

MGS 2006: AFP Lecture 4

Functional Reactive Programming and Arrows

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Functional Reactive Programming (1)

Functional Reactive Programming (FRP):

- Paradigm for reactive programming in a functional setting.
- Originated from Functional Reactive Animation (Fran) (Elliott & Hudak).
- Has evolved in a number of directions and into different concrete implementations.
- (Usually) continuous notion of time and additional support for discrete events.

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Reactive programming

Reactive systems:

- Input arrives *incrementally* while system is running.
- Output is generated in response to input in an interleaved and *timely* fashion.

Contrast **transformational systems**.

The notions of

- time
- time-varying values, or **signals**

are inherent and central for reactive systems.

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Functional Reactive Programming (2)

Yampa:

- The most recent Yale FRP implementation.
- **Embedding** in Haskell (a Haskell library).
- **Arrows** used as the basic structuring framework.
- **Continuous time**.
- Discrete-time signals modelled by continuous-time signals and an option type.
- Advanced **switching constructs** allows for highly dynamic system structure.

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Related languages

FRP related to:

- Synchronous languages, like Esterel, Lucid Sychrone.
- Modeling languages, like Simulink, Modelica.

Distinguishing features of FRP:

- First class reactive components.
- Allows highly dynamic system structure.
- Supports hybrid (mixed continuous and discrete) systems.

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Yampa?

Yampa is a river with long calmly flowing sections and abrupt whitewater transitions in between.



A good metaphor for hybrid systems!

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FRP applications

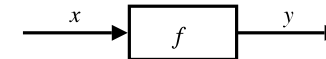
Some domains where FRP has been used:

- Graphical Animation (Fran: Elliott, Hudak)
- Robotics (Frob: Peterson, Hager, Hudak, Elliott, Pembeci, Nilsson)
- Vision (FVision: Peterson, Hudak, Reid, Hager)
- GUIs (Fruit: Courtney)
- Hybrid modeling (Nilsson, Hudak, Peterson)

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Signal functions

Key concept: **functions on signals**.



Intuition:

Signal $\alpha \approx \text{Time} \rightarrow \alpha$

$x :: \text{Signal } T1$

$y :: \text{Signal } T2$

$f :: \text{Signal } T1 \rightarrow \text{Signal } T2$

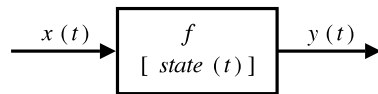
Additionally: **causality** requirement.

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Signal functions and state

Alternative view:

Signal functions can encapsulate **state**.



$state(t)$ summarizes input history $x(t')$, $t' \in [0, t]$.

Functions on signals are either:

- **Stateful**: $y(t)$ depends on $x(t)$ and $state(t)$
- **Stateless**: $y(t)$ depends only on $x(t)$

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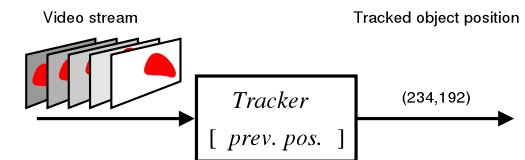
Signal functions in Yampa

- **Signal functions** are **first class entities**.
Intuition: $SF \alpha \beta \approx Signal \alpha \rightarrow Signal \beta$
- **Signals** are **not** first class entities: they only exist indirectly through signal functions.
- The second-class nature of signals allows causality to be exploited for an efficient implementation.

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Example: Video tracker

Video trackers are typically stateful signal functions:

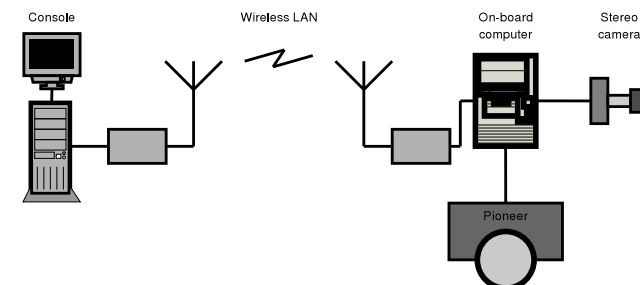


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Example: Robotics (1)

[PPDP'02, with Izzet Pabeci and Greg Hager, Johns Hopkins University]

