Functional Reactivity: Eschewing the Imperative

An Overview of Functional Reactive Programming in the Context of Yampa

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The notions of

- time
- time-varying values, or ***signals***

are inherent and central to reactive systems.

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Thus, besides timeliness, difficulties related to development of concurrent, parallel, and distributed programming are also inherent.

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> - express systems directly as ***transformations*** of such entities
> - adopt system-wide ***logical*** time, abstracting away processing delays (hence ***synchronous***)

. . .

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- many problems related to imperative idioms for concurrency and synchronisation simply vanishes.

Contrast programming with values at isolated points in time in a fundamentally temporally agnostic setting.

The Synchronous Approach (3)

The synchronous languages were invented in France in the 1980s. The first ones were:

- Esterel
- Lustre
- Signal

Have been very successful; e.g. lots of industrial applications.

Many new languages and variations since then.

Functional Reactive Programming

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Functional Reactive Programming (FRP):

- Paradigm for reactive, concurrent programming in purely declarative (functional) setting.
- Originated from Functional Reactive Animation (Fran) (Elliott & Hudak).
- Has evolved in a number of directions and into different concrete implementations.
- (Usually) continuous notion of time and additional support for discrete events.

FRP applications

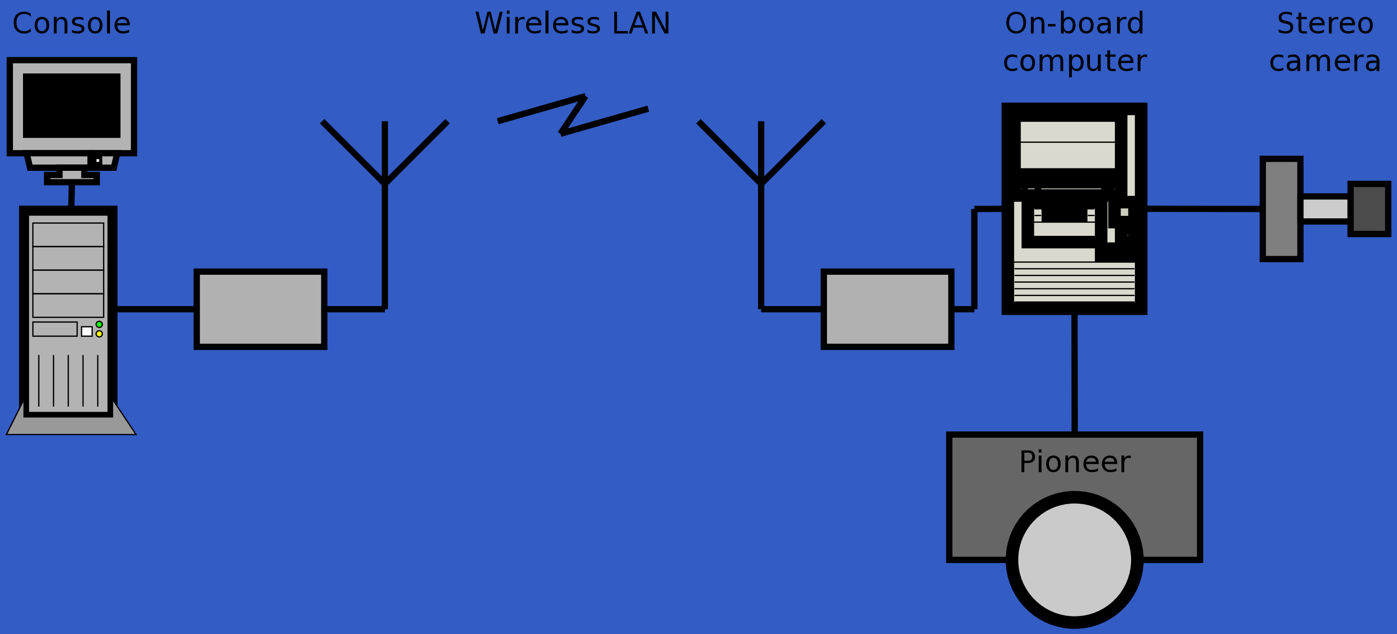
Some domains where FRP or FRP-like ideas have been used:

- Graphical Animation
- Robotics
- Vision
- GUIs
- Hybrid modeling
- Video games
- Sensor networks
- Audio processing and generation
- Financial, event-based systems

Example: Robotics (1)

[PPDP’02, with Izzet Pembeci and Greg Hager, Johns Hopkins University]

Hardware setup:



Example: Robotics (2)

Related approaches

FRP related to:

- Synchronous languages, like Esterel, Lucid Synchrone.
- Modeling languages, like Simulink, Modelica.

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Distinguishing features of FRP:

- First class reactive components.
- Allows highly dynamic system structure.
- Supports hybrid (mixed continuous and discrete) systems.

Yampa

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- ***Embedding*** in Haskell (a Haskell library).
- ***Arrows*** used as the basic structuring framework.
- Notionally ***continuous time***.
- Discrete-time signals modelled by continuous-time signals and an option type, allowing for ***hybrid*** systems.
- Advanced ***switching constructs*** allows for highly dynamic system structure.

Yampa?

Yampa?

Y et

A nother

M ostly

P ointless

A cronym

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???

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No . . .

Yampa?

