MGS 2006: AFP Lecture 4

Functional Reactive Programming and Arrows

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Reactive programming

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The notions of

- time
- time-varying values, or ***signals***

are inherent and central for reactive systems.

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- Paradigm for reactive programming in a functional setting.
- Originated from Functional Reactive Animation (Fran) (Elliott & Hudak).
- Has evolved in a number of directions and into different concrete implementations.
- (Usually) continuous notion of time and additional support for discrete events.

Functional Reactive Programming (2)

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- The most recent Yale FRP implementation.

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- Discrete-time signals modelled by continuous-time signals and an option type.

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- The most recent Yale FRP implementation.
- ***Embedding*** in Haskell (a Haskell library).
- ***Arrows*** used as the basic structuring framework.
- ***Continuous time***.
- Discrete-time signals modelled by continuous-time signals and an option type.
- Advanced ***switching constructs*** allows for highly dynamic system structure.

Related languages

FRP related to:

- Synchronous languages, like Esterel, Lucid Synchrone.
- Modeling languages, like Simulink, Modelica.

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Distinguishing features of FRP:

- First class reactive components.
- Allows highly dynamic system structure.
- Supports hybrid (mixed continuous and discrete) systems.

FRP applications

Some domains where FRP has been used:

- Graphical Animation (Fran: Elliott, Hudak)
- Robotics (Frob: Peterson, Hager, Hudak, Elliott, Pembeci, Nilsson)
- Vision (FVision: Peterson, Hudak, Reid, Hager)
- GUIs (Fruit: Courtney)
- Hybrid modeling (Nilsson, Hudak, Peterson)

Yampa?

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Yet
Another
Mostly
Pointless
Acronym

Yampa?

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???

Yampa?

Yet
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???

No . . .

Yampa?

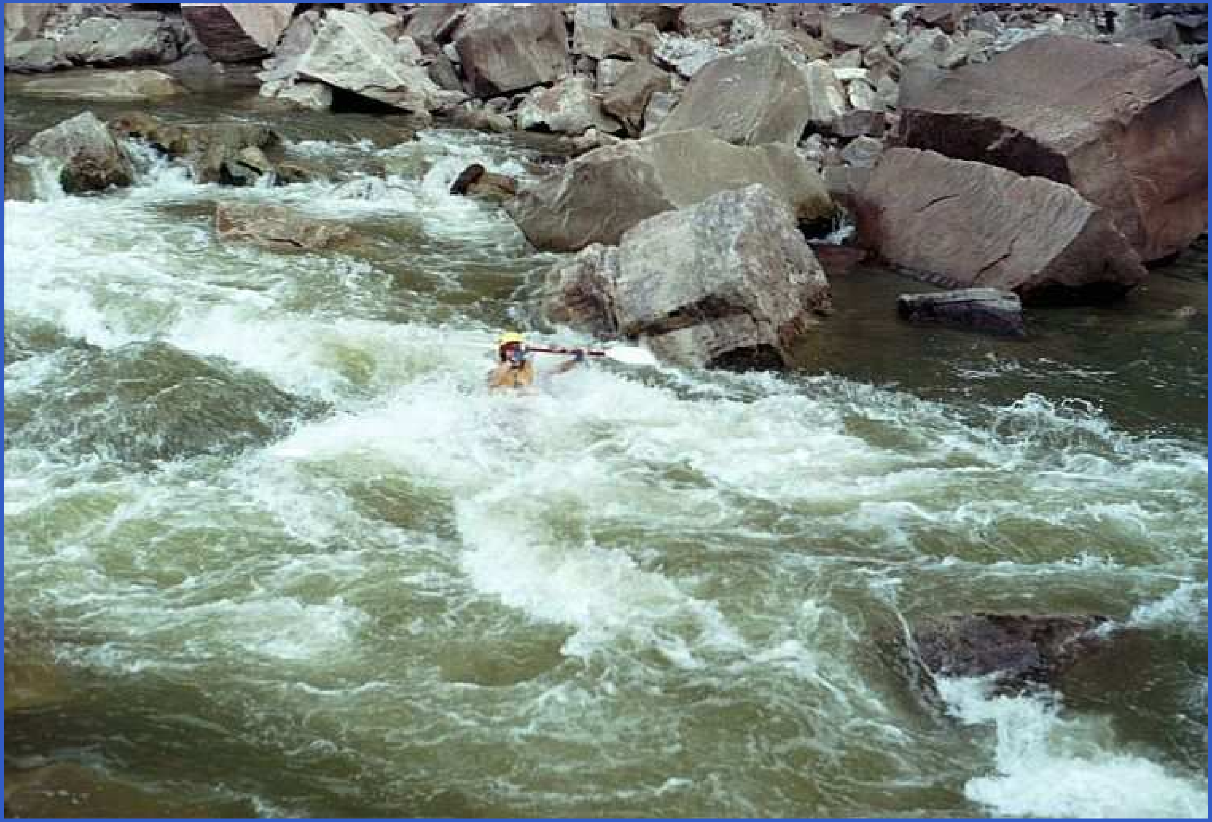
Yampa is a river . . .