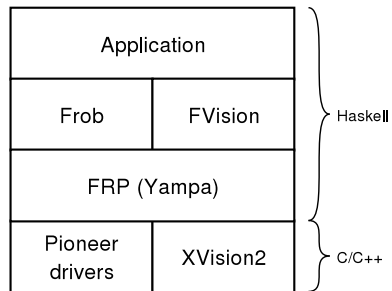
Hardware setup:



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Example: Robotics (2)

Software architecture:



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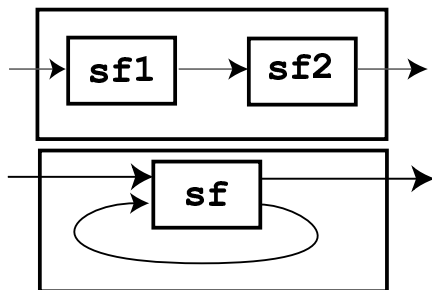
Example: Robotics (3)



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Yampa and Arrows (1)

Systems are described by combining signal functions (forming new signal functions):



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Yampa and Arrows (2)

Yampa uses John Hughes' **arrow** framework: the signal function type is an arrow.

Signal function instances of core combinators:

- $\text{arr} :: (a \rightarrow b) \rightarrow \text{SF } a \ b$
- $\text{>>>} :: \text{SF } a \ b \rightarrow \text{SF } b \ c \rightarrow \text{SF } a \ c$
- $\text{first} :: \text{SF } a \ b \rightarrow \text{SF } (a,c) \ (b,c)$
- $\text{loop} :: \text{SF } (a,c) \ (b,c) \rightarrow \text{SF } a \ b$

Enough to express any conceivable “wiring”.

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Arrows, Monads, and FRP (1)

- Like monads, arrows represent a form of effectful computations.
- In fact, some arrows, those that support an **apply** operation, are also monads (but not vice versa).

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The arrow syntactic sugar

Using the basic combinators directly is often very cumbersome. Ross Paterson's syntactic sugar for arrows provides a convenient alternative:

```
proc pat -> do [ rec ]
  pat1 <- sfexp1 -< exp1
  pat2 <- sfexp2 -< exp2
  ...
  patn <- sfexpn -< expn
  returnA -< exp
```

Also: `let pat = exp ≡ pat <- arr id -< exp`

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Arrows, Monads, and FRP (2)

- Could Yampa be based on monads instead?

NO! Essentially because

```
(>>=) :: Monad m =>
      m a -> (a -> m b) -> m b
```

implies that a new signal function would have to be computed at every point in time, depending on the result of the first computation. This does not make much sense in a dataflow setting.

- But possibly on **co-monads** (Uustalu, Vene 2005)

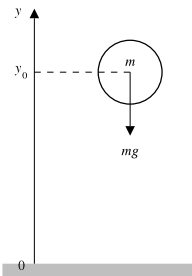
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Some further basic signal functions

- `identity :: SF a a`
`identity = arr id`
- `constant :: b -> SF a b`
`constant b = arr (const b)`
- `integral :: VectorSpace a s => SF a a`
- `time :: SF a Time`
`time = constant 1.0 >>> integral`
- `(^<<) :: (b->c) -> SF a b -> SF a c`
`f (^<<) sf = sf >>> arr f`

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A bouncing ball



$$y = y_0 + \int v dt$$

$$v = v_0 + \int -9.81$$

On impact:

$$v = -v(t-)$$

(fully elastic collision)

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Modelling the bouncing ball: part 1

Free-falling ball:

```
type Pos = Double
```

```
type Vel = Double
```

```
fallingBall ::
```

```
    Pos -> Vel -> SF () (Pos, Vel)
```

```
fallingBall y0 v0 = proc () -> do
```

```
    v <- (v0 +) ^<< integral -< -9.81
```

```
    y <- (y0 +) ^<< integral -< v
```

```
    returnA -< (y, v)
```

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Events

Conceptually, **discrete-time** signals are only defined at discrete points in time, often associated with the occurrence of some **event**.

Yampa models discrete-time signals by lifting the **range** of continuous-time signals:

```
data Event a = NoEvent | Event a
```

Discrete-time signal = `Signal (Event a)`.