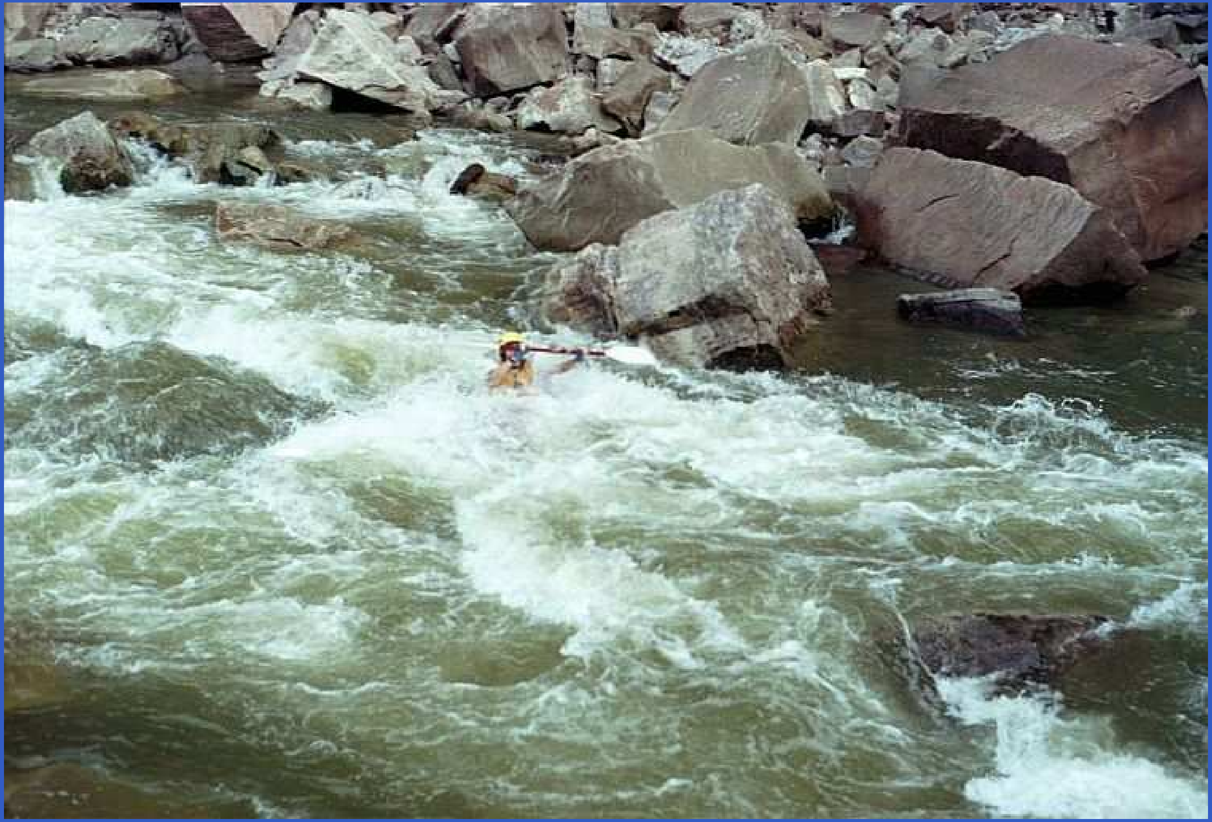
Yampa is a river . . .

Yampa?

. . . with long calmly flowing sections . . .

Yampa?

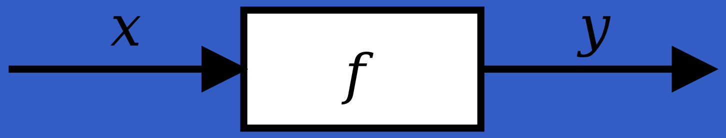
. . . and abrupt whitewater transitions in between.



A good metaphor for hybrid systems!

Signal functions

Key concept: ***functions on signals***.



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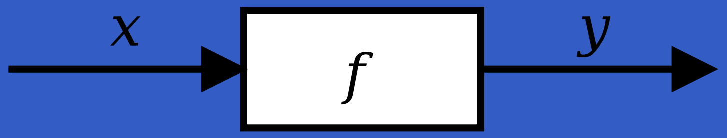

Intuition:

$$\texttt{Signal}\ \alpha \approx \texttt{Time} \rightarrow \alpha$$
$$x :: \texttt{Signal T1}$$
$$y :: \texttt{Signal T2}$$
$$f :: \texttt{Signal T1} \rightarrow \texttt{Signal T2}$$

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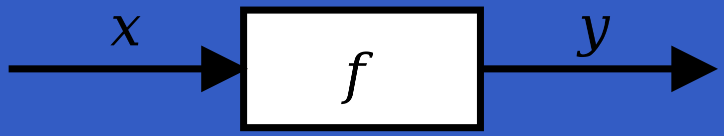

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Additionally: ***causality*** requirement.

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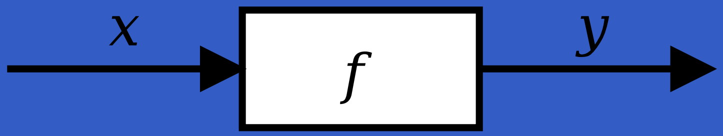

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Additionally: ***causality*** requirement.

Signal functions are ***first class entities*** in Yampa:

$\texttt{SF}\ \alpha\ \beta \approx \texttt{Signal}\ \alpha\ \rightarrow \texttt{Signal}\ \beta$

Signal functions and state

Alternative view:

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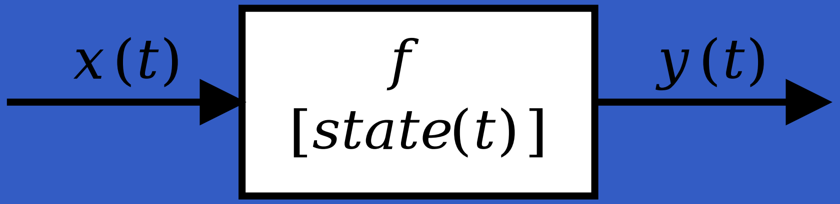
Signal functions can encapsulate ***state***.



$state(t)$ summarizes input history $x(t')$, $t' \in [0, t]$.

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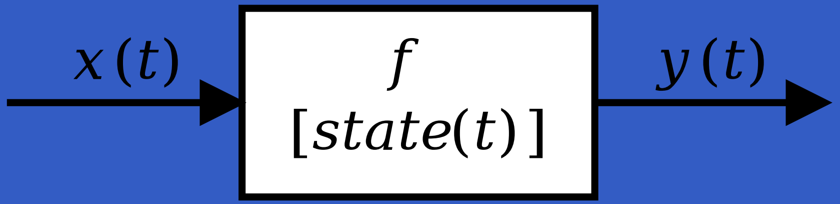