Yampa?

. . . with long calmly flowing sections . . .

Yampa?

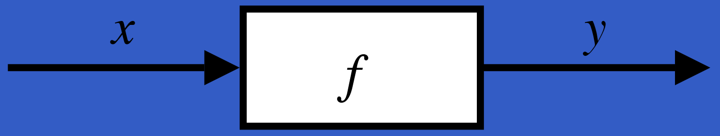
. . . and abrupt whitewater transitions in between.



A good metaphor for hybrid systems!

Signal functions

Key concept: ***functions on signals***.



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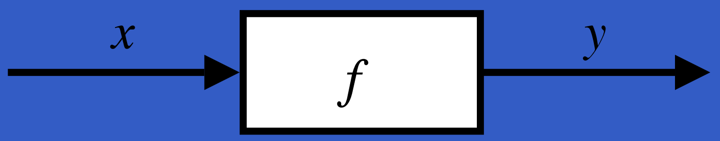

Intuition:

$$\texttt{Signal}\ \alpha \approx \texttt{Time} \rightarrow \alpha$$
$$x\ \texttt{::}\ \texttt{Signal T1}$$
$$y\ \texttt{::}\ \texttt{Signal T2}$$
$$f\ \texttt{::}\ \texttt{Signal T1} \rightarrow \texttt{Signal T2}$$

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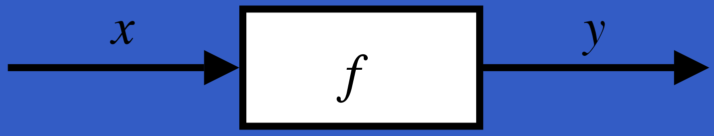

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Additionally: ***causality*** requirement.

Signal functions and state

Alternative view:

Signal functions and state

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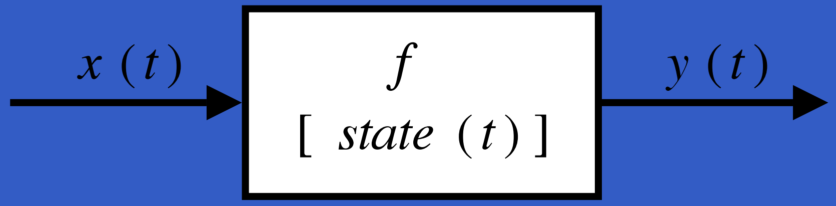
Signal functions can encapsulate ***state***.



$state(t)$ summarizes input history $x(t')$, $t' \in [0, t]$.

Signal functions and state

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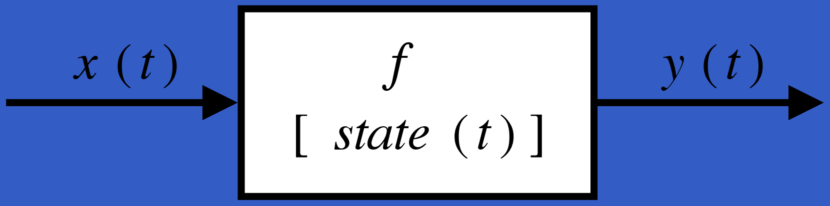


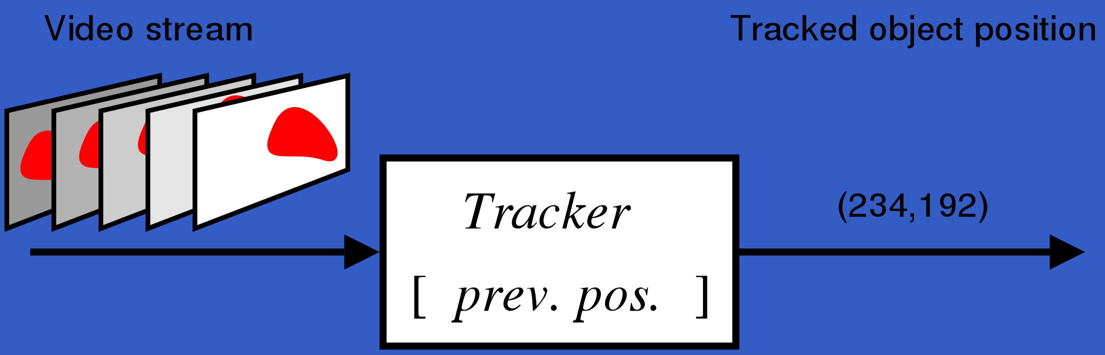
$state(t)$ summarizes input history $x(t')$, $t' \in [0, t]$.

