Functional Reactivity:
Eschewing the Imperative

An Overview of Functional Reactive Programming in the Context of Yampa

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Reactive programming (1)

:

- Input arrives while system is running.
- Output is generated in response to input in an interleaved and fashion.

Contrast .

The notions of

- time
- time-varying values, or

are inherent and central to reactive systems.

Reactive Programming (2)

Reactive systems are

- generally concurrent
- often parallel
- often distributed

Thus, besides timeliness, difficulties related to development of concurrent, parallel, and distributed programming are also inherent.

The Synchronous Approach (1)

The “synchronous realisation” (France, 1980s):

> If we heed the observation that time-varying values are central to reactive programming and
>
> - express systems directly as of such entities
> - adopt system-wide time, abstracting away processing delays (hence)
>
> . . .

The Synchronous Approach (2)

. . . then:

- systems can be described declaratively at a very high level of abstraction
- simple, deterministic semantics, facilitates reasoning
- many problems related to imperative idioms for concurrency and synchronisation simply vanishes.

Contrast programming with values at isolated points in time in a fundamentally temporally agnostic setting.

The Synchronous Approach (3)

The synchronous languages were invented in France in the 1980s. The first ones were:

- Esterel
- Lustre
- Signal

Have been very successful; e.g. lots of industrial applications.

Many new languages and variations since then.

Functional Reactive Programming

Functional Reactive Programming (FRP):

- Paradigm for reactive, concurrent programming in purely declarative (functional) setting.
- Originated from Functional Reactive Animation (Fran) (Elliott & Hudak).
- Has evolved in a number of directions and into different concrete implementations.
- (Usually) continuous notion of time and additional support for discrete events.

FRP applications

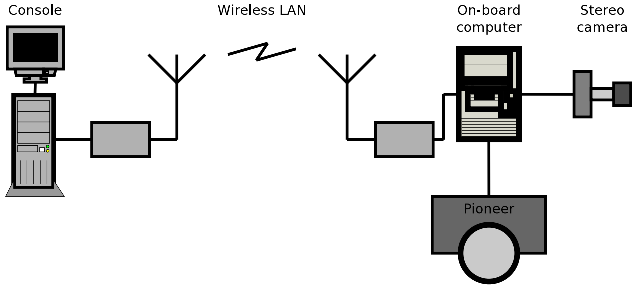
Some domains where FRP or FRP-like ideas have been used:

- Graphical Animation
- Robotics
- Vision
- GUIs
- Hybrid modeling
- Video games
- Sensor networks
- Audio processing and generation
- Financial, event-based systems

Example: Robotics (1)

[PPDP'02, with Izzet Pembeci and Greg Hager, Johns Hopkins University]

Hardware setup:



Example: Robotics (2)

Related approaches

FRP related to:

- Synchronous languages, like Esterel, Lucid Synchrone.
- Modeling languages, like Simulink, Modelica.

Distinguishing features of FRP:

- First class reactive components.
- Allows highly dynamic system structure.
- Supports hybrid (mixed continuous and discrete) systems.

Yampa

- An FRP system originating at Yale
- in Haskell (a Haskell library).
- used as the basic structuring framework.
- Notionally .
- Discrete-time signals modelled by continuous-time signals and an option type, allowing for systems.
- Advanced allows for highly dynamic system structure.

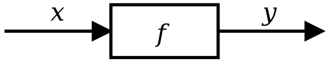
Yampa?

Yampa is a river with long calmly flowing sections and abrupt whitewater transitions in between.

A good metaphor for hybrid systems!

Signal functions

Key concept: .



Intuition:

Signal α $\approx$ Time$\rightarrow\alpha$
x :: Signal T1
y :: Signal T2
f :: Signal T1 $\rightarrow$Signal T2

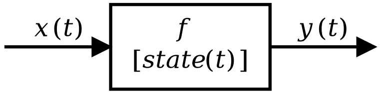
Additionally: requirement.

are ***first class entities*** in Yampa:
SF α β $\approx$ Signal α $\rightarrow$Signal β

Signal functions and state

Alternative view:

Signal functions can encapsulate ***state***.