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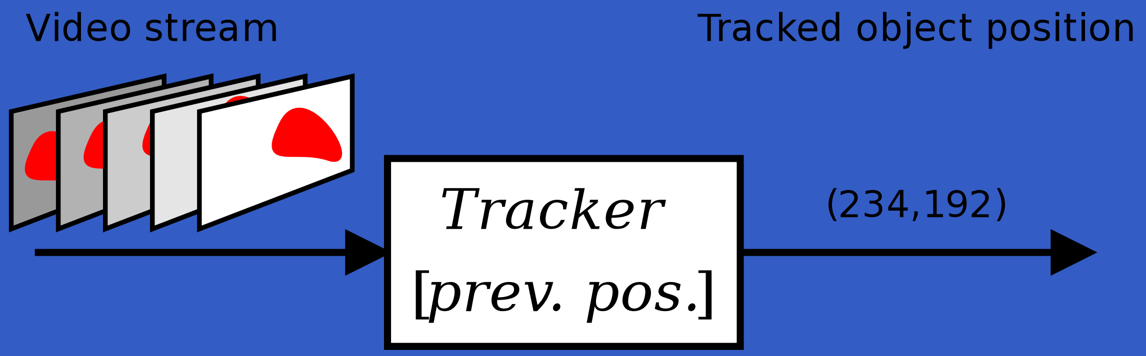
Functions on signals are either:

- ***Stateful***: $y(t)$ depends on $x(t)$ and $state(t)$
- ***Stateless***: $y(t)$ depends only on $x(t)$

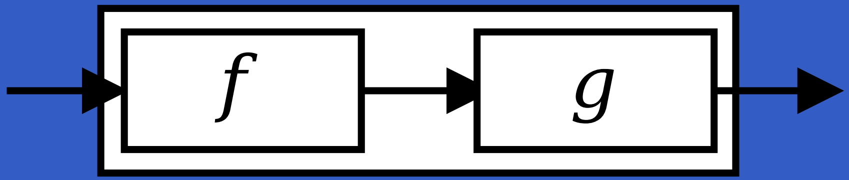
Example: Video tracker

Video trackers are typically stateful signal functions:



Building systems (1)

How to build systems? Think of a signal function as a ***block***. Blocks have inputs and outputs and can be combined into larger blocks. For example, serial composition:



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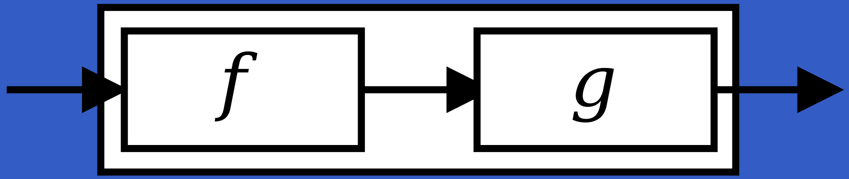
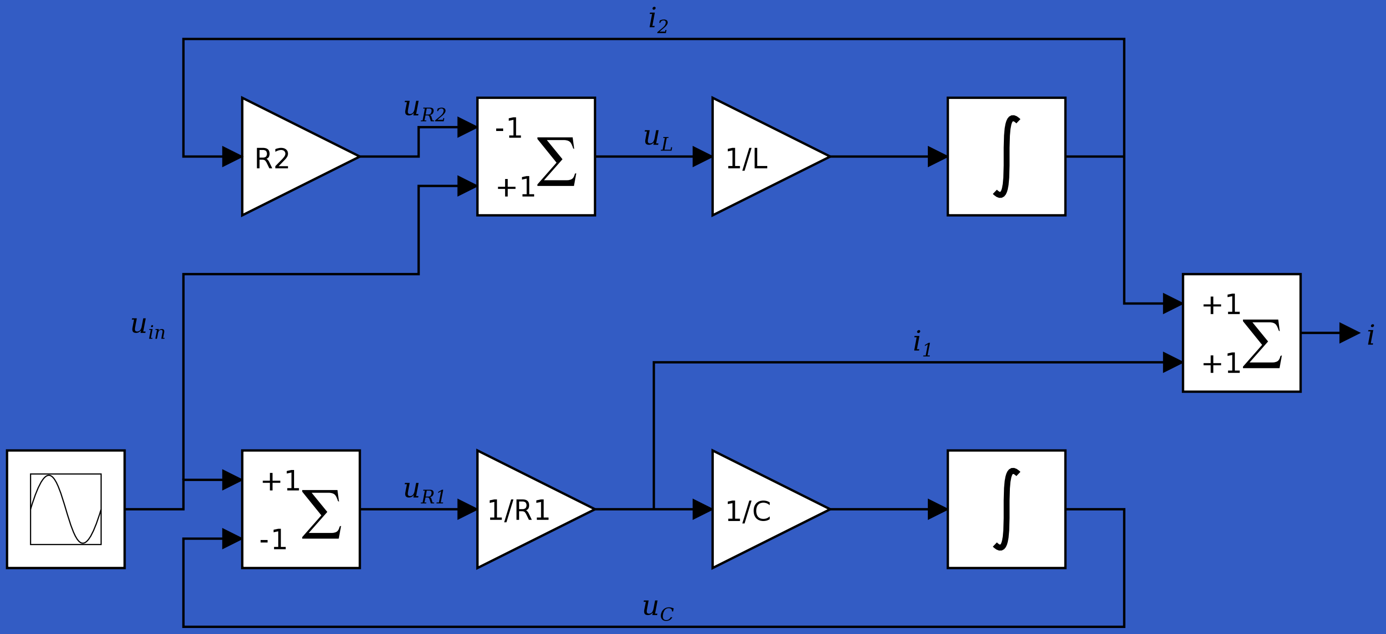


A *combinator* can be defined that captures this idea:

```
(>>>) :: SF a b -> SF b c -> SF a c
```

Building systems (2)

But systems can be complex:



