G54FOP: Lecture 16

Denotational Semantics and Domain Theory II

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This Lecture

- Denotational semantics for small imperative language.
- Introduction to semantics of loops and recursion.

Imperative Language (1)

Syntax of expressions:

e	$\rightarrow$		*expressions:*
		x	*variable*
	$\mid$	n	*constant natural number,* $\mathbb{N}$
	$\mid$	`true`	*constant true*
	$\mid$	`false`	*constant false*
	$\mid$	`not` e	*logical negation*
	$\mid$	e `&&` e	*logical conjunction*
	$\mid$	$\ldots$	

Imperative Language (2)

e	$\rightarrow$		*expressions:*
		$\ldots$	
	$\mid$	e `+` e	*addition*
	$\mid$	e `-` e	*subtraction*
	$\mid$	e `=` e	*numeric equality test*
	$\mid$	e `<` e	*numeric less than test*

Imperative Language (3)

Syntax of commands:

$c \rightarrow$		*commands:*
$\mid$	`skip`	*no operation*
$\mid$	x `:=` e	*assignment*
$\mid$	c `;` c	*sequence*
$\mid$	`if` e `then` c `else` c	*conditional*
$\mid$	`while` e `do` c	*iteration*

Semantics of Expressions (1)

We take the ***semantic domain*** to be $\mathbb{N}$ for simplicity.

We need a way to give meaning to ***variables***. A ***store*** maps a variable name to its value:

$$\Sigma = x \rightarrow \mathbb{N}$$
$$\sigma : \Sigma$$

Semantics of Expressions (2)

We then need two ***semantic functions***, one for expressions (have no side effects in this language), one for commands.

Starting with the one for expressions:

$$\mathrm{E}[\![\cdot]\!] : e \rightarrow (\Sigma \rightarrow \mathbb{N})$$

(Note: $e \rightarrow (\Sigma \rightarrow \mathbb{N}) = e \rightarrow \Sigma \rightarrow \mathbb{N}$ etc.)

(Definition on whiteboard)

Semantics of Commands

A command is executed for its ***effects***: given a state, executing a command results in a new state. A command is a ***state transformer***.

In our case, the state comprises only the store:

$$\Sigma = x \rightarrow \mathbb{N}$$

Thus, type of state transformer: $\Sigma \rightarrow \Sigma$.

Semantic function for commands:

$$\mathrm{C}[\![\cdot]\!] : c \rightarrow (\Sigma \rightarrow \Sigma)$$ [Not correct yet!]

(Definition on whiteboard)