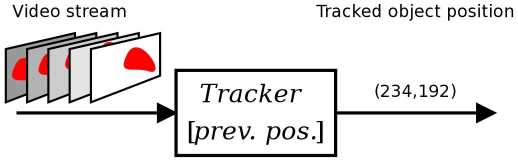
$state(t)$ summarizes input history $x(t')$, $t' \in [0, t]$.

Functions on signals are either:

- : $y(t)$ depends on $x(t)$ and $state(t)$
- : $y(t)$ depends only on $x(t)$

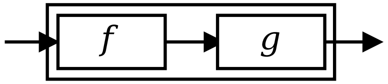
Example: Video tracker

Video trackers are typically stateful signal functions:



Building systems (1)

How to build systems? Think of a signal function as a . Blocks have inputs and outputs and can be combined into larger blocks. For example, serial composition:

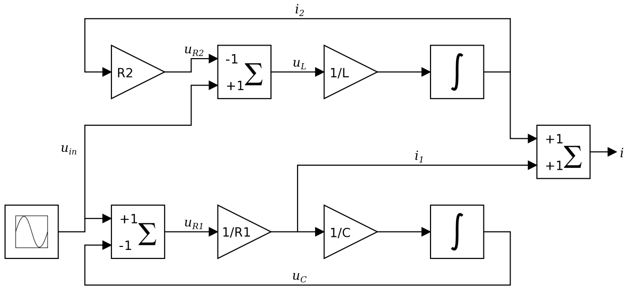


A *combinator* can be defined that captures this idea:

```
(>>>) :: SF a b -> SF b c -> SF a c
```

Building systems (2)

But systems can be complex:



Arrows

Yampa uses John Hughes’ framework:

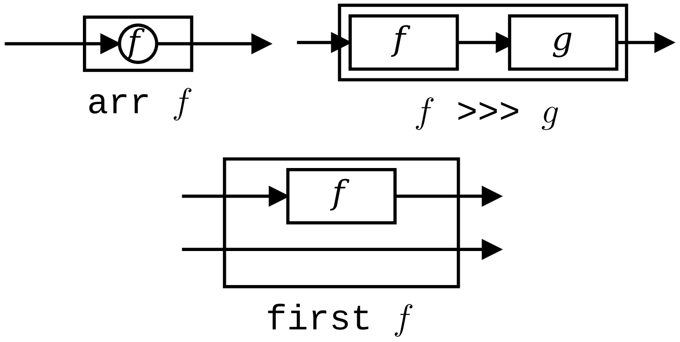
- Abstract data type interface for function-like types (or “blocks”, if you prefer).
- Particularly suitable for types representing process-like computations.
- Provides a minimal set of “wiring” combinators.

What is an arrow? (1)

- A a of arity two.
- Three operators:
  - :
    ```
    arr :: (b->c) -> a b c
    ```
  - :
    ```
    (>>>) :: a b c -> a c d -> a b d
    ```
  - :
    ```
    first :: a b c -> a (b,d) (c,d)
    ```
- A set of that must hold.

What is an arrow? (2)

These diagrams convey the general idea:



Some arrow laws

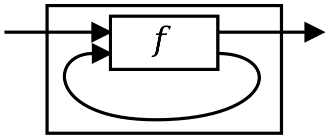
```
(f >>> g) >>> h  =  f >>> (g >>> h)
  arr (f >>> g)  =  arr f >>> arr g
   arr id >>> f  =  f
              f  =  f >>> arr id
  first (arr f)  =  arr (first f)
first (f >>> g)  =  first f >>> first g
```

Being able to use simple algebraic laws like these greatly facilitates reasoning about programs.