Associating information with an event occurrence:

```
tag :: Event a -> b -> Event b
```

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Some basic event sources

- `never :: SF a (Event b)`
- `now :: b -> SF a (Event b)`
- `after :: Time -> b -> SF a (Event b)`
- `repeatedly :: Time -> b -> SF a (Event b)`
- `edge :: SF Bool (Event ())`

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Modelling the bouncing ball: part 2

Detecting when the ball goes through the floor:

```
fallingBall' ::
  Pos -> Vel
  -> SF () ((Pos, Vel), Event (Pos, Vel))
fallingBall' y0 v0 = proc () -> do
  yv@(y, _) <- fallingBall y0 v0 -< ()
  hit      <- edge          -< y <= 0
  returnA -< (yv, hit 'tag' yv)
```

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The basic switch

Idea:

- Allows one signal function to be replaced by another.
- Switching takes place on the first occurrence of the switching event source.

```
switch ::
  SF a (b, Event c)
  -> (c -> SF a b)
  -> SF a b
```

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Switching

Q: How and when do signal functions “start”?

- A:
- **Switchers** “apply” a signal functions to its input signal at some point in time.
 - This creates a “running” signal function **instance**.
 - The new signal function instance often replaces the previously running instance.

Switchers thus allow systems with **varying structure** to be described.

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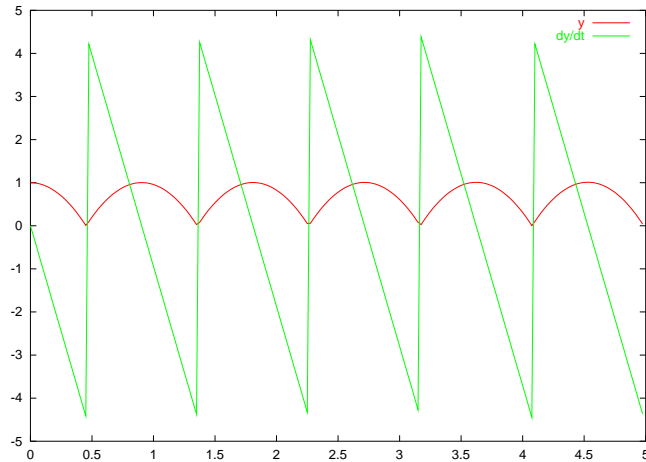
Modelling the bouncing ball: part 3

Making the ball bounce:

```
bouncingBall :: Pos -> SF () (Pos, Vel)
bouncingBall y0 = bbAux y0 0.0
  where
    bbAux y0 v0 =
      switch (fallingBall' y0 v0) $ \(y,v) ->
        bbAux y (-v)
```

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Simulation of bouncing ball

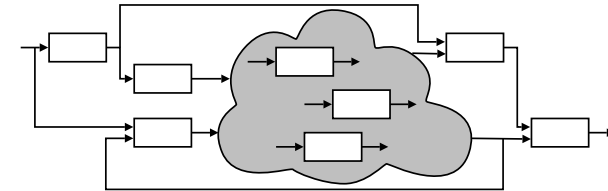


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Highly dynamic system structure?

Basic switch allows one signal function to be replaced by another.

- What about more general structural changes?



- What about state?

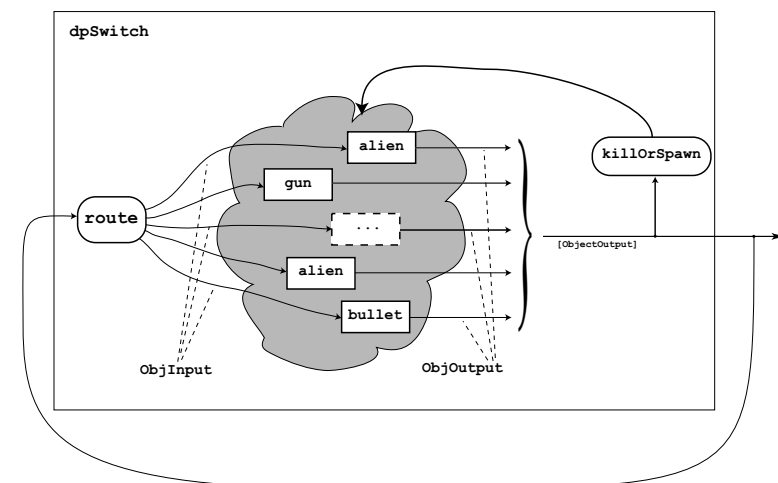
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Example: Space Invaders



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Overall game structure



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Dynamic signal function collections

Idea:

- Switch over **collections** of signal functions.
- On event, “freeze” running signal functions into collection of signal function **continuations**, preserving encapsulated **state**.
- Modify collection as needed and switch back in.