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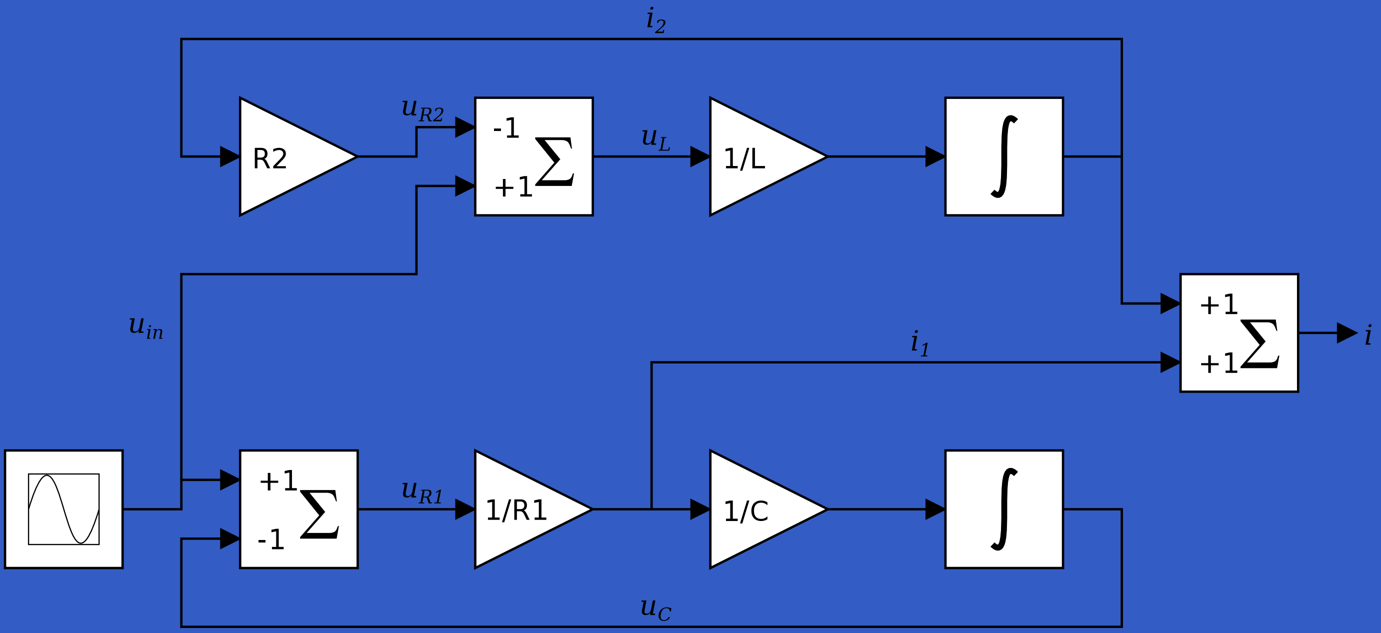

How many and what combinators do we need to be able to describe arbitrary systems?

Arrows

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- Abstract data type interface for function-like types (or “blocks”, if you prefer).
- Particularly suitable for types representing process-like computations.
- Provides a minimal set of “wiring” combinators.

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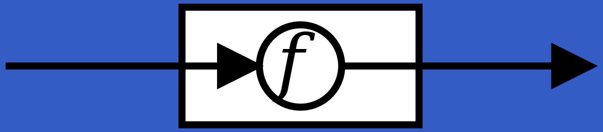
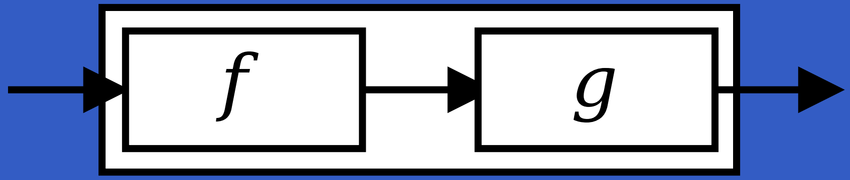
    ```
- A set of ***algebraic laws*** that must hold.

What is an arrow? (2)

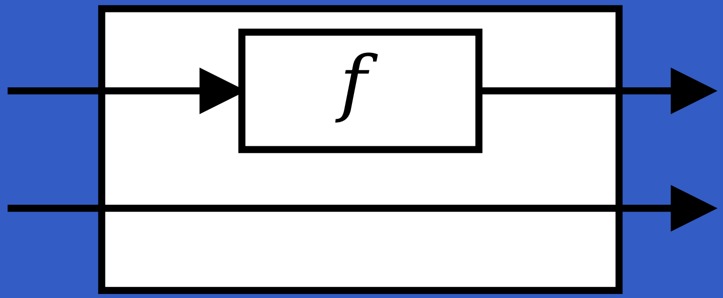
These diagrams convey the general idea:



`arr` f



f `>>>` g



`first` f

Some arrow laws

```
(f >>> g) >>> h = f >>> (g >>> h)
  arr (f >>> g) = arr f >>> arr g
   arr id >>> f = f
              f = f >>> arr id
  first (arr f) = arr (first f)
first (f >>> g) = first f >>> first g
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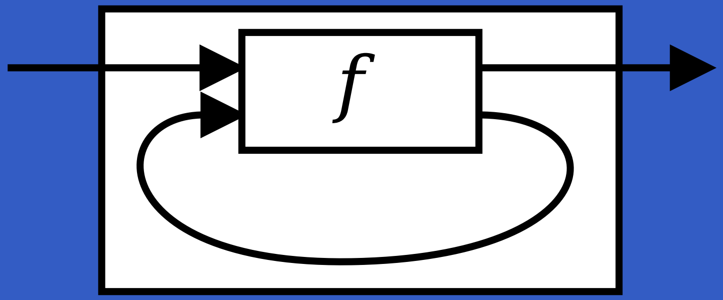
Being able to use simple algebraic laws like these greatly facilitates reasoning about programs.

The `loop` combinator

Another important operator is `loop`: a fixed-point operator used to express recursive arrows or ***feedback***:



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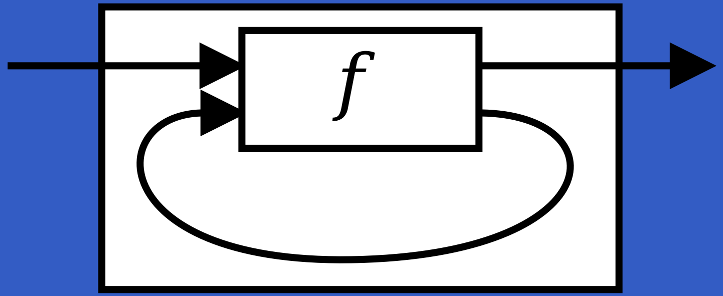


`loop` f

Remarkably, the four combinators `arr`, `>>>`, `first`, and `loop` suffice for expressing ***any conceivable wiring!***

Some more arrow combinators (1)

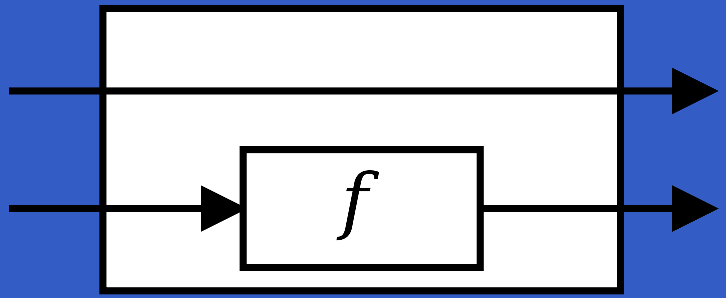
```
second :: Arrow a =>
    a b c -> a (d,b) (d,c)

