MGS 2012: FUN Lecture 4

More about Monads

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This Lecture

- Monads in Haskell
- Some standard monads
- Combining effects: monad transformers

Monads in Haskell

In Haskell, the notion of a monad is captured by a ***Type Class***:

```
class Monad m where
        return :: a -> m a
        (>>=)  :: m a -> (a -> m b) -> m b
```

Allows names of the common functions to be overloaded and sharing of derived definitions.

The Maybe Monad in Haskell

```
instance Monad Maybe where
    -- return :: a -> Maybe a
    return = Just

    -- (>>=) :: Maybe a -> (a -> Maybe b)
    --           -> Maybe b
    Nothing  >>= _ = Nothing
    (Just x) >>= f = f x
```

Exercise 1: A State Monad in Haskell

Haskell 2010 does not permit type synonyms to be instances of classes. Hence we have to define a new type:

```
newtype S a = S (Int -> (a, Int))

unS :: S a -> (Int -> (a, Int))
unS (S f) = f
```

Provide a `Monad` instance for S.

Exercise 1: Solution

```
instance Monad S where
    return a = S (\s -> (a, s))

    m >>= f = S $ \s ->
        let (a, s’) = unS m s
        in unS (f a) s’
```

Monad-specific Operations (1)

To be useful, monads need to be equipped with additional operations specific to the effects in question. For example:

```
fail :: String -> Maybe a
fail s = Nothing


catch :: Maybe a -> Maybe a -> Maybe a
m1 ‘catch‘ m2 =
    case m1 of
        Just _  -> m1
        Nothing -> m2
```

Monad-specific Operations (2)

Typical operations on a state monad:

```
set :: Int -> S ()
set a = S (\_ -> ((), a))

get :: S Int
get = S (\s -> (s, s))
```

Moreover, need to “run” a computation. E.g.:

```
runS :: S a -> a
runS m = fst (unS m 0)
```

The do-notation (1)

Haskell provides convenient syntax for programming with monads:

```
do
     a <- exp₁
     b <- exp₂
     return exp₃
```

is syntactic sugar for

```
exp₁ >>= \a ->
exp₂ >>= \b ->
return exp₃
```

The do-notation (2)

Computations can be done solely for effect, ignoring the computed value:

do

exp_1

exp_2

`return` exp_3

is syntactic sugar for

exp_1 `>>= \_ ->`

exp_2 `>>= \_ ->`

`return` exp_3

The do-notation (3)

A `let`-construct is also provided:

```
do
    let a = $exp_1$
        b = $exp_2$
    return $exp_3$
```

is equivalent to

```
do
    a <- return $exp_1$
    b <- return $exp_2$
    return $exp_3$
```

Numbering Trees in do-notation

```
numberTree :: Tree a -> Tree Int
numberTree t = runS (ntAux t)
    where
        ntAux :: Tree a -> S (Tree Int)
        ntAux (Leaf _) = do
            n <- get
            set (n + 1)
            return (Leaf n)
        ntAux (Node t1 t2) = do
            t1’ <- ntAux t1
            t2’ <- ntAux t2
            return (Node t1’ t2’)
```

The Compiler Fragment Revisited (1)

Given a suitable “Diagnostics” monad `D` that collects error messages, `enterVar` can be turned from this:

```
enterVar :: Id -> Int -> Type -> Env
            -> Either Env ErrorMgs
```

into this:

```
enterVarD :: Id -> Int -> Type -> Env
             -> D Env
```

(Suffix “D” just to remind us the types have changed.)

The Compiler Fragment Revisited (2)

And then `identDefs` from

```
identDefs ::
    Int -> Env -> [(Id, Type, Exp ())]
    -> ([(Id, Type, Exp Attr)],
          Env,
          [ErrorMsg])
```

into

```
identDefsD ::
    Int -> Env -> [(Id, Type, Exp ())]
    -> D ([(Id, Type, Exp Attr)], Env)
```

with the function definition changing from . . .

The Compiler Fragment Revisited (2)

```
identDefs l env [] = ([], env, [])
identDefs l env ((i,t,e) : ds) =
  ((i,t,e') : ds', env'', ms1++ms2++ms3)
  where
    (e', ms1) = identAux l env e
    (env', ms2) =
         case enterVar i l t env of
            Left env' -> (env', [])
            Right m   -> (env,  [m])
    (ds', env'', ms3) =
       identDefs l env' ds
```

The Compiler Fragment Revisited (3)

into this:

```
identDefsD l env [] = return ([], env)
identDefsD l env ((i,t,e) : ds) = do
    e'             <- identAuxD l env e
    env'           <- enterVarD i l t env
    (ds', env'')   <- identDefsD l env' ds
    return ((i,t,e') : ds', env'')
```

The Compiler Fragment Revisited (4)

Compare with the “core” identified earlier!

```
identDefs l env [] = ([], env)
identDefs l env ((i,t,e) : ds) =
  ((i,t,e’) : ds’, env’’)
  where
    e’           = identAux l env e
    env’         = enterVar i l t env
    (ds’, env’’) = identDefs l env’ ds
```

The monadic version is very close to this “ideal”, without sacrificing functionality, clarity, or pureness!