The `loop` combinator

Another important operator is `loop`: a fixed-point operator used to express recursive arrows or

:



Remarkably, the four combinators `arr`, `>>>`, `first`, and `loop` suffice for expressing

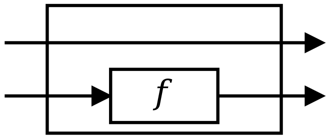
---

Some more arrow combinators (1)

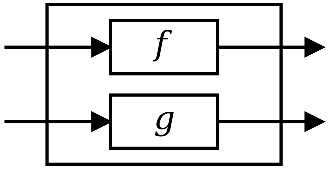
```
second :: Arrow a =>
    a b c -> a (d,b) (d,c)

(***) :: Arrow a =>
    a b c -> a d e -> a (b,d) (c,e)

(&&&) :: Arrow a =>
    a b c -> a b d -> a b (c,d)
```

Some more arrow combinators (2)

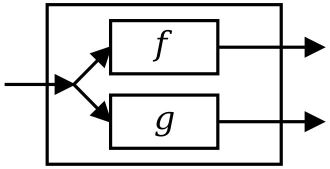
As diagrams:



`second` f



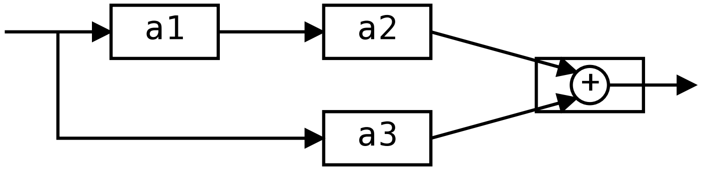
f `***` g



f `&&&` g

Example: A Simple Network

A simple network:



```
a1, a2, a3 ::  A Double Double
```

One way to express it using arrow combinators:

```
circuit_v1 ::  A Double Double
circuit_v1 = (a1 &&& arr id)
             >>> (a2 *** a3)
             >>> arr (uncurry (+))
```

The arrow do notation (1)

Using the basic combinators directly can be cumbersome. Ross Paterson's ***do***-notation for arrows provides a convenient alternative. Only

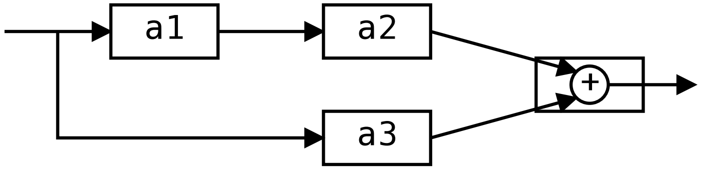
!

`proc` pat `-> do [ rec ]`
  pat_1 `<-` $sfexp_1$ `-<` exp_1
  pat_2 `<-` $sfexp_2$ `-<` exp_2
  `...`
  pat_n `<-` $sfexp_n$ `-<` exp_n
  `returnA -<` exp

Also: `let` pat `=` exp $\equiv$ pat `<- arr id -<` exp

The arrow do notation (2)

Let us redo the example using this notation:



```
circuit_v4 :: A Double Double
circuit_v4 = proc x -> do
    y1 <- a1 -< x
    y2 <- a2 -< y1
    y3 <- a3 -< x
    returnA -< y2 + y3
```

Yampa and Arrows

The Yampa signal function type is an arrow.

