MGS 2005
Functional Reactive Programming

Lecture 1: Introduction to FRP, Yampa, and Arrows

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Outline

- Brief introduction to FRP and Yampa
- Signal functions
- Arrows

Reactive programming

Reactive systems:

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The notions of

- time
- time-varying values, or ***signals***

are inherent and central for reactive systems.

Functional Reactive Programming

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- Paradigm for reactive programming in a functional setting.
- Originated from Functional Reactive Animation (Fran) (Elliott & Hudak).
- Has evolved in a number of directions and into different concrete implementations.

FRP applications

Some domains where FRP has been used:

- Graphical Animation (Fran: Elliott, Hudak)
- Robotics (Frob: Peterson, Hager, Hudak, Elliott, Pembeci, Nilsson)
- Vision (FVision: Peterson, Hudak, Reid, Hager)
- GUIs (Fruit: Courtney)
- Hybrid modeling (Nilsson, Hudak, Peterson)

Key FRP features

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- Synchronous: all system parts operate in synchrony.
- Support for hybrid (mixed continuous and discrete time) systems.
- Allows dynamic system structure.

Related languages and paradigms

FRP related to:

- Synchronous languages, like Esterel, Lucid Synchrone.

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- Synchronous languages, like Esterel, Lucid Synchrone.
- Modeling languages, like Simulink, Modelica.

Yampa

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- The most recent Yale FRP implementation. People:
  - Antony Courtney
  - Paul Hudak
  - Henrik Nilsson
  - John Peterson

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- The most recent Yale FRP implementation. People:
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  - John Peterson
- A Haskell ***combinator library***, a.k.a. ***Domain-Specific Embedded Language*** (DSEL).

Yampa

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- Structured using ***arrows***.

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Yampa

What is ***Yampa***?

- Structured using ***arrows***.
- ***Continuous-time*** signals (conceptually)
- Option type ***Event*** to handle discrete-time signals.
- Advanced ***switching constructs*** to describe systems with dynamic structure.

Yampa?

Yampa?

Y et

A nother

M ostly

P ointless

A cronym

Yampa?

Yet

Another

Mostly

Pointless

Acronym

???

Yampa?

Yet
Another
Mostly
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???

No . . .

Yampa?

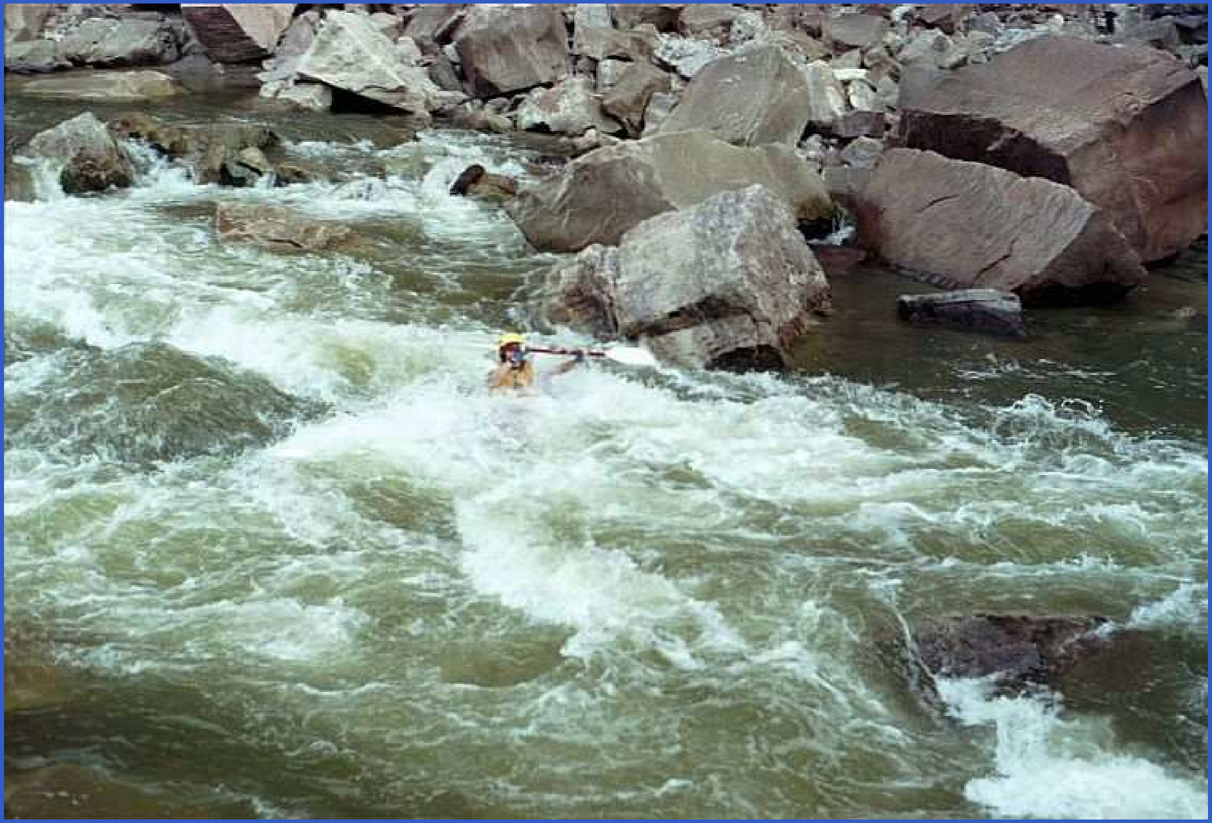
Yampa is a river . . .