(***) :: Arrow a =>
    a b c -> a d e -> a (b,d) (c,e)

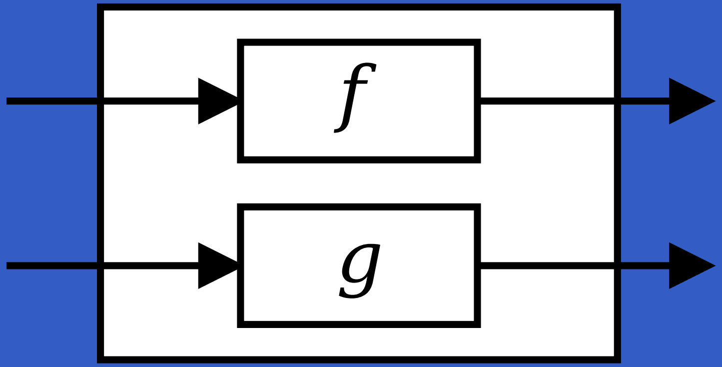
(&&&) :: Arrow a =>
    a b c -> a b d -> a b (c,d)
```

Some more arrow combinators (2)

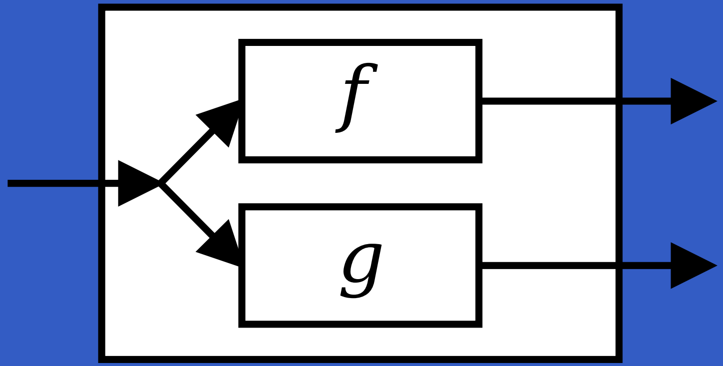
As diagrams:



second f



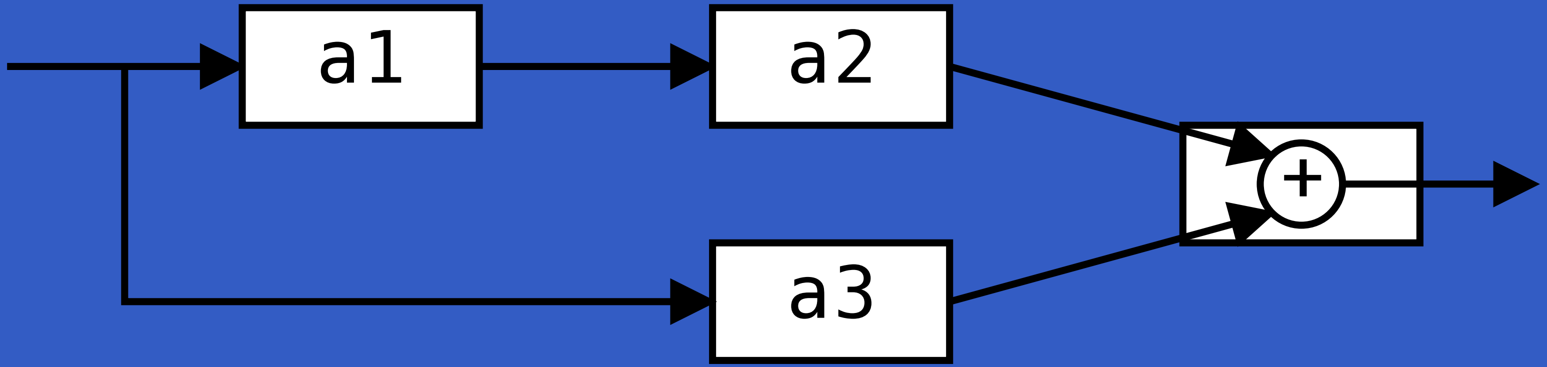
f *** g



f &&& g

Example: A Simple Network

A simple network:



```
a1, a2, a3 ::  A Double Double
```

One way to express it using arrow combinators:

```
circuit_v1 ::  A Double Double
circuit_v1 = (a1 &&& arr id)
             >>> (a2 *** a3)
             >>> arr (uncurry (+))
```

The arrow do notation (1)

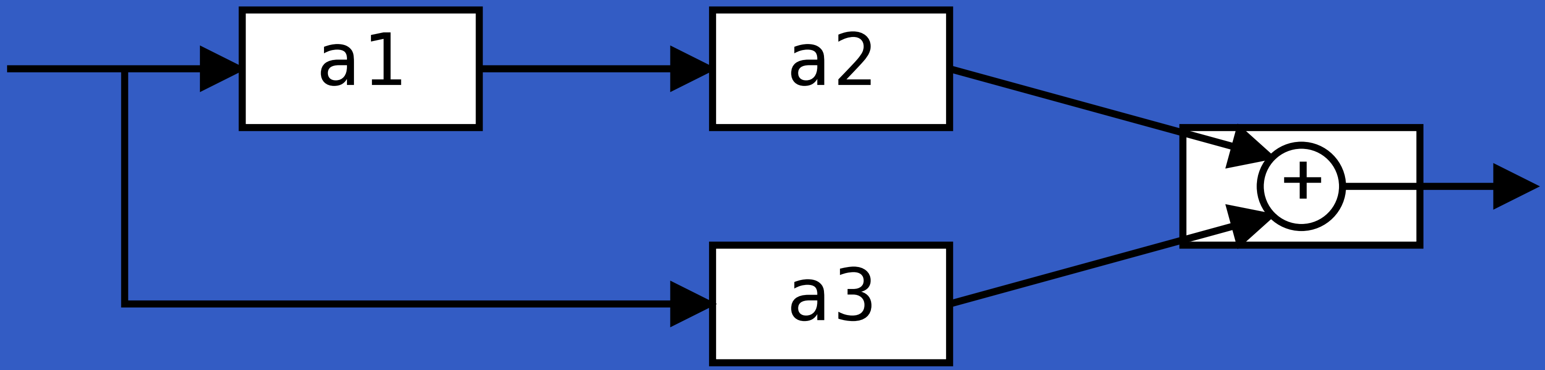
Using the basic combinators directly can be cumbersome. Ross Paterson's ***do***-notation for arrows provides a convenient alternative. Only ***syntactic sugar***!

