Formal Models of Social Influence and Conflict

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Abstract. Social influence is the process in which an agent is under pressure to form her opinion on an issue based on the opinions expresses by her peers. An obvious reaction to social influence is to change ones opinions to conform to the pressure. Social influence has been formally studied as a method that helps information spread in a network. However, social influence can also be a force that changes the relations between agents, not only their opinions. We are interested in pursuing this comparatively under-explored aspect of social influence by formally modelling social influence. Within our framework different models of social influence can be defined that are not by design conflict free.

1 Introduction

Consider a society of individuals that perpetually expresses their opinions on various issues. Who "listens" to whom in this society is described by a network and everyone in this society knows who "listens" to whom. To "listen" is taken as an indication of agreeing with the opinion being expressed. How do individuals in such a society influence each-other's opinions and behaviour is a question that has traditionally been studied by researchers in social science [5, 9, 13, 1, 6, 12].

Recently the problem of social influence has also drawn the attention of researchers in artificial intelligence and multi-agent systems. The works [11, 3, 4, 8, 2], for example, are making some important contributions towards advancing the state of the art in social influence research. The motivation for this development can easily be found in the increased distributivity of computation and the ubiquity of social network on-line services as a medium for commerce and broadcasting.

Social influence plays a substantial role in several phenomena that occur in social networks. In [11, 3, 4] it is studied how social influence is affecting the opinions and beliefs of the agents in the network. In [2] an epistemic dimension of influence is introduced to model the limitations of agents to have a complete knowledge of who "listens" to whom in a network. In [7] the accent is on models of strategic reasoning in situations of social influence. Studies such as [11, 3, 4, 2, 7] use logic based models of social influence which allow for agent based reasoning to be analysed in a social network setting. In all of these works, social networks are static and it is the private and public opinions of the networked agents that change.

There are three proposed logic-based models of social influence which can be encountered in the literature, two of which such that a conflicting social influence cannot be exerted on an agent. An agent can either be under social influenced to adopt an opinion about an issue or not. However, it cannot happen that an agent is both influenced to adopt opinion φ and an opinion $\neg\varphi$ at the same time, nor that the issues are logically related. Although the threshold model introduced in [2] does allow for a conflicting influence to occur, this is not explicitly considered as a situation of interest. We find that conflicting social influence can occur and it needs to be adequately modelled. Our contribution is to propose social influence models that allow for conflicts to be represented.

In particular, we introduce a new model of influence based on a well known concept in social network analysis called "Simmelian tie" introduced by Krackhardt in [10]. While all existing logic-based social influence models look only to the adjacent agents to determine existence of social influence, our Simmelian model also takes in to account relations among the adjacent agents. Our approach to modelling social influence allows for different models to be directly compared.

2 Models of social influence

Our goal is to formalise and analyse the phenomenon of *social influence*. We intuitively understand social influence to be the pressure on an agent to not express public support for a given issue.

We define N to be a finite, non-empty set of unique agent identifiers. Further we have a finite, non-empty set $\mathcal{I}$ of relevant issues, well formed propositional logic formulas. We use $\mathcal{I}^+$ to denote the consistent set of non-negated formulas in $\mathcal{I}$ and we have $\mathcal{I}^- = \{\neg\varphi | \varphi \in \mathcal{I}^+\}$ with $\mathcal{I}^+ \cap \mathcal{I}^- = \emptyset$. We use $\sim \varphi$ to be $\neg\varphi$ if $\varphi \in \mathcal{I}^+$ and $\sim \varphi$ to be φ' when $\varphi = \neg\varphi'$ otherwise. We model the publicly expressed opinions by a *support function*.

Definition 1 (Support function). *Given a set of agents N and relevant issues $\mathcal{I}$, a support function* $\mathrm{pro} : \mathcal{I} \to 2^N$ *maps every relevant issue to the set of agents which publicly support it. We require that* $\mathrm{pro}(\varphi) \cap \mathrm{pro}(\sim \varphi) = \emptyset$*. For any $S \subset \mathcal{I}$, $\varphi \in \mathcal{I}$, if $S \models \varphi$, and $i \in \mathrm{pro}(\psi)$ for all $\psi \in S$, then $i \in \mathrm{pro}(\varphi)$.*

A social network is typically modelled as a graph in which the nodes are agents and there exists an edge between two agents if there is some form of social relationship between them. Because we regard social networks to be symmetric, irreflexive relations over the agents, we model every edge $e \in E$ as a set $e \subseteq N$ of size two ($|e| = 2$).