Functions on signals are either:

- ***Stateful***: $y(t)$ depends on $x(t)$ and $state(t)$
- ***Stateless***: $y(t)$ depends only on $x(t)$

Example: Video tracker

Video trackers are typically stateful signal functions:



Signal functions in Yampa

- ***Signal functions*** are ***first class entities***.
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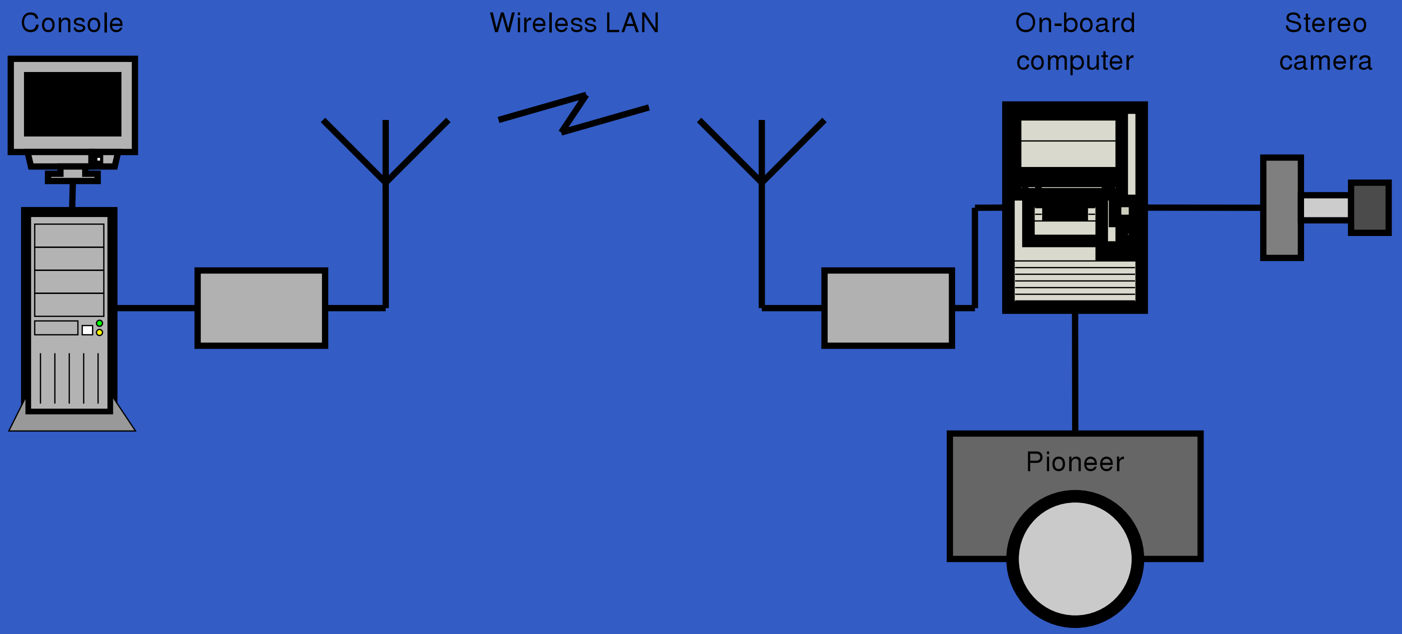
Signal functions in Yampa

- ***Signal functions*** are ***first class entities***. Intuition: $\texttt{SF}\ \alpha\ \beta \approx \texttt{Signal}\ \alpha \rightarrow \texttt{Signal}\ \beta$
- ***Signals*** are ***not*** first class entities: they only exist indirectly through signal functions.
- The second-class nature of signals allows causality to be exploited for an efficient implementation.

Example: Robotics (1)

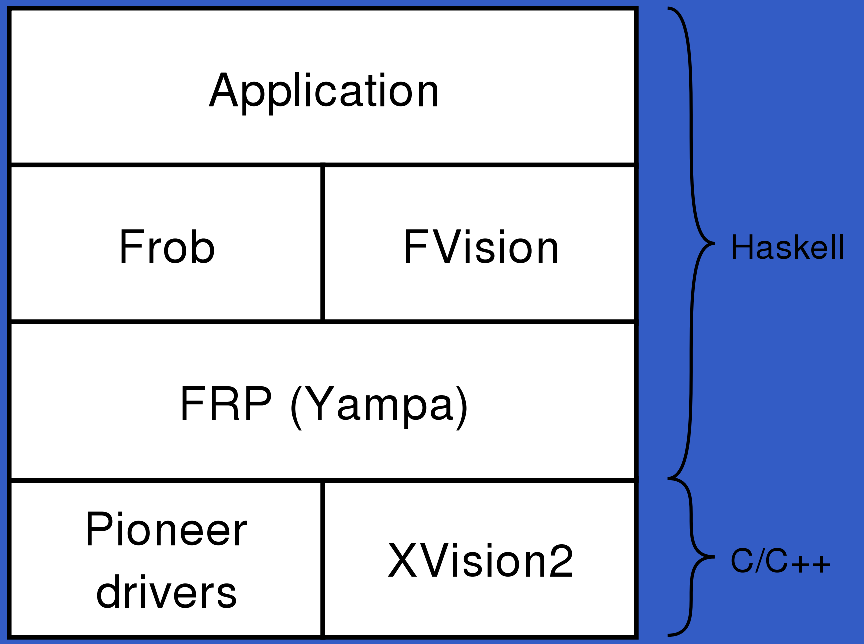
[PPDP'02, with Izzet Pembeci and Greg Hager, Johns Hopkins University]