Yampa?

. . . with long calmly flowing sections . . .

Yampa?

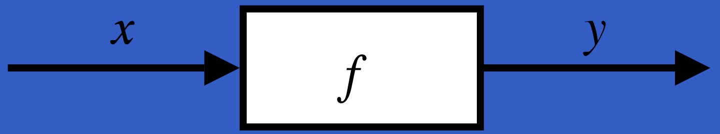
. . . and abrupt whitewater transitions in between.



A good metaphor for hybrid systems!

Signal functions (1)

Key concept: ***functions on signals***.



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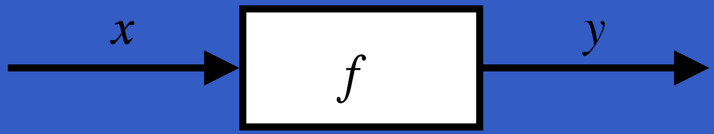

Intuition:

$$\texttt{Signal}\ \alpha \approx \texttt{Time}\rightarrow\alpha$$
$$x :: \texttt{Signal T1}$$
$$y :: \texttt{Signal T2}$$
$$f :: \texttt{Signal T1} \rightarrow \texttt{Signal T2}$$

Signal functions (2)

Additionally, ***causality*** required: output at time t must be determined by input on interval $[0, t]$.

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Signal functions are said to be

- ***pure*** or ***stateless*** if output at time t only depends on input at time t
- ***impure*** or ***stateful*** if output at time t depends on input over the interval $[0, t]$.

Signal functions in Yampa

- ***Signal functions*** are ***first class entities***.
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Signal functions in Yampa

- ***Signal functions*** are ***first class entities***.
  Intuition: $\texttt{SF}\ \alpha\ \beta \approx \texttt{Signal}\ \alpha \rightarrow \texttt{Signal}\ \beta$
- ***Signals*** are ***not*** first class entities: they only exist indirectly through signal functions.

Signal functions and state

Alternative view:

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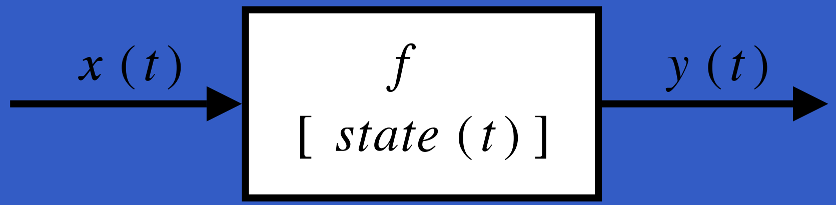
Signal functions can encapsulate ***state***.



$state(t)$ summarizes input history $x(t')$, $t' \in [0, t]$.
Thus, really a kind of ***process***.