Definition 2 (Social network). *Given a set of agents N and a set of relevant issues $\mathcal{I}$, a* social network *is a tuple $G = (N, \mathcal{I}, \mathrm{pro}, E)$ where* pro *is a support function and $E \subseteq 2^N$ is a set of edges, set of sets of agents of size two. Given a social network $G = (N, \mathcal{I}, \mathrm{pro}, E)$ we define the neighbours of agent $i \in N$ to be $n(i) = \{j \in N \mid \{i, j\} \in E\}$. The subset of i's neighbours which support (or "like") an opinion $\varphi \in \mathcal{I}$ is $l(i, \varphi) = n(i) \cap \mathrm{pro}(\varphi)$.*

Intuitively, we model a social network as a node-labelled graph over the agents. When depicting a social network, we label each node with its name $i \in N$ together with the issues she supports. Let G be a social network over four agents $N = \{a, b, c, d\}$ with $E = \{\{a, b\}, \{a, c\}, \{a, d\}\}$. The social network can be depicted as in Figure 1. We see that the agents which support φ are $\mathrm{pro}(\varphi) = \{b, c\}$. The neighbours of a are $n(a) = \{b, c, d\}$ and the neighbours of a which support φ are $l(a, \varphi) = \{b, c\}$.

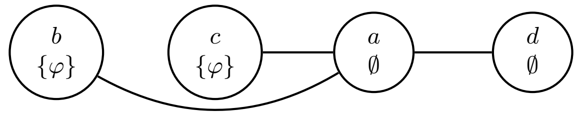

Fig. 1: An example of a social network

Social influence exerted on an agent comes from her neighbours. The influence sources are those subsets of neighbours without which no social influence would exit. Given an agent $i \in N$ and $\varphi \in \mathcal{I}$ we define "sources of influence" as *set of pivotal sets* $\Omega(i, \varphi)$. We use pivotal sets as a proxy to modelling social influence. Intuitively, a pivotal set $A \in \Omega(i, \varphi)$ is a set of agents s.t., if all edges between i and the agents in A were removed, i would no longer experience social influence on φ.

Definition 3 (Social Influence Model). *A* social influence model *for a social network* $G = (N, \mathcal{I}, \mathrm{pro}, E)$ *is an agent indexed family of functions:* $\Omega^i : \mathcal{I} \to \mathcal{P}(\mathcal{P}(n(i)))$*, where* n *is the function identifying the neighbours of agent* i*. We denote* $\Omega^i(\varphi)$ *as* $\Omega(i, \varphi)$*. The set* $\Omega(i, \varphi)$ *is called* the set of pivotal sets for i regarding φ*.*

The idea behind the social influence model is that it defines the presence of social influence by how it could be avoided. That is, when considering a social network G, we need to express the social influence exerted on an agent i by describing the sets of agents with which i needs to stop interacting if she wants to avoid that influence. The *pivotal set* $A \in \Omega(i, \varphi)$ should reflect that *ceteris paribus*, after agent i has dropped all ties to the members of A, i is no longer under pressure to support φ. Further more, the *social model* should contain exactly these pivotal sets. Notice that if $\emptyset \in \Omega(i, \varphi)$, then the agent i can avoid the social influence towards supporting φ not making any changes to the network. Thus we can determine whether influence is exerted upon i regarding φ simply by determining whether or not $\emptyset \in \Omega(i, \varphi)$.

We define four models of social influence, two obtained by generalising existing models from the literature: Threshold social influence (TSI) Ω_t, Opposition Sensitive Threshold social influence (OS-TSI) Ω_o, Accumulated Tolerance Social Influence (ATSI) Ω_a and Simmelian Social Influence (SSI) Ω_s. The TSI and OS-TIS models reflect the case when social influence emerges due to the proportion of neighbours that expressed support for an issue.

