Functional Hybrid Modeling from an Object-Oriented Perspective

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Background (1)

- ***Functional Reactive Programming*** (FRP) integrates notions suitable for ***causal*** hybrid modelling with functional programming.
- ***Yampa*** is an instance of FRP embedded in Haskell.
- One central idea: ***first-class*** reactive components (or models):
  - enables highly structurally dynamic systems to be described declaratively;
  - opens up for meta-modelling without additional language layers.

Background (2)

- Additional interesting aspects:
  - full power of a modern functional language available;
  - polymorphic type system;
  - well-understood underlying semantics.

Functional Hybrid Modelling (1)

- Our goal with ***Functional Hybrid Modelling*** (FHM) is to combine an FRP-approach with non-causal modelling yielding:
  - a powerful, fully-declarative, non-causal modelling language supporting highly structurally dynamic systems;
  - a semantic framework for studying modelling and simulation languages supporting structural dynamism.

Functional Hybrid Modelling (2)

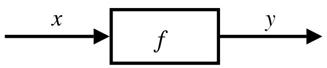
- The idea of FHM goes back a few years (PADL 2003). UK research funding (EPSRC) secured very recently. Thus still work in very early stages.

The Rest of the Talk

- A brief introduction to FRP/Yampa as a background.
- Sketch the key ideas of how this may be generalized to a non-causal setting.

Signal functions

Key concept: ***functions on signals*** (first class).



Intuition:

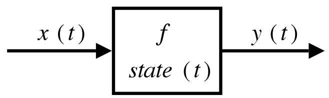
Signal $\alpha \approx$ Time $\rightarrow \alpha$
x :: Signal T1
y :: Signal T2
SF α β $\approx$ Signal α $\rightarrow$ Signal β
f :: SF T1 T2

Additionally, ***causality*** required: output at time t must be determined by input on interval $[0, t]$.

Signal functions and state

Alternative view:

Signal functions can encapsulate ***state***.



$state(t)$ summarizes input history $x(t')$, $t' \in [0, t]$.

From this perspective, signal functions are:

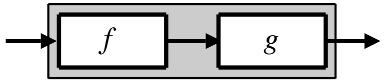
- ***stateful*** if $y(t)$ depends on $x(t)$ and $state(t)$
- ***stateless*** if $y(t)$ depends only on $x(t)$

Integral is an example of a stateful signal function.

Programming with signal functions

In Yampa, systems are described by combining signal functions (forming new signal functions).

For example, serial composition:



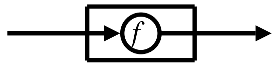
A *combinator* can be defined that captures this:

$$(\ggg) :: SF\ a\ b \rightarrow SF\ b\ c \rightarrow SF\ a\ c$$

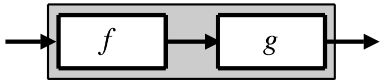
Note: plain function operating on first-class signal function.

The Arrow framework (1)

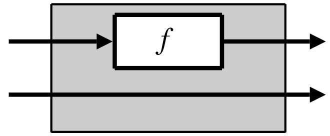
These diagrams convey the general idea:



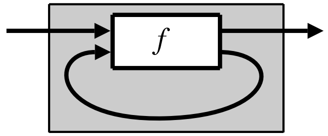
$arr\, f$



$\ggg$



$first\, f$

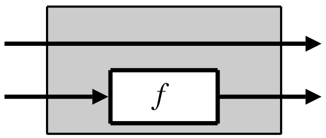


$loop\, f$

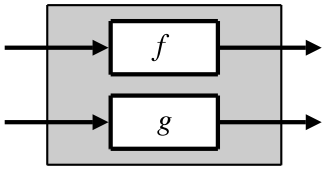
$$first :: SF\ a\ b \rightarrow SF\ (a, c)\ (b, c)$$
$$loop :: SF\ (a, c)\ (b, c) \rightarrow SF\ a\ b$$

The Arrow framework (2)

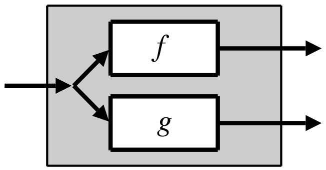
Some derived combinators:



$second\, f$



$f *** g$



$f \&\&\& g$

Example: Constructing a network

The Arrow notation

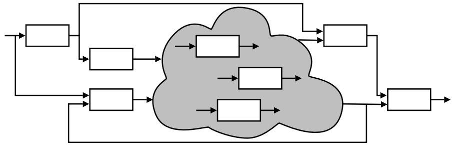
Switching

Some switching combinators:

- $switch :: SF\ a\ (b, Event\ c) \rightarrow (c \rightarrow SF\ a\ b) \rightarrow SF\ a\ b$
- $pSwitchB :: Functor\ col \Rightarrow col\ (SF\ a\ b) \rightarrow SF\ (a, col\ b)\ (Event\ c) \rightarrow (col\ (SF\ a\ b) \rightarrow c \rightarrow SF\ a\ (col\ b)) \rightarrow SF\ a\ (col\ b)$

What makes Yampa different?