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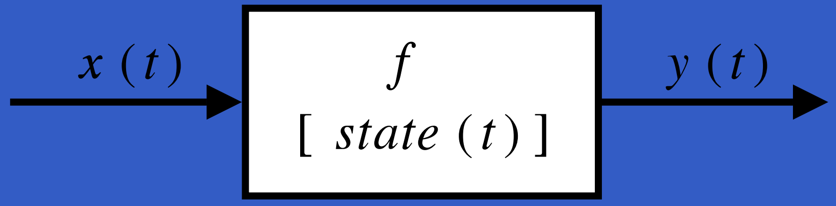

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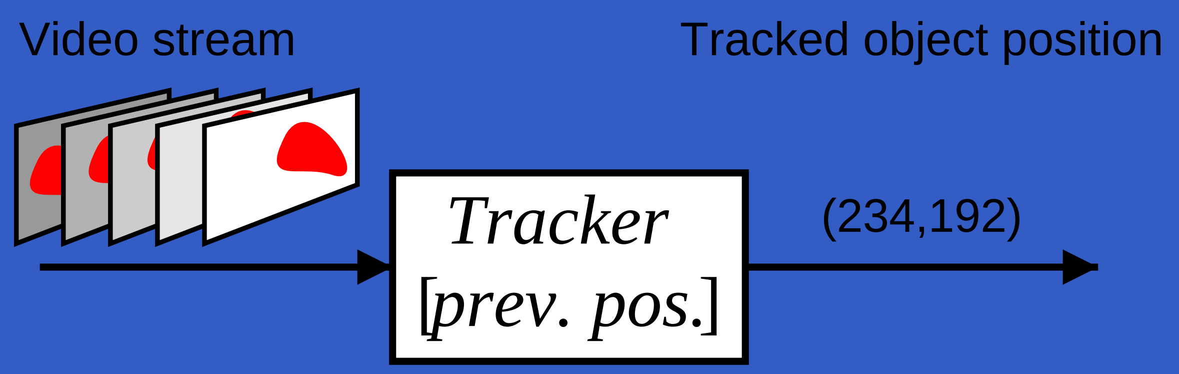
Thus, really a kind of ***process***.

From this perspective, signal functions are:

- ***stateful*** if $y(t)$ depends on $x(t)$ and $state(t)$
- ***stateless*** if $y(t)$ depends only on $x(t)$

Example: Video tracker

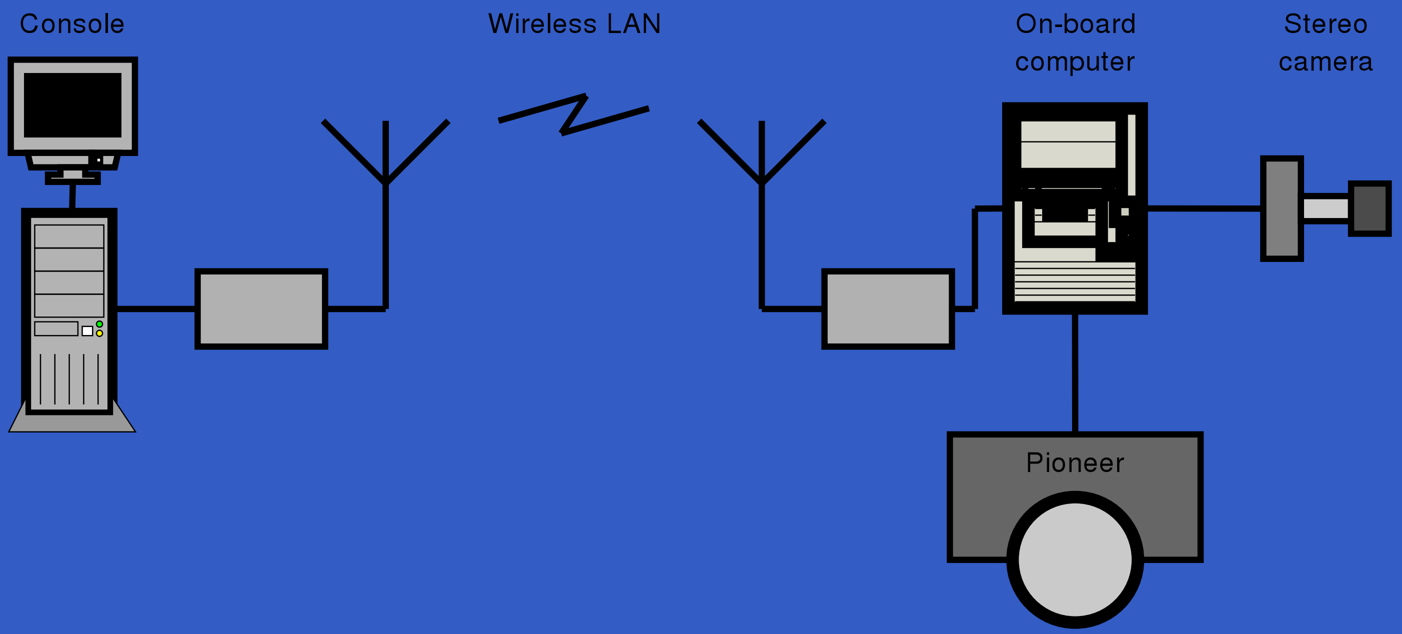
Video trackers are typically stateful signal functions:



Example: Robotics (1)

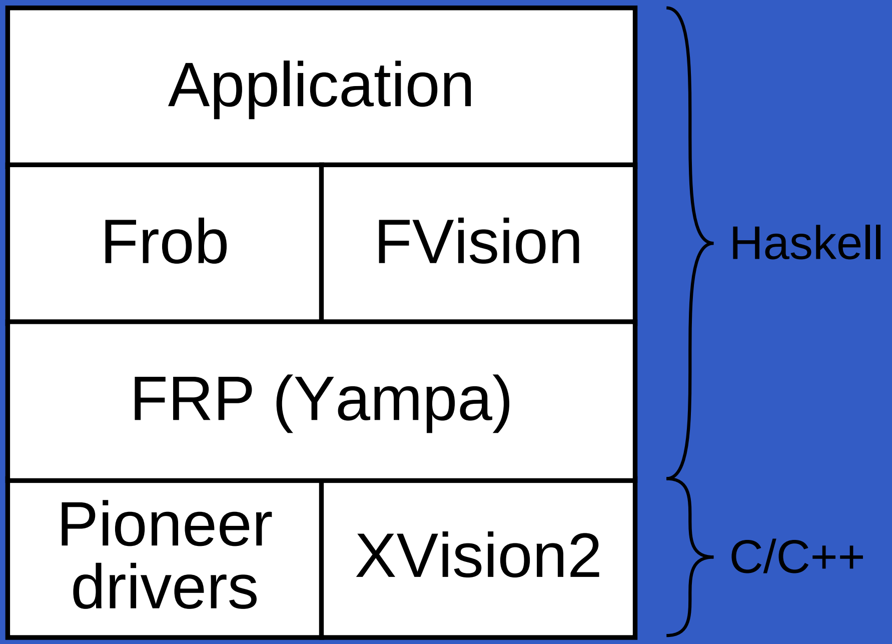
[PPDP'02, with Izzet Pembeci and Greg Hager, Johns Hopkins University]

Hardware setup:



Example: Robotics (2)

Software architecture:



Example: Robotics (3)

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In Yampa, systems are described by combining signal functions (forming new signal functions).

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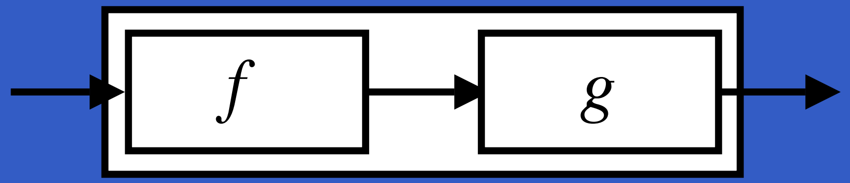