The List Monad

Computation with many possible results, “nondeterminism”:

```
instance Monad [] where
    return a = [a]
    m >>= f  = concat (map f m)
    fail s   = []
```

Example:

```
x <- [1, 2]
y <- ['a', 'b']
return (x,y)
```

Result:

```
[(1,'a'),(1,'b'),
 (2,'a'),(2,'b')]
```

The Reader Monad

Computation in an environment:

```
instance Monad ((->) e) where
    return a = const a
    m >>= f  = \e -> f (m e) e

getEnv :: ((->) e) e
getEnv = id
```

The Haskell IO Monad

In Haskell, IO is handled through the IO monad. IO is ***abstract***! Conceptually:

```
newtype IO a = IO (World -> (a, World))
```

Some operations:

```
putChar     :: Char -> IO ()
putStr      :: String -> IO ()
putStrLn    :: String -> IO ()
getChar     :: IO Char
getLine     :: IO String
getContents :: String
```

Monad Transformers (1)

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For example: State and Error/Partiality?

We could implement a suitable monad from scratch:

```
newtype SE s a = SE (s -> Maybe (a, s))
```

Monad Transformers (2)

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- Not always obvious how: e.g., should the combination of state and error have been

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- Not always obvious how: e.g., should the combination of state and error have been

```
newtype SE s a = SE (s -> (Maybe a, s))
```

- Duplication of effort: similar patterns related to specific effects are going to be repeated over and over in the various combinations.

Monad Transformers (3)

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- A library of monad transformers can be developed, each adding a specific effect (state, error, . . .), allowing the programmer to mix and match.

Monad Transformers (3)

Monad Transformers can help:

- A ***monad transformer*** transforms a monad by adding support for an additional effect.
- A library of monad transformers can be developed, each adding a specific effect (state, error, . . .), allowing the programmer to mix and match.
- A form of ***aspect-oriented programming***.

Monad Transformers in Haskell (1)

- A ***monad transformer*** maps monads to monads. Represented by a type constructor T of the following kind:

```
T :: (* -> *) -> (* -> *)
```

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- A ***monad transformer*** maps monads to monads. Represented by a type constructor T of the following kind:

```
T :: (* -> *) -> (* -> *)
```

- Additionally, a monad transformer ***adds*** computational effects. A mapping `lift` from computations in the underlying monad to computations in the transformed monad is needed:

```
lift :: M a -> T M a
```

Monad Transformers in Haskell (2)

- These requirements are captured by the following (multi-parameter) type class:

```
class (Monad m, Monad (t m))
      => MonadTransformer t m where
    lift :: m a -> t m a
```

Classes for Specific Effects

A monad transformer adds specific effects to ***any*** monad. Thus the effect-specific operations needs to be overloaded. For example:

```
class Monad m => E m where
    eFail :: m a
    eHandle :: m a -> m a -> m a

class Monad m => S m s | m -> s where
    sSet :: s -> m ()
    sGet :: m s
```

The Identity Monad

We are going to construct monads by successive transformations of the identity monad:

```
newtype I a = I a
unI (I a) = a

instance Monad I where
    return a = I a
    m >>= f = f (unI m)

runI :: I a -> a
runI = unI
```

The Error Monad Transformer (1)

```
newtype ET m a = ET (m (Maybe a))
unET (ET m) = m
```

Any monad transformed by ET is a monad:

```
instance Monad m => Monad (ET m) where
    return a = ET (return (Just a))

    m >>= f = ET $ do
        ma <- unET m
        case ma of
            Nothing -> return Nothing
            Just a  -> unET (f a)
```

The Error Monad Transformer (2)

We need the ability to run transformed monads:

```
runET :: Monad m => ET m a -> m a
runET etm = do
    ma <- unET etm
    case ma of
        Just a  -> return a
        Nothing -> error "Should not happen"
```

ET is a monad transformer:

```
instance Monad m =>
         MonadTransformer ET m where
    lift m = ET (m >>= \a -> return (Just a))
```

The Error Monad Transformer (3)

Any monad transformed by ET is an instance of E:

```
instance Monad m => E (ET m) where
    eFail = ET (return Nothing)
    m1 ‘eHandle‘ m2 = ET $ do
        ma <- unET m1
        case ma of
            Nothing -> unET m2
            Just _  -> return ma
```

The Error Monad Transformer (4)

A state monad transformed by ET is a state monad:

```
instance S m s => S (ET m) s where
    sSet s = lift (sSet s)
    sGet = lift sGet
```

Exercise 2: Running Transf. Monads

Let

```
ex2 = eFail ‘eHandle‘ return 1
```

1. Suggest a possible type for `ex2`. (Assume `1 :: Int`.)
2. Given your type, use the appropriate combination of “run functions” to run `ex2`.