Hardware setup:



Example: Robotics (2)

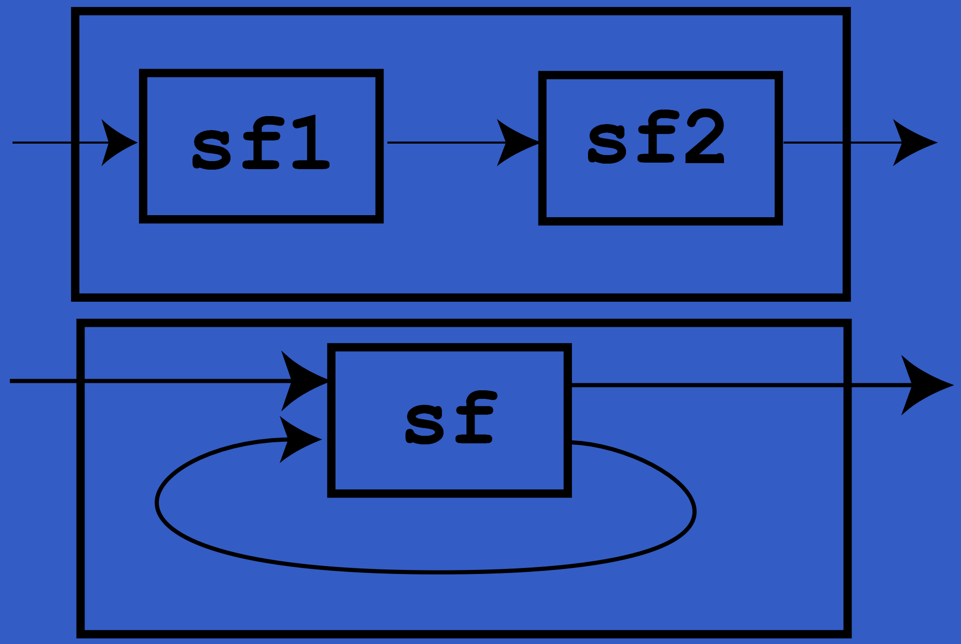
Software architecture:



Example: Robotics (3)

Yampa and Arrows (1)

Systems are described by combining signal functions (forming new signal functions):



Yampa and Arrows (2)

Yampa uses John Hughes' ***arrow*** framework: the signal function type is an arrow.

Signal function instances of core combinators:

- `arr :: (a -> b) -> SF a b`
- `>>> :: SF a b -> SF b c -> SF a c`
- `first :: SF a b -> SF (a,c) (b,c)`
- `loop :: SF (a,c) (b,c) -> SF a b`

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Enough to express any conceivable "wiring".

Arrows, Monads, and FRP (1)

- Like monads, arrows represent a form of effectful computations.
- In fact, some arrows, those that support an ***apply*** operation, are also monads (but not vice versa).

Arrows, Monads, and FRP (2)

- Could Yampa be based on monads instead?

  NO! Essentially because

  ```
  (>>=) :: Monad m =>
               m a -> (a -> m b) -> m b
  ```

  implies that a new signal function would have to be computed at every point in time, depending on the result of the first computation. This does not make much sense in a dataflow setting.

- But possibly on ***co-monads*** (Uustalu, Vene 2005)

The arrow syntactic sugar

Using the basic combinators directly is often very cumbersome. Ross Paterson's syntactic sugar for arrows provides a convenient alternative:

`proc` pat `-> do [ rec ]`
    pat_1 `<-` $sfexp_1$ `-<` exp_1
    pat_2 `<-` $sfexp_2$ `-<` exp_2
    `...`
    pat_n `<-` $sfexp_n$ `-<` exp_n
    `returnA -<` exp