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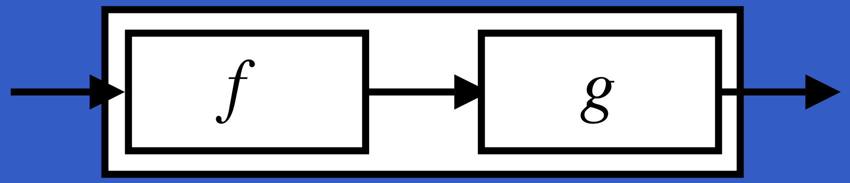
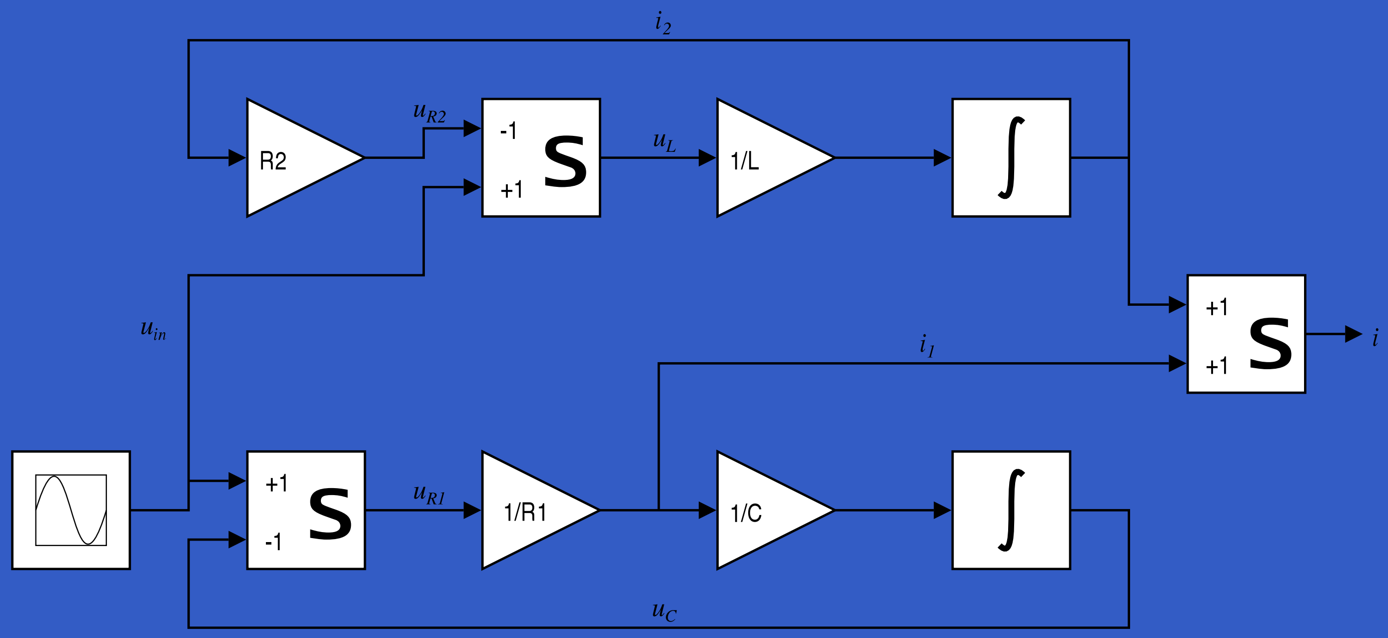


A *combinator* can be defined that captures this idea:

```
(>>>) :: SF a b -> SF b c -> SF a c
```

Yampa and Arrows (2)

But systems can be complex:



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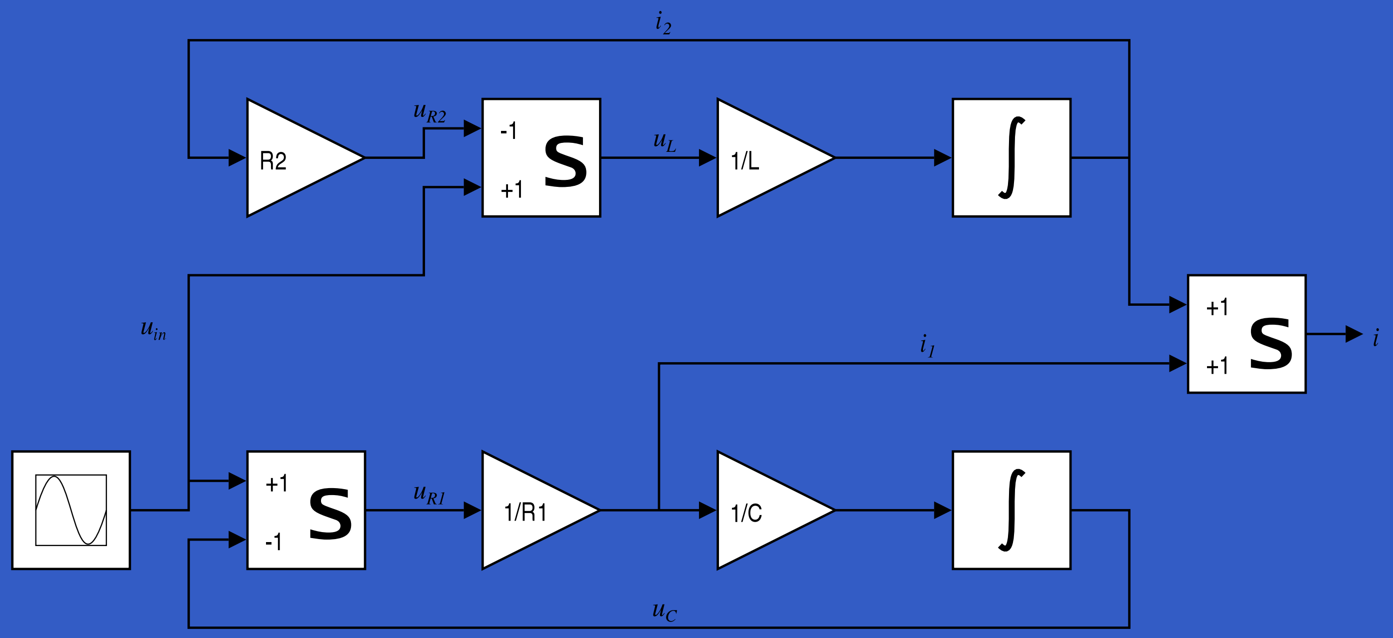

How many and what combinators do we need to be able to describe arbitrary systems?

Yampa and Arrows (3)

John Hughes' ***arrow*** framework:

- Abstract data type interface for function-like types.

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Yampa and Arrows (3)

John Hughes' ***arrow*** framework:

- Abstract data type interface for function-like types.
- Particularly suitable for types representing process-like computations.
- Related to ***monads***, since arrows are computations, but more general.
- Provides a minimal set of "wiring" combinators.

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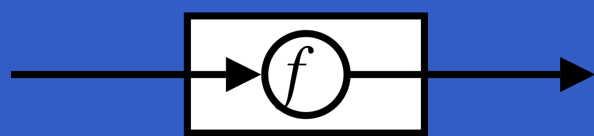
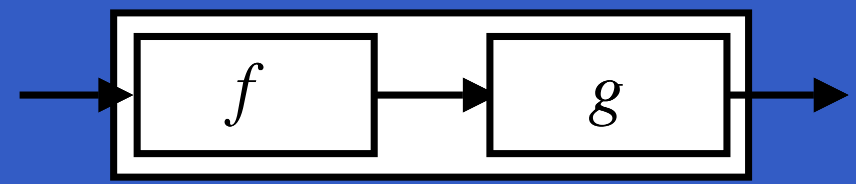
- A set of ***algebraic laws*** that must hold.

What is an arrow? (2)

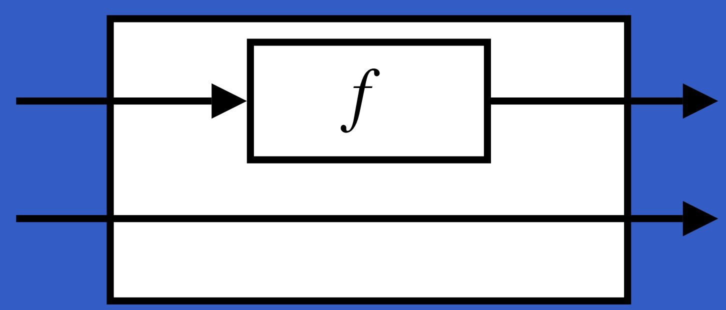
These diagrams convey the general idea:



`arr` f



f `>>>` g



`first` f

The `Arrow` class

In Haskell, a ***type class*** is used to capture these ideas (except for the laws):

```
class Arrow a where
    arr   :: (b -> c) -> a b c
    (>>>) :: a b c -> a c d -> a b d
    first :: a b c -> a (b,d) (c,d)
```

Functions are arrows (1)

Functions are a simple example of arrows. The arrow type constructor is just `(->)` in that case.

Exercise 1: Suggest suitable definitions of

- `arr`
- `(>>>)`
- `first`

for this case!

(We have not looked at what the laws are yet, but they are “natural”.)

