

G54FOP: Lecture 7

The Untyped λ -Calculus I: Introduction

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- Pure notion of effective computation procedure: *all* computation reduced to function definition and application.
- Invented in the 1920s by Alonzo Church.
- Cf. other formalisations of the notion of effective computation; e.g., the Turing machine.
- The λ -calculus and Turing Machines are equivalent in that they capture the exact same notion of what “computation” means.

The λ -Calculus: What is it? (2)

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- The Church-Turing Hypothesis: The λ -calculus, Turing Machines, etc. coincides with our intuitive understanding of what “computation” means.
- The λ -calculus is important because it is at once:
 - very simple, yet in essence a practically useful programming language
 - mathematically precise, allowing for formal reasoning.

Key Idea

λ -abstraction (or anonymous function):

$$\lambda x . t$$

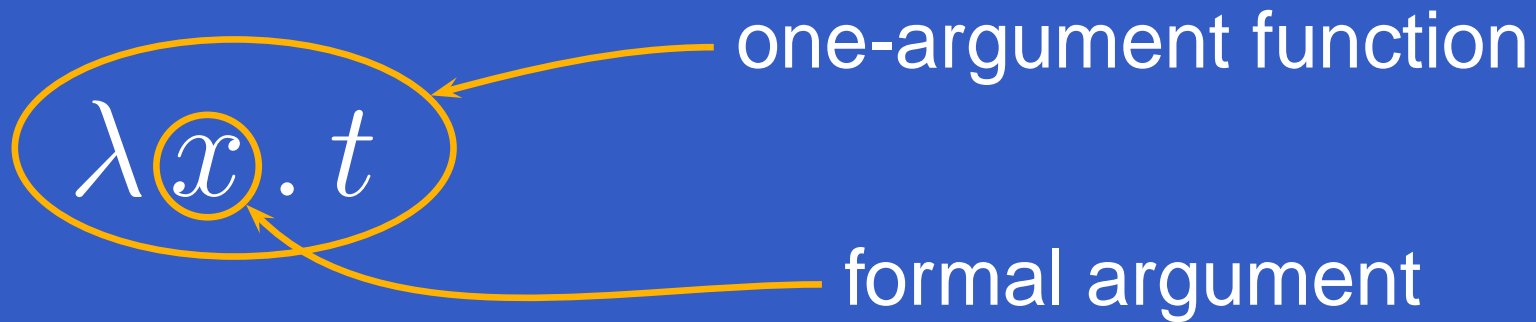
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λ -abstraction (or anonymous function):

$\lambda x . t$ ← one-argument function

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- λ -abstractions often named for convenience. E.g. $I \equiv \lambda x.x$. **Just an abbreviation!**
So e.g. $F \equiv \lambda x.(\dots F \dots)$ **not** valid def. Why?

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- A λ -term with **no free** variables is called **closed**. Otherwise **open**.
- A closed λ -term is called a **combinator**.

Exercise

In the following:

- Which variables are free and which are bound?
- Which terms are open and which are closed?

(a) x

(b) $\lambda x.x$

(c) $\lambda x.y$

(d) $\lambda x.\lambda y.x y$

(e) $(\lambda x.x) x$

(f) $\lambda x.\lambda y.(\lambda x.x y) (\lambda z.x y)$

Operational Semantics (1)

Sole means of computation: **β -reduction** or **function application**:

$$(\lambda x.t_1) t_2 \xrightarrow{\beta} [x \mapsto t_2]t_1$$

where

$$[x \mapsto t_2]t_1$$

means “term t_1 with all **free** occurrences of x (with respect to t_1) replaced by t_2 .”

Subtle problems concerning **name clashes** will be considered later.

Operational Semantics (2)

A term that can be β -reduced is called a ***(β -)redex***.

Exercise: Underline the redexes in

$(\lambda x.x) ((\lambda x.x) (\lambda z.(\lambda x.x) z))$