Also: `let` pat `=` exp $\equiv$ pat `<- arr id -<` exp

Some further basic signal functions

- ```
  identity :: SF a a
  identity = arr id
  ```
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  constant :: b -> SF a b
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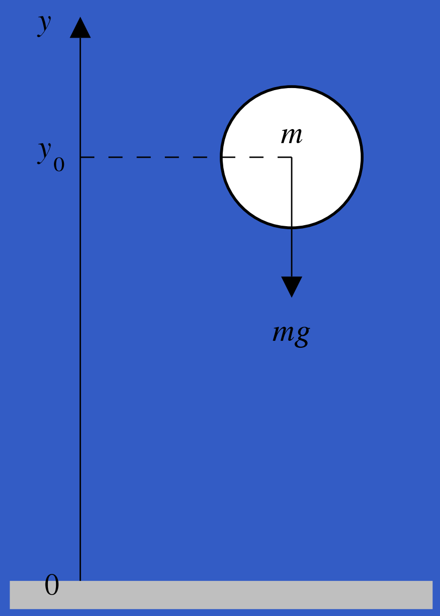
  time :: SF a Time
  time = constant 1.0 >>> integral
  ```
- ```
  (^<<) :: (b->c) -> SF a b -> SF a c
  f (^<<) sf = sf >>> arr f
  ```
  ```

A bouncing ball



$$y = y_0 + \int v \, \mathrm{d}t$$

$$v = v_0 + \int -9.81$$

On impact:

$$v = -v(t-)$$

(fully elastic collision)

Modelling the bouncing ball: part 1

Free-falling ball:

```
type Pos = Double
type Vel = Double

fallingBall ::
    Pos -> Vel -> SF () (Pos, Vel)
fallingBall y0 v0 = proc () -> do
    v <- (v0 +) ^<< integral -< -9.81
    y <- (y0 +) ^<< integral -< v
    returnA -< (y, v)
```

Events

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```
data Event a = NoEvent | Event a
```

Discrete-time signal $=$ `Signal` (`Event` α).

Associating information with an event occurrence:

```
tag :: Event a -> b -> Event b
```

Some basic event sources

- `never :: SF a (Event b)`
- `now :: b -> SF a (Event b)`
- `after :: Time -> b -> SF a (Event b)`
- ```
  repeatedly ::
          Time -> b -> SF a (Event b)
  ```
- `edge :: SF Bool (Event ())`
  ```

Modelling the bouncing ball: part 2

Detecting when the ball goes through the floor:

```
fallingBall' ::
    Pos -> Vel
    -> SF () ((Pos,Vel), Event (Pos,Vel))
fallingBall' y0 v0 = proc () -> do
    yv@(y, _) <- fallingBall y0 v0 -< ()
    hit       <- edge              -< y <= 0
    returnA -< (yv, hit `tag` yv)
```

Switching

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- The new signal function instance often replaces the previously running instance.

Switchers thus allow systems with ***varying structure*** to be described.

The basic switch

Idea:

- Allows one signal function to be replaced by another.
- Switching takes place on the first occurrence of the switching event source.

```
switch ::
    SF a (b, Event c)
   -> (c -> SF a b)
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Initial SF with event source

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```

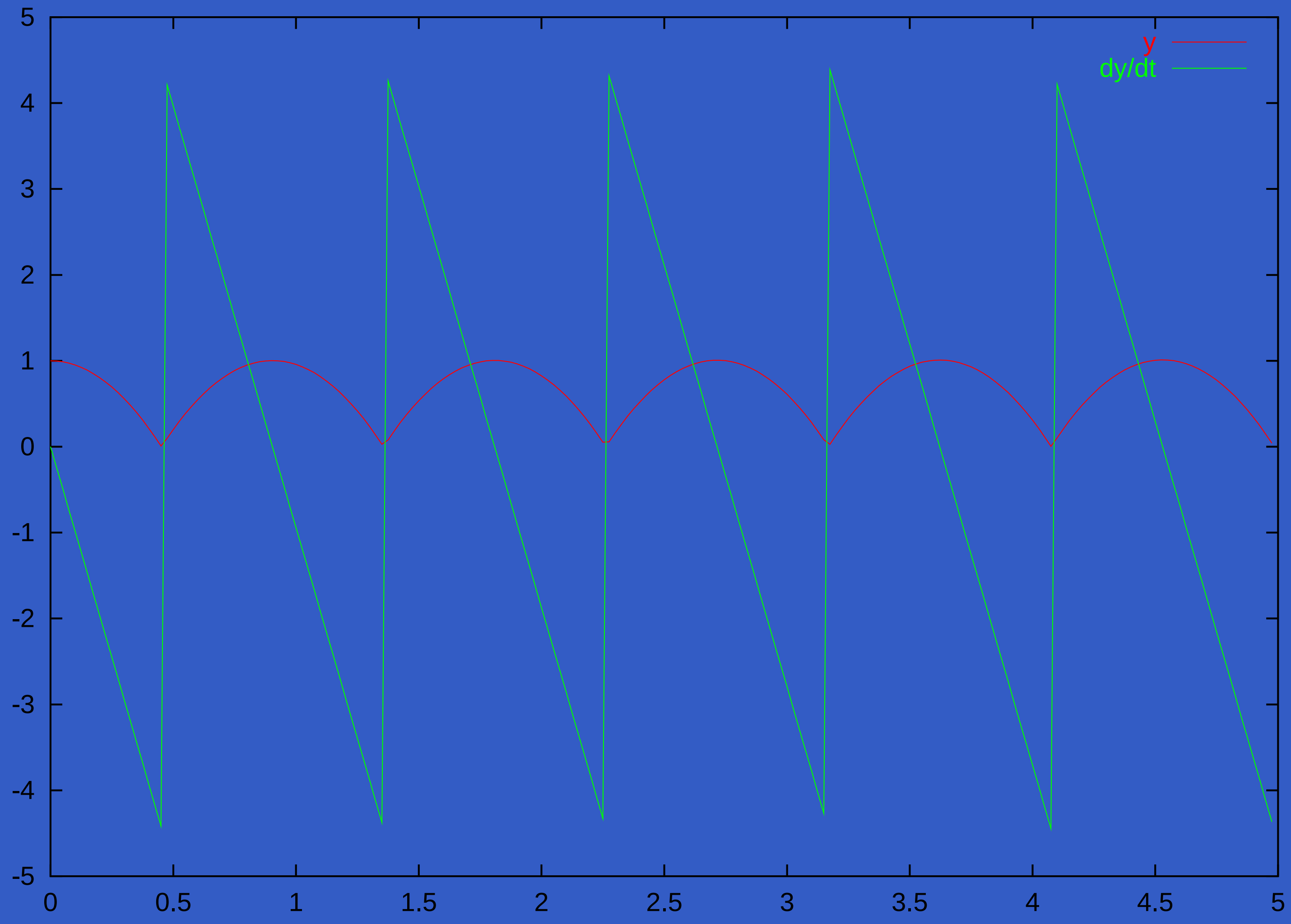
Function yielding SF to switch into

Modelling the bouncing ball: part 3

Making the ball bounce:

```
bouncingBall :: Pos -> SF () (Pos, Vel)
bouncingBall y0 = bbAux y0 0.0
  where
    bbAux y0 v0 =
      switch (fallingBall' y0 v0) $ \(y,v) ->
      bbAux y (-v)
```

Simulation of bouncing ball



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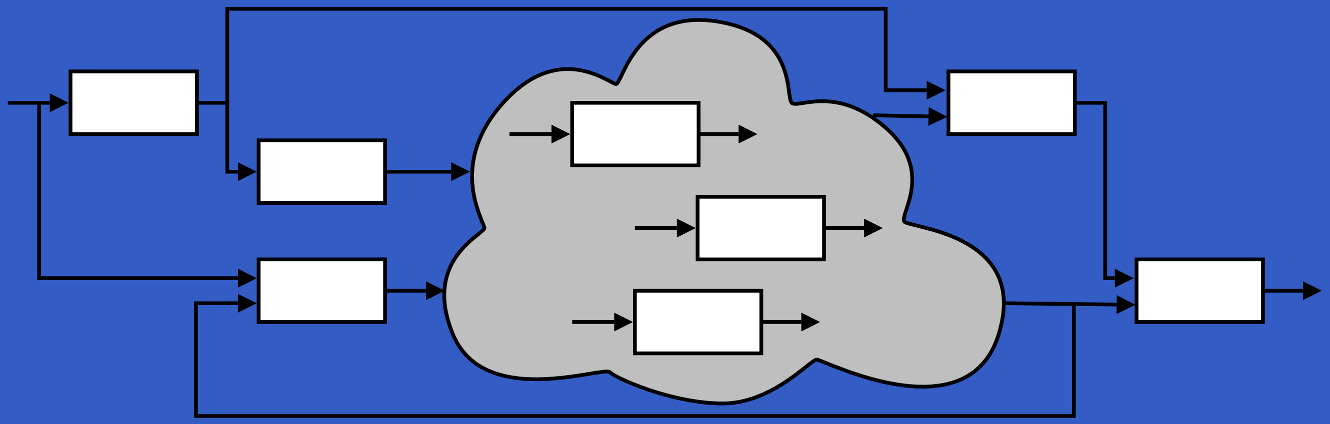
- What about more general structural changes?



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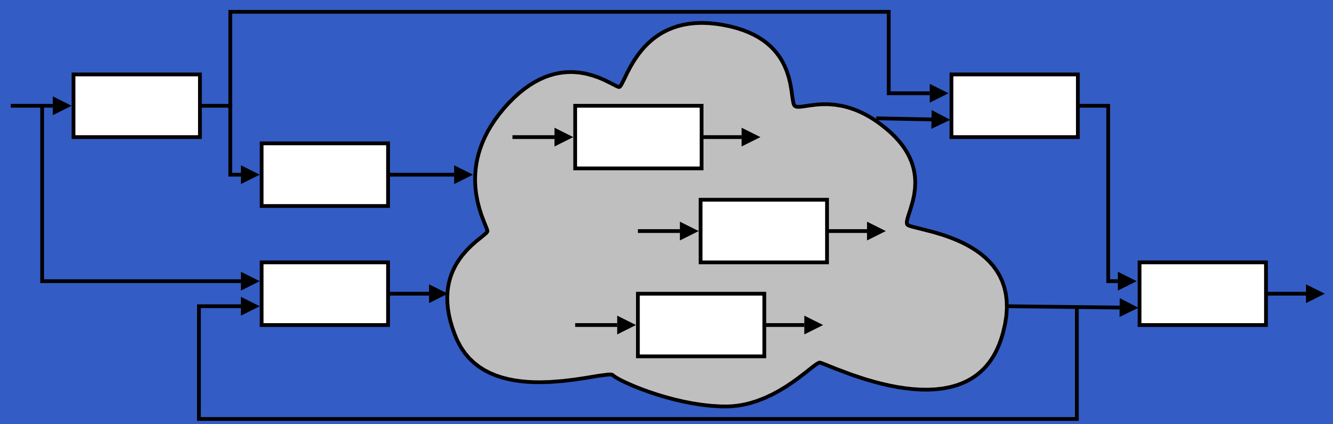
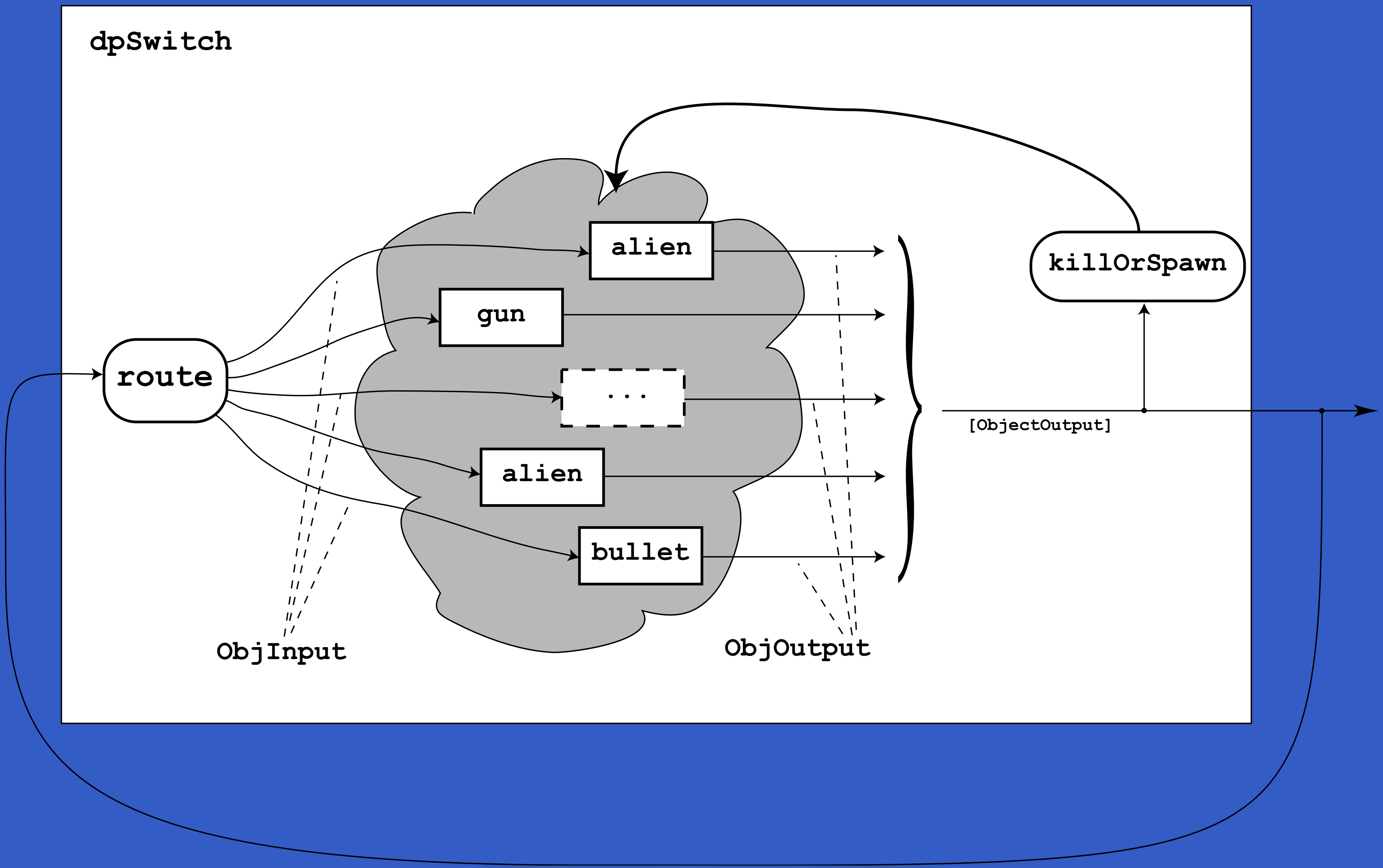


- What about state?

Example: Space Invaders

Overall game structure



Dynamic signal function collections

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- Switch over ***collections*** of signal functions.
- On event, “freeze” running signal functions into collection of signal function ***continuations***, preserving encapsulated ***state***.
- Modify collection as needed and switch back in.

dpSwitch

Need ability to express:

- How input routed to each signal function.
- When collection changes shape.
- How collection changes shape.

