

G54FOP: Handout, Lecture 12

Types and Type Systems I

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Values

$v \rightarrow$	true	values:	true value
	false		false value
	nv		numeric value
$nv \rightarrow$		numeric values:	
	0		zero value
	succ nv		successor value

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Example Language: Abstract Syntax

$t \rightarrow$	true	terms:	constant true
	false		constant false
	if t then t else t		conditional
	0		constant zero
	succ t		successor
	pred t		predecessor
	iszero t		zero test

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Dynamic Semantics (1)

Small-step operational semantics:

$t \rightarrow t'$ Read: t evaluates to t' in one step

if true then t_2 **else** $t_3 \rightarrow t_2$ (E-IFTRUE)

if false then t_2 **else** $t_3 \rightarrow t_3$ (E-IFFALSE)

$$\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3}$$
 (E-IF)

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Dynamic Semantics (2)

$$\frac{t_1 \longrightarrow t'_1}{\text{succ } t_1 \longrightarrow \text{succ } t'_1} \quad (\text{E-SUCC})$$

$$\text{pred } 0 \longrightarrow 0 \quad (\text{E-PREDZERO})$$

$$\text{pred } (\text{succ } nv_1) \longrightarrow nv_1 \quad (\text{E-PREDSUCC})$$

$$\frac{t_1 \longrightarrow t'_1}{\text{pred } t_1 \longrightarrow \text{pred } t'_1} \quad (\text{E-PRED})$$

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Types

$T \rightarrow$ *types:*

- Bool** *type of booleans*
- | **Nat** *type of natural numbers*

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Dynamic Semantics (3)

$$\text{iszero } 0 \longrightarrow \text{true} \quad (\text{E-ISZEROZERO})$$

$$\text{iszero } (\text{succ } nv_1) \longrightarrow \text{false} \quad (\text{E-ISZEROSUCC})$$

$$\frac{t_1 \longrightarrow t'_1}{\text{iszero } t_1 \longrightarrow \text{iszero } t'_1} \quad (\text{E-ISZERO})$$

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Typing Rules

$$\text{true} : \text{Bool} \quad (\text{T-TRUE})$$

$$\text{false} : \text{Bool} \quad (\text{T-FALSE})$$

$$\frac{t_1 : \text{Bool} \quad t_2 : T \quad t_3 : T}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T} \quad (\text{T-IF})$$

$$0 : \text{Nat} \quad (\text{T-ZERO})$$

$$\frac{t_1 : \text{Nat}}{\text{succ } t_1 : \text{Nat}} \quad (\text{T-SUCC})$$

$$\frac{t_1 : \text{Nat}}{\text{pred } t_1 : \text{Nat}} \quad (\text{T-PRED})$$

$$\frac{t_1 : \text{Nat}}{\text{iszero } t_1 : \text{Bool}} \quad (\text{T-ISZERO})$$

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