```
proc pat -> do [ rec ]
    pat₁ <- sfexp₁ -< exp₁
    pat₂ <- sfexp₂ -< exp₂
    ...
    patₙ <- sfexpₙ -< expₙ
    returnA -< exp
```

Also: `let` pat = exp $\equiv$ pat `<- arr id -<` exp

The arrow do notation (2)

Let us redo the example using this notation:



```
circuit_v4 :: A Double Double
circuit_v4 = proc x -> do
    y1 <- a1 -< x
    y2 <- a2 -< y1
    y3 <- a3 -< x
    returnA -< y2 + y3
```

Yampa and Arrows

The Yampa signal function type is an arrow.

Signal function instances of the core combinators:

- `arr :: (a -> b) -> SF a b`
- `>>> :: SF a b -> SF b c -> SF a c`
- `first :: SF a b -> SF (a,c) (b,c)`
- `loop :: SF (a,c) (b,c) -> SF a b`

Some further basic signal functions

-
  ```
  identity :: SF a a
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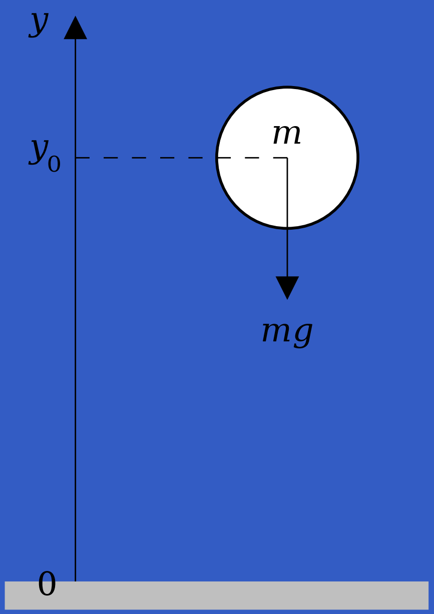
- `time :: SF a Time`
  `time = constant 1.0 >>> integral`
- `(^<<) :: (b->c) -> SF a b -> SF a c`
  `f (^<<) sf = sf >>> arr f`

A bouncing ball



$$\begin{aligned} y &= y_0 + \int v \,\mathrm{d}t \\ v &= v_0 + \int -9.81 \end{aligned}$$

On impact:

$$v = -v(t-)$$

(fully elastic collision)

Modelling the bouncing ball: part 1

Free-falling ball:

```
type Pos = Double
type Vel = Double

fallingBall ::
    Pos -> Vel -> SF () (Pos, Vel)
fallingBall y0 v0 = proc () -> do
    v <- (v0 +) ˆ<< integral -< -9.81
    y <- (y0 +) ˆ<< integral -< v
    returnA -< (y, v)
```

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```
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Associating information with an event occurrence:

```
tag :: Event a -> b -> Event b
```

Modelling the bouncing ball: part 2

Detecting when the ball goes through the floor:

```
fallingBall’ ::
    Pos -> Vel
    -> SF () ((Pos,Vel), Event (Pos,Vel))
fallingBall’ y0 v0 = proc () -> do
    yv@(y, _) <- fallingBall y0 v0 -< ()
    hit       <- edge              -< y <= 0
    returnA -< (yv, hit ‘tag‘ yv)
```

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- This creates a “running” signal function ***instance***.
- The new signal function instance often replaces the previously running instance.

Switchers thus allow systems with ***varying structure*** to be described.

The basic switch

Idea:

- Allows one signal function to be replaced by another.
- Switching takes place on the first occurrence of the switching event source.

```
switch ::
    SF a (b, Event c)
   -> (c -> SF a b)
   -> SF a b
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Initial SF with event source

The basic switch

Idea:

- Allows one signal function to be replaced by another.
- Switching takes place on the first occurrence of the switching event source.