Signal function instances of the core combinators:

- `arr ::  (a -> b) -> SF a b`
- `>>> ::  SF a b -> SF b c -> SF a c`
- `first ::  SF a b -> SF (a,c) (b,c)`
- `loop ::  SF (a,c) (b,c) -> SF a b`

Some further basic signal functions

-
  ```
  identity :: SF a a
  identity = arr id
  ```

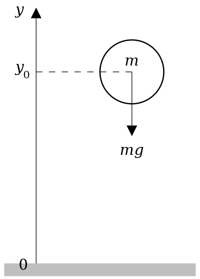
-
  ```
  constant :: b -> SF a b
  constant b = arr (const b)
  ```

-
  ```
  integral :: VectorSpace a s=>SF a a
  ```

-
  ```
  time :: SF a Time
  time = constant 1.0 >>> integral
  ```

-
  ```
  (^<<) :: (b->c) -> SF a b -> SF a c
  f (^<<) sf = sf >>> arr f
  ```

A bouncing ball



$$
\begin{aligned}
y &= y_0 + \int v \, \mathrm{d}t \\
v &= v_0 + \int -9.81
\end{aligned}
$$

On impact:

$$v = -v(t-)$$

(fully elastic collision)

Modelling the bouncing ball: part 1

Free-falling ball:

```
type Pos = Double
type Vel = Double

fallingBall ::
    Pos -> Vel -> SF () (Pos, Vel)
fallingBall y0 v0 = proc () -> do
    v <- (v0 +) ˆ<< integral -< -9.81
    y <- (y0 +) ˆ<< integral -< v
    returnA -< (y, v)
```

Events

Conceptually, signals are only defined at discrete points in time, often associated with the occurrence of some ***event***.

Yampa models discrete-time signals by lifting the of continuous-time signals:

```
data Event a = NoEvent | Event a
```

.

Associating information with an event occurrence:

```
tag :: Event a -> b -> Event b
```

Modelling the bouncing ball: part 2

Detecting when the ball goes through the floor:

```
fallingBall’ ::
    Pos -> Vel
    -> SF () ((Pos,Vel), Event (Pos,Vel))
fallingBall’ y0 v0 = proc () -> do
    yv@(y, _) <- fallingBall y0 v0 -< ()
    hit       <- edge              -< y <= 0
    returnA -< (yv, hit ‘tag‘ yv)
```

Switching

Q: How and when do signal functions “start”?

A:
- “apply” a signal functions to its input signal at some point in time.
- This creates a “running” signal function
  .
- The new signal function instance often replaces the previously running instance.

Switchers thus allow systems with to be described.

The basic switch

Idea:

- Allows one signal function to be replaced by another.
- Switching takes place on the first occurrence of the switching event source.

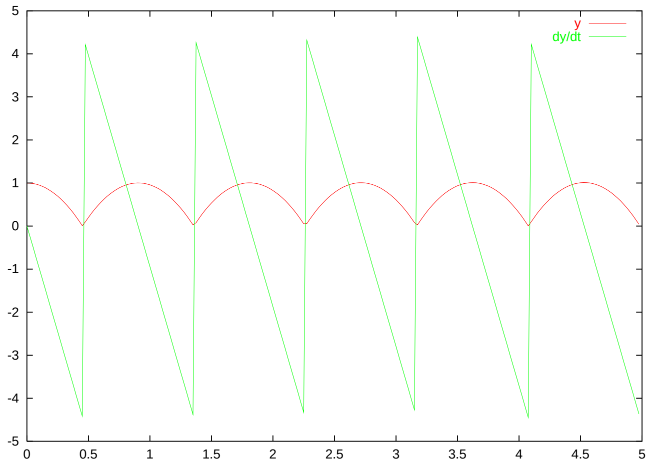
```
switch ::
   SF a (b, Event c)
   -> (c -> SF a b)
   -> SF a b
```

Modelling the bouncing ball: part 3

Making the ball bounce:

```
bouncingBall :: Pos -> SF () (Pos, Vel)
bouncingBall y0 = bbAux y0 0.0
  where
    bbAux y0 v0 =
      switch (fallingBall’ y0 v0) $ \(y,v) ->
      bbAux y (-v)
```

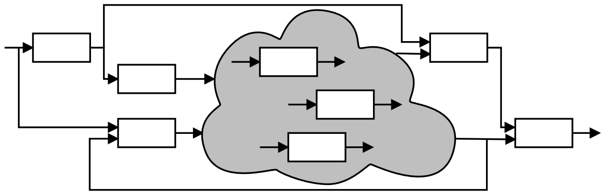
Simulation of bouncing ball



Highly dynamic system structure?

