrow} \\ abcAde &\underset{rm}{\Leftarrow} \\ aAde &\underset{rm}{\Leftarrow} \\ aABe &\underset{rm}{\Leftarrow} \\ S & \end{aligned}$$

LR Parsing & Left/Right Recursion (1)

Remark: Note how the ***right-recursive*** production

$$A \to bcA$$

causes symbols bc to pile up on the parse stack until a reduction by

$$A \to c$$

can occur, in turn allowing the stacked symbols to be reduced away.

LR Parsing & Left/Right Recursion (2)

Even clearer if considering parsing of a string like

$$abcbcbccde \quad \text{or} \quad abcbcbcbcbccde$$

Exercise: Try parsing these!

Left-recursion allows reduction to happen sooner, thus keeping the size of the parse stack down. This is why left-recursive grammars often are preferred for LR parsing.

LR(1) Grammars (1)

- In practice, LR(0) tends to be a bit too restrictive.
- If we add one symbol of “lookahead” by determining the set of ***terminals that possibly could follow a handle*** being reduced by a production $A \to \beta$, then a wider class of grammars can be handled.
- Such grammars are called ***LR(1)***.

LR(1) Grammars (2)

Idea:

- Associate a ***lookahead set*** with items:

$$A \to \alpha \cdot \beta, \{a_1, a_2, \ldots, a_n\}$$

- On reduction, a complete item is ***only valid*** if the next input symbol belongs to its lookahead set.
- Thus it is OK to have two or more simultaneously valid complete items, as long as their lookahead sets are ***disjoint***.

(Similar to ***predictive*** recursive-descent parsing.)