```
dpSwitch :: Functor col =>
    (forall sf . (a -> col sf -> col (b,sf)))
   ->  col (SF b c)
   ->  SF (a, col c) (Event d)
   ->  (col (SF b c) -> d -> SF a (col c))
   -> SF a (col c)
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Routing function

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```
dpSwitch :: Functor col =>
    (forall sf . (a -> col sf -> col (b,sf)))
    -> col (SF b c)        <- Initial collection
    -> SF (a, col c) (Event d)
    -> (col (SF b c) -> d -> SF a (col c))
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Event source

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Function yielding SF to switch into

Describing the alien behavior (1)

```
type Object = SF ObjInput ObjOutput

alien :: RandomGen g =>
  g -> Position2 -> Velocity -> Object
alien g p0 vyd = proc oi -> do
  rec
    -- Pick a desired horizontal position
    rx   <- noiseR (xMin, xMax) g -< ()
    smpl <- occasionally g 5 ()   -< ()
    xd   <- hold (point2X p0) -< smpl ‘tag‘ rx
    ...
```

Describing the alien behavior (2)

```
...
-- Controller
let axd = 5 * (xd - point2X p)
              - 3 * (vector2X v)
    ayd = 20 * (vyd - (vector2Y v))
    ad  = vector2 axd ayd
    h   = vector2Theta ad
...
```

Describing the alien behavior (3)

```
...
-- Physics
let a = vector2Polar
            (min alienAccMax
                 (vector2Rho ad))
            h
vp  <- iPre v0    -< v
ffi <- forceField -< (p, vp)
v   <- (v0 ^+^) ^<< impulseIntegral
          -< (gravity ^+^ a, ffi)
p   <- (p0 .+^) ^<< integral -< v
...
```

Describing the alien behavior (4)

```
    ...
    -- Shields
    sl  <- shield -< oiHit oi
    die <- edge   -< sl <= 0

returnA -< ObjOutput {
               ooObsObjState = oosAlien p h v,
               ooKillReq     = die,
               ooSpawnReq    = noEvent
             }
where
    v0 = zeroVector
```

Other functional approaches?

Transition function operating on world model with explicit state (e.g. Asteroids by Lüth):

- Model snapshot of world with ***all*** state components.
- Transition function takes input and current world snapshot to output and the next world snapshot.

One could also use this technique ***within*** Yampa to avoid switching over dynamic collections.

Why use Yampa, then?

- Yampa provides a lot of functionality for programming with time-varying values:
  - Captures common patterns.
  - Carefully designed to facilitate reuse.
- Yampa allows state to be nicely encapsulated by signal functions:
  - Avoids keeping track of all state globally.
  - Adding more state usually does not imply any major changes to type or code structure.

State in `alien`

Each of the following signal functions used in `alien` encapsulate state:

- `noiseR`
- `occasionally`
- `hold`
- `iPre`
- `forceField`
- `impulseIntegral`
- `integral`
- `shield`
- `edge`

Why not imperative, then?

If state is so important, why not stick to imperative/object-oriented programming where we have “state for free”?

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  - High abstraction level.
  - Referential transparency, algebraic laws: formal reasoning ought to be simpler.

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If state is so important, why not stick to imperative/object-oriented programming where we have “state for free”?

- Advantages of declarative programming retained:
  - High abstraction level.
  - Referential transparency, algebraic laws: formal reasoning ought to be simpler.
- Synchronous approach avoids “event-call-back soup”, meaning robust, easy-to-understand semantics.

Obtaining Yampa

Yampa 0.92 is available from

`http://www.haskell.org/yampa`

Reading

- John Hughes. Generalising monads to arrows. *Science of Computer Programming*, 37:67–111, May 2000
- John Hughes. Programming with arrows. In *Advanced Functional Programming*, 2004. To be published by Springer Verlag.
- Henrik Nilsson, Antony Courtney, and John Peterson. Functional reactive programming, continued. In *Proceedings of the 2002 Haskell Workshop*, pp. 51–64, October 2002.

Reading (2)

- Paul Hudak, Antony Courtney, Henrik Nilsson, and John Peterson. Arrows, robots, and functional reactive programming. In *Advanced Functional Programming*, 2002. LNCS 2638, pp. 159–187.
- Tarmo Uustalu and Varmo Vene. The Essence of Dataflow Programming. 2005