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Describing the alien behavior (1)

```
type Object = SF ObjInput ObjOutput
```

```
alien :: RandomGen g =>
```

```
  g -> Position2 -> Velocity -> Object
```

```
alien g p0 vvd = proc oi -> do
```

```
  rec
```

```
    -- Pick a desired horizontal position
```

```
    rx  <- noiser (xMin, xMax) g -< ()
```

```
    smpl <- occasionally g 5 () -< ()
```

```
    xd  <- hold (point2X p0) -< smpl `tag` rx
```

```
    ...
```

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dpSwitch

Need ability to express:

- How input routed to each signal function.
- When collection changes shape.
- How collection changes shape.

```
dpSwitch :: Functor col =>
  (forall sf . (a -> col sf -> col (b,sf)))
-> col (SF b c)
-> SF (a, col c) (Event d)
-> (col (SF b c) -> d -> SF a (col c))
-> SF a (col c)
```

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Describing the alien behavior (2)

```
...
```

```
-- Controller
```

```
let axd = 5 * (xd - point2X p)
```

```
      - 3 * (vector2X v)
```

```
    ayd = 20 * (vyd - (vector2Y v))
```

```
    ad  = vector2 axd ayd
```

```
    h   = vector2Theta ad
```

```
...
```

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Describing the alien behavior (3)

```
...
-- Physics
let a = vector2Polar
      (min alienAccMax
       (vector2Rho ad))
      h
vp  <- iPre v0    -< v
ffi <- forceField -< (p, vp)
v   <- (v0 ^+^)^ << impulseIntegral
      -< (gravity ^+^ a, ffi)
p   <- (p0 .+^)^ << integral -< v
...
```

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Other functional approaches?

Transition function operating on world model with explicit state (e.g. Asteroids by Lüth):

- Model snapshot of world with **all** state components.
- Transition function takes input and current world snapshot to output and the next world snapshot.

One could also use this technique **within** Yampa to avoid switching over dynamic collections.

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Describing the alien behavior (4)

```
...
-- Shields
sl  <- shield -< oiHit oi
die <- edge   -< sl <= 0

returnA -< ObjOutput {
      ooObsObjState = oosAlien p h v,
      ooKillReq     = die,
      ooSpawnReq    = noEvent
    }
where
  v0 = zeroVector
```

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Why use Yampa, then?

- Yampa provides a lot of functionality for programming with time-varying values:
 - Captures common patterns.
 - Carefully designed to facilitate reuse.
- Yampa allows state to be nicely encapsulated by signal functions:
 - Avoids keeping track of all state globally.
 - Adding more state usually does not imply any major changes to type or code structure.

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State in alien

Each of the following signal functions used in alien encapsulate state:

- `noiser`
- `occasionally`
- `hold`
- `iPre`
- `forceField`
- `impulseIntegral`
- `integral`
- `shield`
- `edge`

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Obtaining Yampa

Yampa 0.92 is available from

<http://www.haskell.org/yampa>

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Why not imperative, then?

If state is so important, why not stick to imperative/object-oriented programming where we have “state for free”?

- Advantages of declarative programming retained:
 - High abstraction level.
 - Referential transparency, algebraic laws: formal reasoning ought to be simpler.
- Synchronous approach avoids “event-call-back soup”, meaning robust, easy-to-understand semantics.

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Reading

- John Hughes. Generalising monads to arrows. *Science of Computer Programming*, 37:67–111, May 2000
- John Hughes. Programming with arrows. In *Advanced Functional Programming*, 2004. To be published by Springer Verlag.
- Henrik Nilsson, Antony Courtney, and John Peterson. Functional reactive programming, continued. In *Proceedings of the 2002 Haskell Workshop*, pp. 51–64, October 2002.

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Reading (2)

- Paul Hudak, Antony Courtney, Henrik Nilsson, and John Peterson. Arrows, robots, and functional reactive programming. In *Advanced Functional Programming*, 2002. LNCS 2638, pp. 159–187.
- Tarmo Uustalu and Varmo Vene. *The Essence of Dataflow Programming*. 2005