Definition 4 (Threshold social influence (TSI)). *Let* G *be a social network with agents* N *and issues* $\mathcal{I}$*. Let* $i \in N$ *be an agent and let* $\varphi \in N$ *be an issue. An influence threshold is a number* $\theta \in [0, 1]$*. The set of threshold pivotal sets* Ω_t *for* i *and* φ *is defined as*

$$\Omega_t(i, \varphi) = \left\{ A \subseteq n(i) \;\middle|\; \frac{|l(i, \varphi) \setminus A|}{|n(i) \setminus A|} \not\geq \theta \right\}.$$

Each set $A \in \Omega_t(i, \varphi)$ is such that after removing the connections between i and all the agents in A, the proportion of i's neighbours who support φ is bellow the influence threshold θ. For $\theta = 1$ we obtain the Strong Social Influence model of [11]. Originally the TSI model was defined in [2].

Example 1. Let G be the social network such that $\{a, b, c, d\} \in N$ as illustrated in Figure 1. Let us take a quota of $\theta = 3/5$. We have that $\Omega_t(a, \varphi) = \{\{b\}, \{c\}, \{b, c\}, \{b, c, d\}\}$. Clearly, a is expected to support φ because $2/3 > 3/5$ of her neighbours support it. If the neighbourhood of a is changed to either $\{c, d\}$, $\{b, d\}$, $\{d\}$ or the empty set, the agent a would no longer be expected to support φ.

The strong, and threshold model in general, only take into account the neighbours with opinion φ to identify social influence regarding φ. Thus, when the threshold is $\theta = 2/5$ an agent will experience conflicting influence when she has for example five neighbours out of which two support φ, one has no opinion on φ and two support $\neg\varphi$. In contrast, the weak influence model from [11, 4, 3] is different in the since what the non φ supporting neighbours support also matters. In our little example, the agent will no longer be under conflicting influence to support both φ and $\neg\varphi$, this agent will be under no social influence regarding this issue.

We generalise the weak influence model of [11] into a new model which refines the threshold influence model, and we call it *opposition sensitive threshold social influence model* or OS-TSI. The OS-TSI intuitively says social influence exist to support φ when the threshold social influence exists to support φ and none of the neighbours supports $\neg\varphi$. The weak social influence model is obtained as a special case of OS-TSI when $\theta > 0$.

Definition 5 (Opposition Sensitive TSI). *Let G be a social network with agents N and issues $\mathcal{I}$. Let $i \in N$ be an agent and let $\varphi \in N$ be an issue. Asocial influence threshold is a number $\theta \in [0,1]$. The set of threshold pivotal sets Ω_o for i and φ is defined as*

$$\Omega_o(i,\varphi) = \left\{ A \subseteq n(i) \;\middle|\; \frac{|l(i,\varphi) \setminus A|}{|n(i) \setminus A|} \not\geq \theta \text{ or } l(i,\neg\varphi) \setminus A \neq \emptyset \right\}.$$

An agent might not be influenced by proportions, but simply by the objective number of agents that expressed the same opinion. This number t may be seen as a personal tolerance of an agent. We assume always that the tolerance is at most the number of agents in the network. Thus if $|N| > t > n(i)$ we have a model of an agent who is never socially influenced.

Definition 6 (Accumulated Tolerance Social Influence (ATSI)). *Let G be a social network with agents N and issues $\mathcal{I}$. Consider an $i \in N$ and $\varphi \in \mathcal{I}$ and that i's tolerance $0 < t < |N|$. We define the accumulated tolerance pivotal sets Ω_a for i and φ as $\Omega_a(i,\varphi) = \{A \subseteq n(i) \mid |l(i,\varphi) \setminus A| < t\}$.*

Example 2. Consider the same social network(s) as in the previous example given in Figure 1 and let $t = 1$. This means that with only one friend supporting an issue, there is an active influence exerted upon the agent to support it. Now $\Omega_a(i,\varphi) = \{\{b\},\{c\},\{b,c\},\{b,d\},\{c,d\},\{b,c,d\}\}$. In fact, any set of neighbours containing at least b or c will suffice.

We lastly want to define the Simmelian social influence model that captures the idea of [10].

