Exercise 8:

1 Define

$$id2iso : \{A\,B : Type\} \to A = B \to Iso\ A\,B$$

2 Axiom : $\{A\,B : Type\} \longrightarrow isEqv(id2iso_{AB})$

$$id2iso : \{A\ B : Type\} \to A = B \to Iso\ A\,B$$

$$id2iso_{A\,A}\ refl :\equiv$$

$$(\lambda a.a,\ \lambda a.a,\ \lambda a.refl,\ \lambda a.refl)$$

$$Iso\ A\,B :\equiv \sum f : A \to B.$$
$$\Sigma\ g : B \to A.$$
$$\Sigma\ \alpha : (a:A) \to g(f\,a) = a$$
$$\beta : (b:B) \to f(g\,b) = b$$

$$\begin{tikzcd}
A = B \arrow[r, "id2eqv"] \arrow[dr, "id2iso"'] & A \simeq B \ (f,g,\alpha,\beta,h) \arrow[d, "\pi"] \\
& \mathrm{Iso}\ A\ B \ (f,g,\alpha,\beta)
\end{tikzcd}$$

$$(p : A = B) \to id2iso(p) = \pi(\,id2eqv(p))$$

$$refl \longmapsto refl$$

[notes on modelling cat semantics :

type-theoretic fibration categories

or Categories w/ families

or Cat's w/ attributes

one particular model :

model types as ∞-groupoids (eg Kan complexes)

then paths will be 1-cells

Kapulkin - Lumsdaine

"The simplicial model of univalent foundations (after Voevodsky)"

Task: Find a non-coherent iso, ie find

$A, B, e : \mathrm{Iso}\ A\ B$

st e is not in the image of

$\pi : A \simeq B \to \mathrm{Iso}\ A\ B$

$P : A \to \mathrm{Prop}$

$$\begin{tikzcd} \Sigma(a:A).\,Pa \arrow[d] \\ A \end{tikzcd}$$

first step: need type with "interesting" higher structure

simple example of higher ind. type:

data S^1 where
  base : S^1
  loop : base = base

Can show that loop $\neq$ refl

data Nat where
  zero : Nat
  suc : Nat $\to$ Nat

data List A where
  nil : List A
  cons : A $\to$ List A $\to$ List A

data S^1 where

  base : S^1

  loop : base = base

$\mathrm{id} : S^1 \to S^1$ $\qquad \lambda x . x$

~~id~~ :

non-coh iso:

$f :\equiv \mathrm{id}$

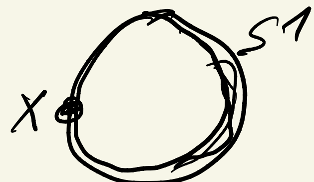
$g :\equiv \mathrm{id}$

$\alpha : (x : S^1) \to \mathrm{id}(\mathrm{id}(x)) = x$

$\alpha\ x :\equiv \mathrm{refl}$

$\beta : (x : S^1) \to x = x$

$\beta\ x :\equiv \text{gen-once}\ x\ \mathrm{refl}$



coh: $\alpha\ x = \beta\ x$

~~ind~~ recursion principle of S^1:

$\mathrm{elim} : \{A : \mathrm{Typ}\}$

$\to (a_0 : A)$

$\to (p : a_0 = a_0)$

$\to S^1 \to A$

st $\mathrm{elim}\ a_0\ \_\ \mathrm{base} \equiv a_0$

$\mathrm{ap}_{\mathrm{elim}\ \cancel{a_0}\ a_0}\ \mathrm{loop} = p$

once : base = base $\to$ base = base

$p \longmapsto p \cdot \mathrm{loop}$

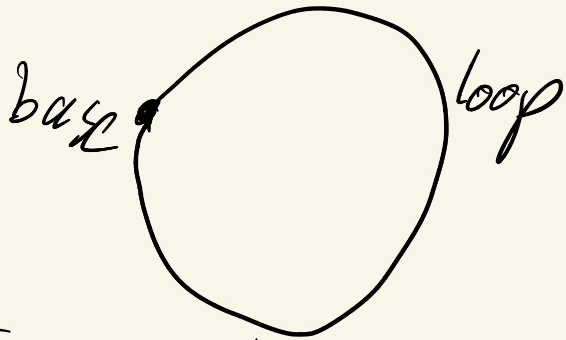
gen-once :

$(x : S^1) \to x = x \longrightarrow x = x$

$p \longmapsto \mathrm{elim}\ \mathrm{once}\ \text{"refl"}$

this is very hand-wavy. Use HoTT book Lem 6.4.2

S^1:



data Torus where

base : Torus

p : base = base

q : base = base

t : $p \cdot q = q \cdot p$

result : Torus $\simeq S^1 \times S^1$