Exercise 2: Solution

```
ex2 :: ET I Int
ex2 = eFail ‘eHandle‘ return 1

ex2result :: Int
ex2result = runI (runET ex2)
```

The State Monad Transformer (1)

```
newtype ST s m a = ST (s -> m (a, s))
unST (ST m) = m
```

Any monad transformed by ST is a monad:

```
instance Monad m => Monad (ST s m) where
    return a = ST (\s -> return (a, s))

    m >>= f = ST $ \s -> do
        (a, s’) <- unST m s
        unST (f a) s’
```

The State Monad Transformer (2)

We need the ability to run transformed monads:

```
runST :: Monad m => ST s m a -> s -> m a
runST stf s0 = do
    (a, _) <- unST stf s0
    return a
```

ST is a monad transformer:

```
instance Monad m =>
         MonadTransformer (ST s) m where
    lift m = ST (\s -> m >>= \a ->
                          return (a, s))
```

The State Monad Transformer (3)

Any monad transformed by ST is an instance of S:

```
instance Monad m => S (ST s m) s where
    sSet s = ST (\_ -> return ((), s))
    sGet   = ST (\s -> return (s, s))
```

An error monad transformed by ST is an error monad:

```
instance E m => E (ST s m) where
    eFail = lift eFail
    m1 ‘eHandle‘ m2 = ST $ \s ->
        unST m1 s ‘eHandle‘ unST m2 s
```

Exercise 3: Effect Ordering

Consider the code fragment

```
ex3a :: (ST Int (ET I)) Int
ex3a = (sSet 42 >> eFail) ‘eHandle‘ sGet
```

Note that the exact same code fragment also can be typed as follows:

```
ex3b :: (ET (ST Int I)) Int
ex3b = (sSet 42 >> eFail) ‘eHandle‘ sGet
```

What is

```
runI (runET (runST ex3a 0))
runI (runST (runET ex3b) 0)
```

Exercise 3: Solution (1)

`runI (runET (runST ex3a 0))` = ***???***

`runI (runST (runET ex3b) 0)` = ***???***

Exercise 3: Solution (1)

```
runI (runET (runST ex3a 0)) = 0
runI (runST (runET ex3b) 0) = ???
```

Exercise 3: Solution (1)

`runI (runET (runST ex3a 0))` $=$ `0`

`runI (runST (runET ex3b) 0)` $=$ `42`

Exercise 3: Solution (1)

```
runI (runET (runST ex3a 0)) = 0
runI (runST (runET ex3b) 0) = 42
```

Why?

Exercise 3: Solution (1)

```
runI (runET (runST ex3a 0)) = 0
runI (runST (runET ex3b) 0) = 42
```

Why? Because:

```
ST s (ET I) a ≅ s -> (ET I) (a, s)
              ≅ s -> I (Maybe (a, s))
              ≅ s -> Maybe (a, s)
ET (ST s I) a ≅ (ST s I) (Maybe a)
              ≅ s -> I (Maybe a, s)
              ≅ s -> (Maybe a, s)
```

Exercise 3: Solution (2)

Note that

$$\texttt{ET (ST s I) a} \cong \texttt{s -> (Maybe a, s)}$$

results in a notion of a ***shared, global*** state, while

$$\texttt{ST s (ET I) a} \cong \texttt{s -> Maybe (a, s)}$$

has a ***transactional*** flavour: only if a computation succeeds will any effects from that computation be taken into account.

Both are natural and useful; hence there is no “right” or “wrong” ordering.

Exercise 4: Alternative ST?

To think about.

Could ST have been defined in some other way, e.g.

```
newtype ST s m a = ST (m (s -> (a, s)))
```

or perhaps

```
newtype ST s m a = ST (s -> (m a, s))
```

Problems with Monad Transformers

- With one transformer for each possible effect, we get a lot of combinations: the number grows quadratically; each has to be instantiated explicitly.
- Jaskelioff (2008,2009) has proposed a possible, more extensible alternative.

Reading (1)

- Nick Benton, John Hughes, Eugenio Moggi. Monads and Effects. In *International Summer School on Applied Semantics 2000*, Caminha, Portugal, 2000.
- Sheng Liang, Paul Hudak, Mark Jones. Monad Transformers and Modular Interpreters. In *Proceedings of the 22nd ACM Symposium on Principles of Programming Languages (POPL'95)*, January 1995, San Francisco, California

Reading (2)

- Mauro Jaskelioff. Monatron: An Extensible Monad Transformer Library. In *Implementation of Functional Languages (IFL'08)*, 2008.
- Mauro Jaskelioff. Modular Monad Transformers. In *European Symposium on Programming (ESOP'09)*, 2009.