- First class reactive components (signal functions).
- Supports hybrid (mixed continuous and discrete time) systems: option type $Event$ represents discrete-time signals.
- Supports dynamic system structure through ***switching combinators***:



Example: Space Invaders

Functional Hybrid Modeling

Same conceptual structure as Yampa, but:

- First-class ***relations*** on signals instead of functions on signals to enable non-causal modeling.
- Employ state-of-the-art symbolic and numerical methods for sound and efficient simulation.
- Adapted switch constructs.

First class signal relations

The type for a relation on a signal of type `Signal` α:

SR α

Specific relations use a more refined type; e.g. the derivative relation:

der :: SR (`Real`, `Real`)

Since a signal carrying pairs is isomorphic to a pair of signals, der can be understood as a binary relation on two signals.

Defining relations

The following tentative construct denotes a signal relation:

sigrel $pattern$ **where** $equations$

The pattern introduces ***signal variables*** which at each point in time are going to be bound to to a "sample" of the corresponding signal.

Given $p :: t$, we have:

sigrel p **where** $\ldots$ :: SR t

Equations

Let $e_i :: t_i$ be non-relational expressions possibly introducing new signal variables.

Point-wise equality; the equality must hold for all points in time:

$e_1 = e_2$

Relation "application"; the relation must hold for all points in time:

$sr \diamond e_3$

Here, sr is an ***expression*** having type SR t_3.

Equations: examples

Consider a differential equation like $x' = f(x, y)$. This equation could be written:

$$der \diamond (x, f(x, y))$$

For convenience, ***syntactic sugar*** closer to standard mathematical notation could be considered:

$$\textbf{der}(x) = f(x, y)$$

Here, **der** is ***not*** a pure function operating only on instantaneous signal values since it depends on the history of the signal.

Modeling electrical components (1)

The type `Pin` is assumed to be a record type describing an electrical connection. It has fields v for voltage and i for current.

$$twoPin :: \text{SR}\ (\texttt{Pin}, \texttt{Pin}, \texttt{Voltage})$$
$$twoPin = \textbf{sigrel}\ (p, n, v)\ \textbf{where}$$
$$v = p.v - n.v$$
$$p.i + n.i = 0$$

Modeling electrical components (2)

$$resistor :: \texttt{Resistance} \rightarrow \text{SR}\ (\texttt{Pin}, \texttt{Pin})$$
$$resistor(r) = \textbf{sigrel}\ (p, n)\ \textbf{where}$$
$$twoPin \diamond (p, n, v)$$
$$r \cdot p.i = v$$
$$inductor :: \texttt{Inductance} \rightarrow \text{SR}\ (\texttt{Pin}, \texttt{Pin})$$
$$inductor(l) = \textbf{sigrel}\ (p, n)\ \textbf{where}$$
$$twoPin \diamond (p, n, v)$$
$$l \cdot \textbf{der}(p.i) = v$$

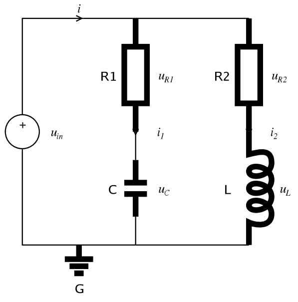
Modeling electrical components (3)

$$capacitor :: \texttt{Capacitance} \rightarrow \text{SR}\ (\texttt{Pin}, \texttt{Pin})$$
$$capacitor(c) = \textbf{sigrel}\ (p, n)\ \textbf{where}$$
$$twoPin \diamond (p, n, v)$$
$$c \cdot \textbf{der}(v) = p.i$$

Modeling an electrical circuit (1)

$$
\begin{aligned}
&simpleCircuit :: \texttt{SR Current} \\
&simpleCircuit = \textbf{sigrel}\ \text{i}\ \textbf{where} \\
&\qquad resistor(1000) \diamond (r1p, r1n) \\
&\qquad resistor(2200) \diamond (r2p, r2n) \\
&\qquad capacitor(0.00047) \diamond (cp, cn) \\
&\qquad inductor(0.01) \diamond (lp, ln) \\
&\qquad vSourceAC(12) \diamond (acp, acn) \\
&\qquad ground \diamond gp \\
&\qquad \ldots
\end{aligned}
$$

Modeling an electrical circuit (2)



$$
\begin{aligned}
&\ldots \\
&\textbf{connect}\ acp, r1p, r2p \\
&\textbf{connect}\ r1n, cp \\
&\textbf{connect}\ r2n, lp \\
&\textbf{connect}\ acn, cn, ln, gp \\
&i = r1p.i + r2p.i
\end{aligned}
$$

Central Research Questions

- Adaptating Yampa's switching constructs, including handling initialization issues.
- Adapting non-causal modelling and simulation methods to a setting with first class signal relations: causality analysis, symbolic processing code generation after each switch.
- Guaranteeing compositional correctness statically through the type system to the extent possible; e.g. employing dependent types to keep track of variable/equation balance.