Basic switch allows one signal function to be replaced by another.

- What about more general structural changes?



- What about state?

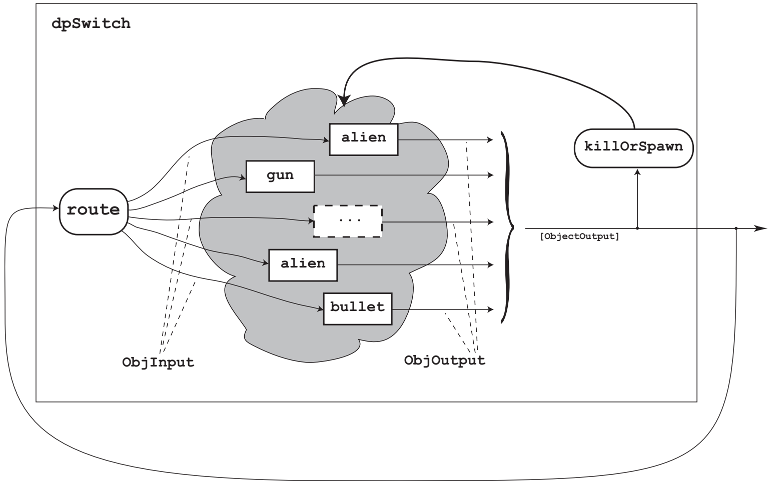
Dynamic signal function collections

Idea:

- Switch over of signal functions.
- On event, “freeze” running signal functions into collection of signal function , preserving encapsulated .
- Modify collection as needed and switch back in.

Example: Space Invaders

Overall game structure



Describing the alien behavior (1)

```
type Object = SF ObjInput ObjOutput

alien :: RandomGen g =>
  g -> Position2 -> Velocity -> Object
alien g p0 vyd = proc oi -> do
  rec

    rx   <- noiseR (xMin, xMax) g -< ()
    smpl <- occasionally g 5 ()   -< ()
    xd   <- hold (point2X p0) -< smpl ‘tag‘ rx
    ...
```

Describing the alien behavior (2)

```
    ...

    let axd = 5 * (xd - point2X p)
              - 3 * (vector2X v)
        ayd = 20 * (vyd - (vector2Y v))
        ad  = vector2 axd ayd
        h   = vector2Theta ad
    ...
```

Describing the alien behavior (3)

```
  ...

  let a = vector2Polar
              (min alienAccMax
                   (vector2Rho ad))
              h
  vp  <- iPre v0    -< v
  ffi <- forceField -< (p, vp)
  v   <- (v0 ^+^) ^<< impulseIntegral
        -< (gravity ^+^ a, ffi)
  p   <- (p0 .+^) ^<< integral -< v
  ...
```

Describing the alien behavior (4)

```
  ...

  sl  <- shield -< oiHit oi
  die <- edge   -< sl <= 0

returnA -< ObjOutput {
             ooObsObjState = oosAlien p h v,
             ooKillReq     = die,
             ooSpawnReq    = noEvent
           }
where
  v0 = zeroVector
```

State in `alien`

Each of the following signal functions used in `alien` encapsulate state:

- `noiseR`
- `occasionally`
- `hold`
- `iPre`
- `forceField`
- `impulseIntegral`
- `integral`
- `shield`
- `edge`

Why not imperative, then?

If state is so important, why not stick to imperative/object-oriented programming where we have “state for free”?

- Advantages of declarative programming retained:
  - High abstraction level.
  - Referential transparency, algebraic laws: formal reasoning is simpler.
- Synchronous approach avoids “event-call-back soup”, meaning robust, easy-to-understand semantics.