Definition 7 (Simmelian influence). *Let $G = (N, \mathcal{I}, \text{pro}, E)$ be a social network, $\varphi \in \mathcal{I}$ and agent $i \in N$. The Simmelian pivotal set of sets $\Omega_s(i,\varphi)$ is $\Omega_s(i,\varphi) = \{A \subseteq n(i) \mid$ there is no $e \in E$ s.t. $e \subseteq l(i,\varphi) \setminus A\}$.*

An agent i has a Simmelian tie with an agent j when $\{i,j\} \in E$ and there exists an agent k such that both $\{i,k\} \in E$ and $\{j,k\} \in E$. A Simmelian influence to support φ is exerted on an agent i when that agent has a Simmelian tie with an agent j that supports φ and there exist an agent k that supports φ with whom both i and j are connected. To identify if there is Simmelian influence over i with respect to φ, we first identify all neighbours of i that support φ, $l(i,\varphi)$. Then we check if there is an edge between any two of the agent in $l(i,\varphi)$; these connected agents together with i form a clique and create the Simmelian influence over i. To remove the influence we need to remove edges so i is in no φ-supporting clique.

Example 3. Consider a social network with seven agents $N = \{a,b,c,d,e,f,g\}$ given on Figure 2.The agent a has five Simmelian ties, given in green, with agents b, c, d, g and f. The set of neighbours for a is $n(a) = \{b,c,d,e,f,g\}$. The set of neighbours who support φ is $l(a,\varphi) = \{b,c,d,e\}$, given in red. The set of Simmelian pivotal sets for a and φ, $\Omega_s(s,\varphi)$, is the set containing $\{b,c\},\{b,d\},\{c,d\}$ and all supersets of these. Namely, if the agent a wishes to not be under Simmelian influence for φ, she then needs to remove at least two of her Simmelian ties with φ-supporting neighbours.

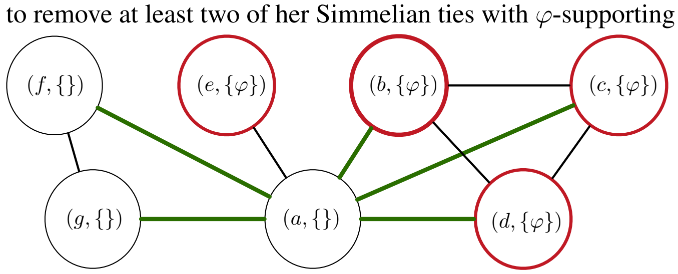

Fig. 2: An example of Simmelian ties and Simmelian influence

Our influence models are designed to capture the possibility of a conflict. Consider the social network on Figure 2 and assume that agents f and g both support issue $\neg\varphi$. In this case, agent a would experience Simmelian influence to support φ from her Simmelian ties with b, c and d and another Simmelian influence to support $\neg\varphi$ from her Simmelian ties with f and g. A rational agent would, in this situation, try to resolve this conflict.

3 Resolving conflicts

Consider the social network on Figure 2 and assume that agents f and g both support issue $\neg\varphi$. In this case, agent a would experience Simmelian influence to support φ from her Simmelian ties with b, c and d and another Simmelian influence to support $\neg\varphi$ from her Simmelian ties with f and g.

We noted that under our social influence models, with the exception of OS-TSI, conflicting social influences to be experienced by the same agent. To be more specific, this is true in the case when the influence threshold is $\theta < 0.5$ and the tolerance is $t < n(i)/2$. To be even more specific, this is the case under the assumption that the issues in $\mathcal{I}$ are *logically independent*.

Example 4. Consider a social network graph G of seven agents as given in Figure 3. Under the TSI and OS-TSI models, for $\theta = 2/6$, the agent a is under the social influence to accept φ, $\neg\psi$, and $\varphi \to \psi$. This set of opinions is an inconsistent set of formulas.

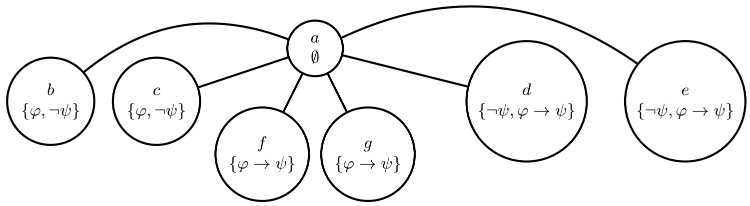

Fig. 3: Conflict of social influence under the OS-TSI model

Notice that if either of agents f and g also supported φ, there would have not been OS-TSI social conflict for a since there would have been no social influence to adopt $\neg\psi$.