```
switch ::
    SF a (b, Event c)
   -> (c -> SF a b)
   -> SF a b
```

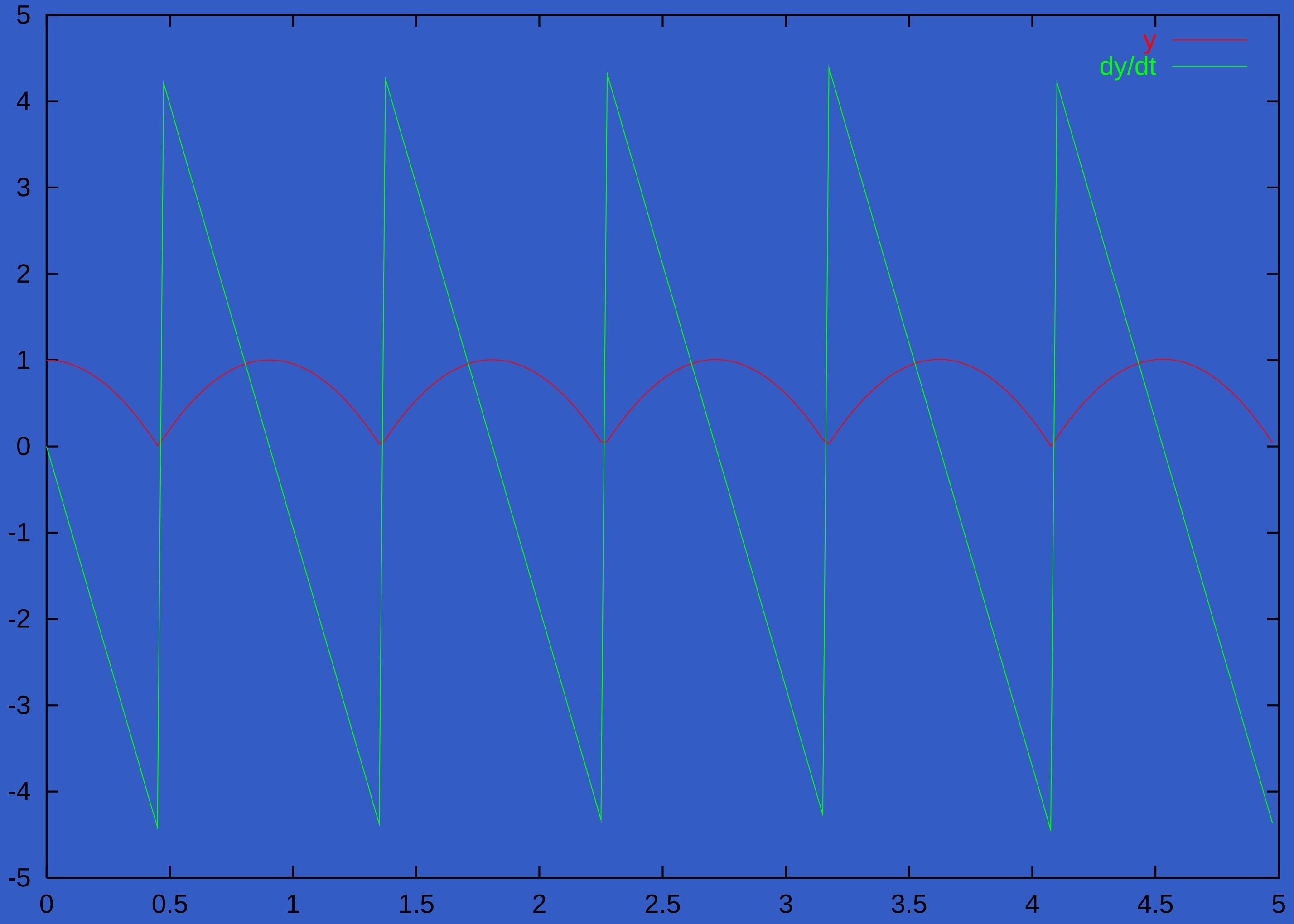
Function yielding SF to switch into

Modelling the bouncing ball: part 3

Making the ball bounce:

```
bouncingBall :: Pos -> SF () (Pos, Vel)
bouncingBall y0 = bbAux y0 0.0
  where
    bbAux y0 v0 =
      switch (fallingBall’ y0 v0) $ \(y,v) ->
      bbAux y (-v)
```

Simulation of bouncing ball



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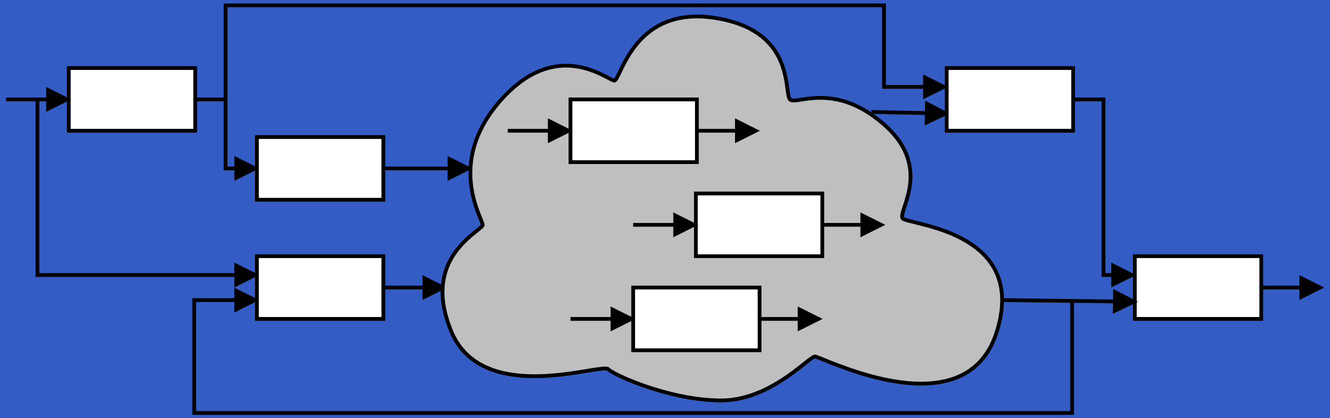

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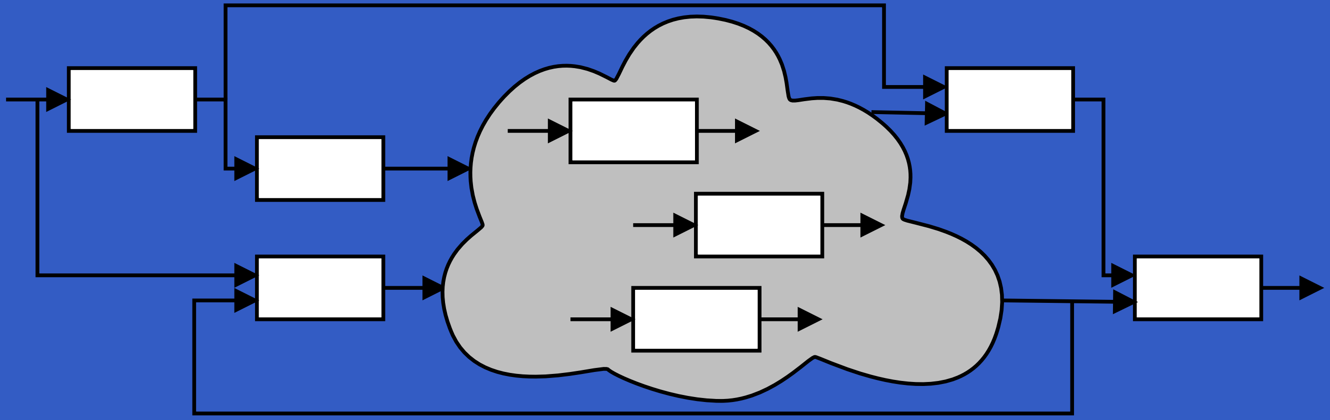


- What about state?

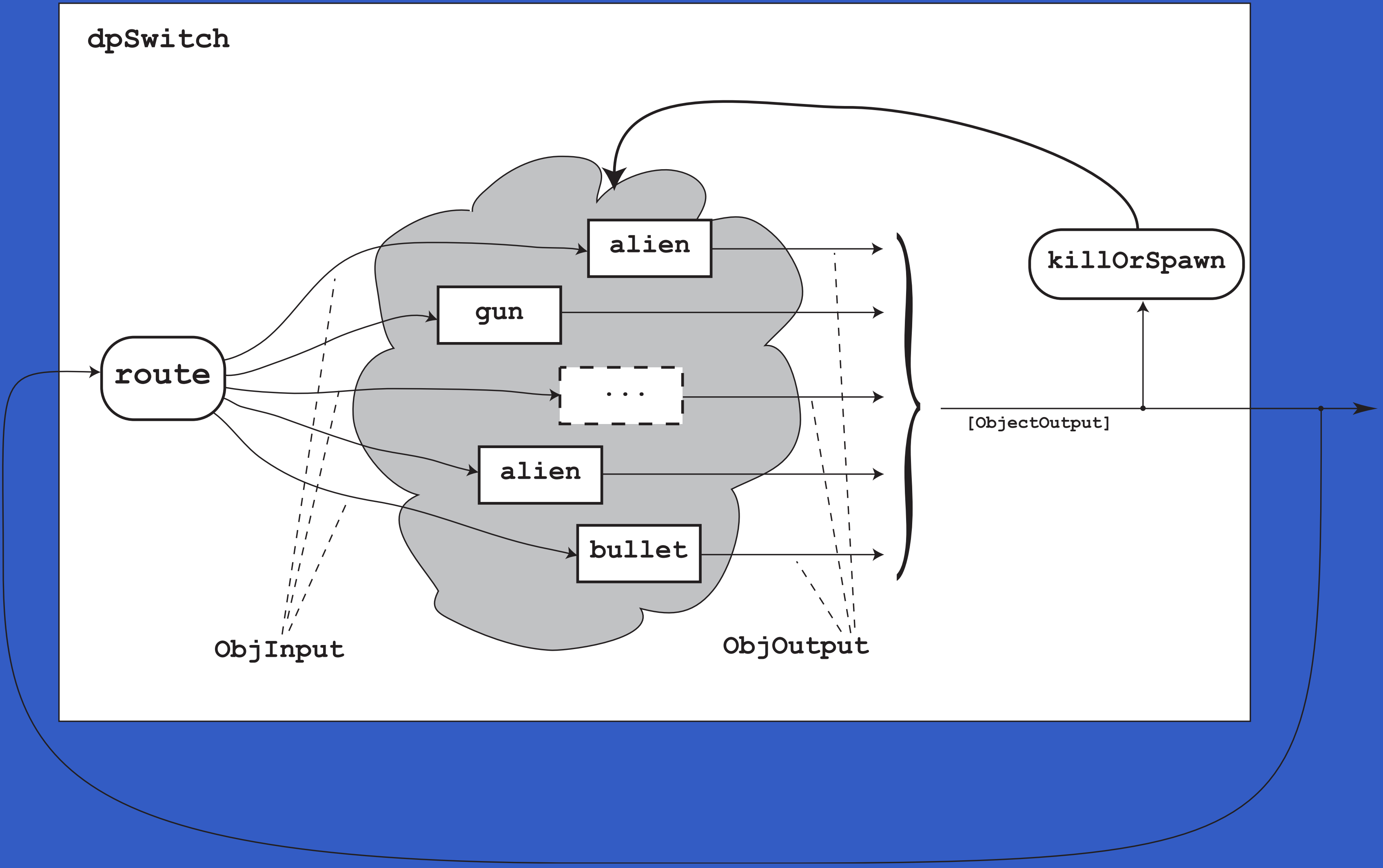
Dynamic signal function collections

Idea:

- Switch over ***collections*** of signal functions.
- On event, “freeze” running signal functions into collection of signal function ***continuations***, preserving encapsulated ***state***.
- Modify collection as needed and switch back in.

Example: Space Invaders

Overall game structure



Describing the alien behavior (1)

```
type Object = SF ObjInput ObjOutput

alien :: RandomGen g =>
  g -> Position2 -> Velocity -> Object
alien g p0 vyd = proc oi -> do
  rec
    -- Pick a desired horizontal position
    rx   <- noiseR (xMin, xMax) g -< ()
    smpl <- occasionally g 5 ()   -< ()
    xd   <- hold (point2X p0) -< smpl ‘tag‘ rx
    ...
```

Describing the alien behavior (2)

```
...
-- Controller
let axd = 5 * (xd - point2X p)
          - 3 * (vector2X v)
    ayd = 20 * (vyd - (vector2Y v))
    ad  = vector2 axd ayd
    h   = vector2Theta ad
...
```

Describing the alien behavior (3)

```
...
-- Physics
let a = vector2Polar
            (min alienAccMax
                 (vector2Rho ad))
            h
vp  <- iPre v0     -< v
ffi <- forceField -< (p, vp)
v   <- (v0 ^+^) ^<< impulseIntegral
      -< (gravity ^+^ a, ffi)
p   <- (p0 .+^) ^<< integral -< v
...
```

Describing the alien behavior (4)

```
        ...
        -- Shields
        sl  <- shield -< oiHit oi
        die <- edge   -< sl <= 0

    returnA -< ObjOutput {
                    ooObsObjState = oosAlien p h v,
                    ooKillReq     = die,
                    ooSpawnReq    = noEvent
                  }
    where
        v0 = zeroVector
```

State in `alien`

Each of the following signal functions used in `alien` encapsulate state:

- `noiseR`
- `occasionally`
- `hold`
- `iPre`
- `forceField`
- `impulseIntegral`
- `integral`
- `shield`
- `edge`

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- Advantages of declarative programming retained:
  - High abstraction level.
  - Referential transparency, algebraic laws: formal reasoning is simpler.
- Synchronous approach avoids “event-call-back soup”, meaning robust, easy-to-understand semantics.