G54FOP: Lecture 16

Denotational Semantics and Domain Theory II

Henrik Nilsson

University of Nottingham, UK

This Lecture

- Denotational semantics for small imperative language.
- Introduction to semantics of loops and recursion.

Imperative Language (1)

Syntax of expressions:

e	$\rightarrow$		*expressions:*
		x	*variable*
	|	n	*constant natural number,* $\mathbb{N}$
	|	**true**	*constant true*
	|	**false**	*constant false*
	|	**not** e	*logical negation*
	|	e **&&** e	*logical conjunction*
	...		

Imperative Language (2)

e	$\rightarrow$		*expressions:*
	...		
	|	e **+** e	*addition*
	|	e **-** e	*subtraction*
	|	e **=** e	*numeric equality test*
	|	e **<** e	*numeric less than test*

Imperative Language (3)

Syntax of commands:

c	$\rightarrow$		*commands:*
	|	**skip**	*no operation*
	|	x **:=** e	*assignment*
	|	c **;** c	*sequence*
	|	**if** e **then** c **else** c	*conditional*
	|	**while** e **do** c	*iteration*

Semantics of Expressions (1)

We take the ***semantic domain*** to be $\mathbb{N}$ for simplicity.

We need a way to give meaning to ***variables***. A ***store*** maps a variable name to its value:

$$\Sigma = x \rightarrow \mathbb{N}$$
$$\sigma : \Sigma$$

Semantics of Expressions (2)

We then need two ***semantic functions***, one for expressions (have no side effects in this language), one for commands.

Starting with the one for expressions:

$$\mathrm{E}[\![\cdot]\!] : e \rightarrow (\Sigma \rightarrow \mathbb{N})$$

(Note: $e \rightarrow (\Sigma \rightarrow \mathbb{N}) = e \rightarrow \Sigma \rightarrow \mathbb{N}$ etc.)

(Definition on whiteboard)

Semantics of Commands

A command is executed for its ***effects***: given a state, executing a command results in a new state. A command is a ***state transformer***.

In our case, the state comprises only the store:

$$\Sigma = x \rightarrow \mathbb{N}$$

Thus, type of state transformer: $\Sigma \rightarrow \Sigma$.

Semantic function for commands:

$$\mathrm{C}[\![\cdot]\!] : c \rightarrow (\Sigma \rightarrow \Sigma) \quad \text{[Not correct yet!]}$$

(Definition on whiteboard)