From Example 4 we see that social conflict can occur in more complex forms than direct social influence to support two contradicting opinions. We therefore give a definition of *conflicting social influences*.

Definition 8 (Conflicting social influence). *Consider a consistent set of formulas $S \subset \mathcal{I}$. We define the closure of S, w.r.t. $\mathcal{I}$, as a set $S^{\mathcal{I}} = \{\varphi \in \mathcal{I} \mid S \models \varphi\}$. Let the set of all social influences experienced by $a \in N$ be $I_\Omega(a) = \{\varphi \in \mathcal{I} \mid \emptyset \notin \Omega(i, \varphi)\}$. We say that an agent $i \in N$ is under conflicting social influences when $I^{\mathcal{I}}_\Omega(a)$ is an inconsistent set of formulas.*

Clearly, a pivotal set $S \in \Omega(i, \varphi)$ can be seen as *influence avoiding actions*, if we let the agent be able to remove edges she is involved in. The agent may opt to drop the edges $\{i, j\}$ for every $j \in S$, in order to avoid experiencing social influence towards supporting φ. However, this does not guarantee that the agent is no longer conflicted. In order to ensure that, we need to define the agent's conflict avoiding actions.

Definition 9 (Conflict avoiding actions). *Given is a social network $G = (N, \mathcal{I}, \mathrm{pro}, E)$, a pivotal sets Ω, and $a \in N$. Let $I_\Omega(a)$ be the set of all social influences experienced by $a \in N$, under a social influence model Ω. We define the minimal conflict set $MC(a)$ as $MC(a) = \{C \subseteq I_\Omega(a) \mid$ there is no $C' \subset C$ s.t. $C \models \bot\}$. The set of avoiding actions can now be defined as*

$$AC(a) := \{S \mid S \in \bigcup_{\varphi \in \bigcup MC(a)} \Omega(a, \varphi)\}.$$

If $C^{\mathcal{I}}_\Omega(a)$ is a consistent set of formulas, then $MC(a) = \emptyset$ and so $AC(a) = \emptyset$.

Recall that to restore consistency to a set it is sufficient to remove one element of its minimally inconsistent subset. This is how the set of avoiding actions is constructed. An agent that is convivial would like to remove a minimal set of its neighbours. To do so, such an agent would find the minimal set of agents $A \in AC(a)$ and remove edges precisely to those agents. We lastly give some properties of social influence models.

Definition 10 (Purely social influence models). *A set of pivotal sets Ω is* purely social *iff for every set of issues $\mathcal{I}$, and social network $(N, \mathcal{I}, \mathrm{pro}, E)$ over these issues, for every $i \in N$ and $\varphi \in \mathcal{I}$, $n(i) \in \Omega(i, \varphi)$.*

Because, of every agent i, $n(i)$ must by definition be an element of $\Omega(i, \varphi)$ for every φ, $n(i)$ is also necessarily in $AC(i)$ which hence is never empty. We look at another property of social influence models.

Definition 11 (Monotonic influence). *A social influence model represented with pivotal sets Ω is monotonic when for every $i \in N$ and $\varphi \in \mathcal{I}$, if $S \in \Omega(i, \varphi)$ then $S' \in \Omega(i, \varphi)$ for every $S \subseteq S' \subseteq n(i)$.*

It is easy to see that the TSI and OS-TSI models are nonmonotonic models, while ATSI and SII are monotonic models. This is because the TSI and OS-TSI models take into account the proportion of neighbours. By removing connections to same "type" of neighbours, say those who support φ, we will never increase the proportion of neighbours that support φ.

4 Future work

We intend to investigate the behaviour of social networks over time with respect to conflicts and of course properties of networks that eventually achieve a conflict free state. The work started by [7] points us to the interesting question of the cost of removing connections with respect to the strategic consequences of these actions. In the accumulated threshold model we provided, every agent had a contribution of 1 towards the agent's tolerance, but a more realistic model could replace this by a more reasoned value. This could be derived from network properties, such as an agent's centrality, or game theoretic notions such as the Shapley-Shubik value [14] of an agent. Further, the framework can be extended to support predicates for more than one concurrent model of social influence to represent the option that different agents in the network can be differently influenced.

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