

# COMP3012/G53CMP: Lecture 8

## *Contextual Analysis: Types and Type Systems I*

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# This Lecture

- Types and Type Systems
- Language safety
- Achieving safety through types
  - Introduction: relation between static and dynamic semantics
  - Operational dynamic semantics of a small example language.

Much of this lecture follows parts of the first few chapters of B. C. Pierce 2002 *Types and Programming Languages* closely.

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A plausible definition (Pierce):

*A type system is a tractable syntactic method for proving the absence of certain program behaviors by classifying phrases according to the kinds of values they compute.*

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- **Static checking** implied as the goal is to **prove** absence of certain errors.
- Done by **classifying** syntactic phrases (or **terms**) according to the **kinds** of value they compute: a type system computes a **static approximation** of the run-time behaviour.

# Types and Type Systems (3)

Example: if known that two program fragments  $exp_1$  and  $exp_2$  compute integers (**classification**), then it is safe to add those numbers together (**absence of errors**):

$$exp_1 + exp_2$$

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Example: if known that two program fragments  $exp_1$  and  $exp_2$  compute integers (**classification**), then it is safe to add those numbers together (**absence of errors**):

$$exp_1 + exp_2$$

Also known that the result is an integer. While not known exactly which integers are involved, at least known they are integers and nothing else (**static approximation**).

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Example. In a dynamically checked language,  $exp_1 + exp_2$  would be evaluated as follows:

- Evaluate  $exp_1$  and  $exp_2$
- Add results together in a manner depending on their types (integer addition, floating point addition, . . . ), or signal error if not possible.

# Types and Type Systems (5)

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For example, typically

*if complex test then S else type error*

will be rejected as ill-typed, even if *complex test* actually always evaluates to true because that cannot be automatically proved in general.

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For example: current main-stream type systems typically

**do check** that arithmetic operations only are done on numbers

**do not check** that the second operand of division is not zero, that array indices are within bounds.

# Types and Type Systems (7)

- The **safety** or **soundness** of a type system must be judged with respect to its own set of run-time errors.

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Other examples: lexical scope rules, visibility attributes (`public`, `protected`, ...).

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- Scheme is a dynamically checked safe language.
- Even statically typed languages usually use some dynamic checks; e.g.:
  - checking of array bounds
  - down-casting (e.g. Java)
  - checking for division by zero
  - pattern-matching failure

# Language Safety (3)

Some examples of statically and dynamically checked safe and unsafe high-level languages:

	<b>Statically chkd</b>	<b>Dynamically chkd</b>
<b>Safe</b>	ML, Haskell, Java	Lisp, Scheme, Perl, Python, Postscript
<b>Unsafe</b>	C, C++	Certain Basic dialects

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- Enforcing disciplined programming  
Type systems are the backbone of
  - Modules
  - Classes



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  - Unlike comments, type signatures will always remain current.
- Efficiency
  - First use of types in computing was to distinguish between integer and floating point numbers.
  - Elimination of many of the dynamic checks that otherwise would have been needed to guarantee safety.

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Increasingly sophisticated type systems, which keep track of more invariants, can help.

But that can make the type systems harder to understand and less automatic.

# Static and Dynamic Semantics

In summary:

- A type system **statically** proves properties about the **dynamic** behaviour of a programs.
- To make precise exactly what these properties are, and formally **prove** that a type system achieves its goals, both the
  - **static semantics**
  - **dynamic semantics**must first be formalized.

# Example Language: Abstract Syntax

Example language. (Will be extended later.)

$t \rightarrow$

*terms:*

**true**

*constant true*

| **false**

*constant false*

| **if**  $t$  **then**  $t$  **else**  $t$

*conditional*

| **0**

*constant zero*

| **succ**  $t$

*successor*

| **pred**  $t$

*predecessor*

| **iszero**  $t$

*zero test*

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- The evaluation rules are going to be such that no evaluation is possible for values.
- A term to which no evaluation rule applies is a **normal form**.
- All values are normal forms.

# Values (2)

$v \rightarrow$

	<b>true</b>	<i>true value</i>
	<b>false</b>	<i>false value</i>
	$nv$	<i>numeric value</i>

$nv \rightarrow$

	<b>0</b>	<i>zero value</i>
	<b>succ</b> $nv$	<i>successor value</i>

# One Step Evaluation Relation (1)

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 $t$  evaluates to  $t'$  in one step.

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$$\frac{t_1 \longrightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad (\text{E-IF})$$

# Evaluation Relation???

- Recall that a **mathematical relation** can be understood as a (possibly infinite) set of pairs of “related things”. For example:

$$\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3)\} \subseteq (\leq)$$



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- The idea of our “one step evaluation relation” is that the “related things” are **terms** and that one term is related to another iff the first evaluates to the second in one step.
- For example:

$$(\text{if true then succ 0 else 0}, \text{succ 0}) \in (\longrightarrow)$$

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$$\frac{Premise_1 \quad Premise_2 \quad \dots \quad Premise_n}{Conclusion}$$

([en.wikipedia.org/wiki/Rule\\_of\\_inference](http://en.wikipedia.org/wiki/Rule_of_inference))

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- If there are no premises, the line is often omitted:

$$\overline{Conclusion} \quad \text{or} \quad Conclusion$$

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- **Schematic** means that universally quantified variables are allowed in the rules. For example:

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- Such a **rule schema** actually stands for an **infinite set** of rules:

...

**if true then 0 else 0  $\longrightarrow$  0**

**if true then succ 0 else 0  $\longrightarrow$  succ 0**

**if true then true else false  $\longrightarrow$  true**

...



# Evaluation Relation???

## (4)

- The **domain** of a variable is often specified by **naming conventions**. E.g. the name of a variable may indicate some specific **syntactic category** such as  $t$ ,  $v$ , or  $nv$ :

$$\frac{t_1 \longrightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \longrightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3}$$

$$\text{pred}(\text{succ } nv_1) \longrightarrow nv_1$$

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$$\frac{t_1 \longrightarrow t'_1}{\mathbf{iszero} \ t_1 \longrightarrow \mathbf{iszero} \ t'_1} \quad (\text{E-ISZERO})$$



# One Step Evaluation Relation (4)

Let's evaluate some terms according to  $\longrightarrow$ :

- `pred (pred 0)`
- `if (iszero (pred (succ 0)))  
 then (pred 0) else (succ 0)`
- `if 0 then 0 else 0`

(On the whiteboard.)

# Values and Stuck Terms

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- **Values** cannot be evaluated further. E.g.:
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- Certain “obviously nonsensical” states are **stuck**: the term cannot be evaluated further, but it is not a value. For example:  
**if 0 then pred 0 else 0**

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These ideas will be made more precise next time.