Functions are arrows (2)

Solution:

- `arr = id`

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  To see this, recall

```
id :: t -> t
arr :: (b->c) -> a b c
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  To see this, recall

  ```
  id :: t -> t
  arr :: (b->c) -> a b c
  ```

  Instantiate with

  ```
  a = (->)
  t = b->c = (->) b c
  ```

Functions are arrows (3)

- `f >>> g = \a -> g (f a)`

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- `f >>> g = \a -> g (f a)` ***or***
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Functions are arrows (3)

- `f >>> g = \a -> g (f a)` ***or***
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- `(>>>) = flip (.)`
- `first f = \(b,d) -> (f b,d)`

Functions are arrows (4)

`Arrow` instance declaration for functions:

```
instance Arrow (->) where
        arr     = id
        (>>>)   = flip (.)
        first f = \(b,d) -> (f b,d)
```

Arrow laws

```
(f >>> g) >>> h = f >>> (g >>> h)
```

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(f >>> g) >>> h = f >>> (g >>> h)
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   arr id >>> f = f
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(f >>> g) >>> h = f >>> (g >>> h)
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```
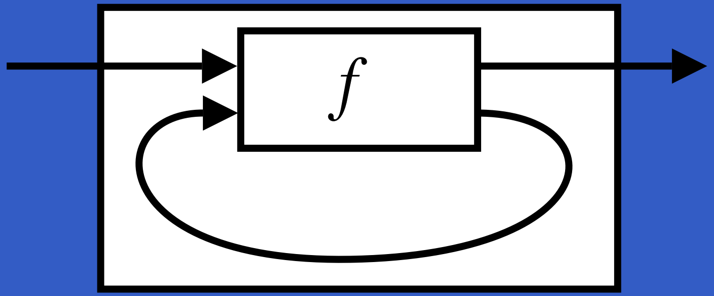

Arrow laws

```
(f >>> g) >>> h = f >>> (g >>> h)
  arr (f >>> g) = arr f >>> arr g
   arr id >>> f = f
              f = f >>> arr id
  first (arr f) = arr (first f)
first (f >>> g) = first f >>> first g
```

Exercise 2: Draw diagrams illustrating the first and last law!

The `loop` combinator (1)

Another important operator is `loop`: a fixed-point operator used to express recursive arrows or ***feedback***:



`loop` f

The `loop` combinator (2)

Not all arrow instances support `loop`. It is thus a method of a separate class:

```
class Arrow a => ArrowLoop a where
    loop :: a (b, d) (c, d) -> a b c
```

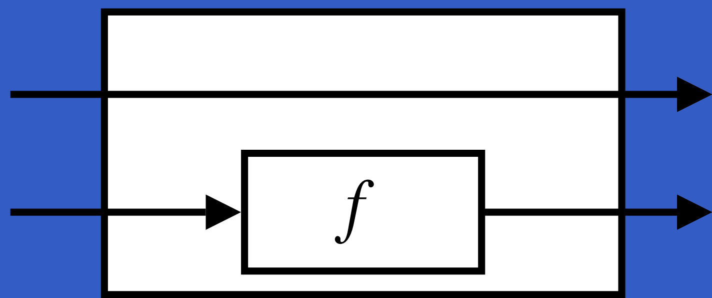
Remarkably, the four combinators `arr`, `>>>`, `first`, and `loop` are sufficient to express any conceivable wiring!

Some more arrow combinators (1)

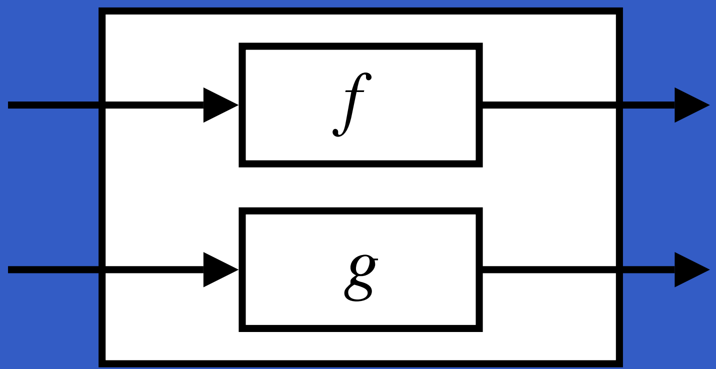
```
second :: Arrow a =>
    a b c -> a (d,b) (d,c)

(***) :: Arrow a =>
    a b c -> a d e -> a (b,d) (c,e)

(&&&) :: Arrow a =>
    a b c -> a b d -> a b (c,d)
```

Some more arrow combinators (2)

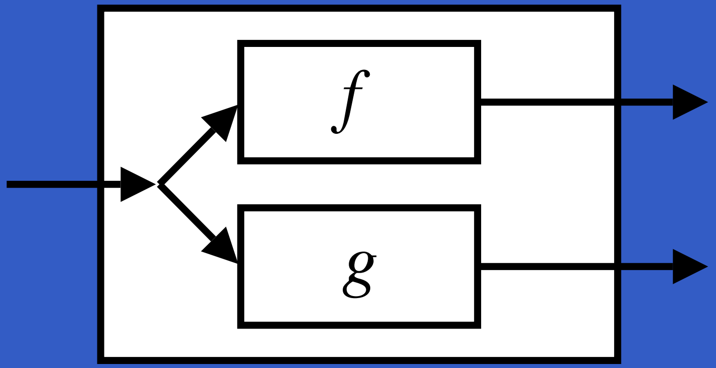
As diagrams:



second f



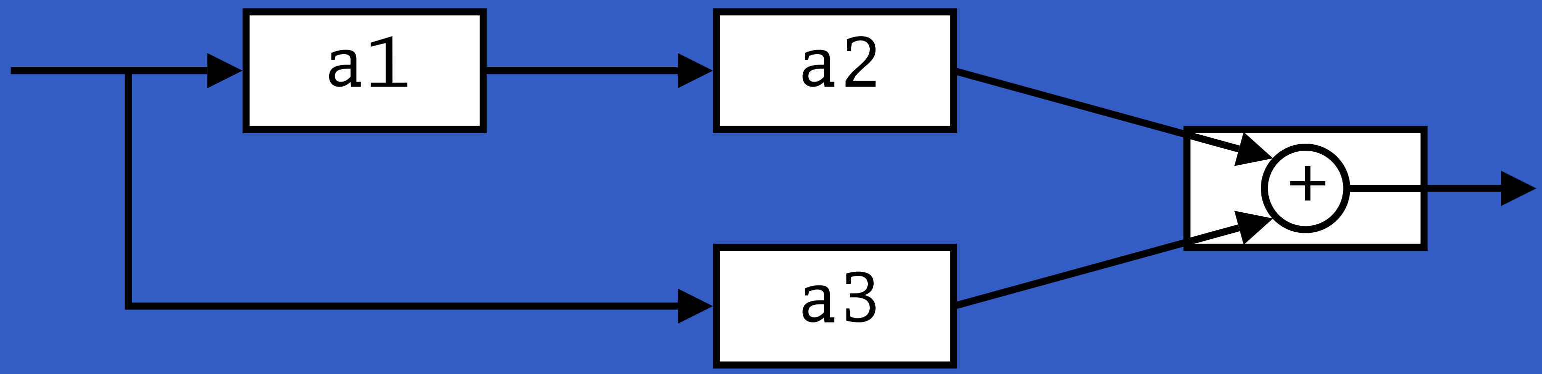
f *** g



f &&& g

Some more arrow combinators (3)

Exercise 3: Describe the following circuit using arrow combinators:



`a1, a2, a3 :: A Double Double`

Exercise 4: The combinators `second`, `(***)`, and `(&&&)` are not primitive, but defined in terms of `arr`, `(>>>)`, and `first`. Suggest suitable definitions!

Reading (1)

- John Hughes. Generalising monads to arrows. *Science of Computer Programming*, 37:67–111, May 2000
- John Hughes. Programming with arrows. In *Advanced Functional Programming*, 2004. To be published by Springer Verlag.
- Henrik Nilsson, Antony Courtney, and John Peterson. Functional reactive programming, continued. In *Proceedings of the 2002 Haskell Workshop*, pp. 51–64, October 2002.

Reading (2)

- Paul Hudak, Antony Courtney, Henrik Nilsson, and John Peterson. Arrows, robots, and functional reactive programming. In *Advanced Functional Programming*, 2002. LNCS 2638, pp. 159–187.