A logical study of agents' distances in social network creations

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1 Introduction

It is commonly accepted that our social contacts affect the way we form our opinions about the world. However, the way these social groups are created has received less attention. This abstract is part of a research project on the logical structure behind the creation of social networks.

Previous works exploring social network creation have relied on a notion of *distance between agents* (the number of features in which they differ), thus following a *(dis)similarity-based* approach. Indeed, while Smets and Velázquez-Quesada (2017a) studies a *threshold* method (agent a will consider agent b part of her social network if and only if their number of distinguishing features is smaller or equal than a given threshold $\theta \in \mathbb{N}$), Smets and Velázquez-Quesada (2017b) uses a *group-size* strategy (a will consider b part of her social network if and only if b is one of the $\lambda \in \mathbb{N}$ agents that are 'closest' to a).

This abstract describes briefly our ongoing work that extends both approaches by generalising their concept of distance. This generalisation takes place by noticing that, while both approaches consider all features as equally relevant for all agents, in real life some features might be more important than others. Thus, the basic models used in the mentioned proposals are extended here with each agent's *priority ordering* over features, reflecting not only the different importance that different features might have, but also the fact that different agents might give precedence to different features.

2 Modelling social networks

The starting point of this proposal and its predecessors is the basic setting of Baltag et al. (2016): a relational 'Kripke' model in which the domain is the set of agents, the accessibility relation represents a social connection from one agent to another, and the atomic valuation describes the features (traits/behaviour/opinions) each agent has. As the goal is to deal with agents which might consider that some features are more important than others, this basic setting is extended with a priority relation on features for each agent.

Let A denote a countable set of agents, and P a *finite* set of features agents might or might not have (with $\mathsf{A} \cap \mathsf{P} = \varnothing$).

Definition 2.1 (Feature-ranked Social Network Model) A *feature-ranked social network model* (in this work, a *social network model*: SNM) is a tuple $M = \langle \mathsf{A}, S, \{\preccurlyeq_a\}_{a \in \mathsf{A}}, V \rangle$ where $S \subseteq \mathsf{A} \times \mathsf{A}$ is the *social* relation (Sab indicates that agent a is socially connected to agent b), each $\preccurlyeq_a \subseteq (\mathsf{P} \times \mathsf{P})$ is a total preorder ($p \preccurlyeq_a q$ indicates that, for agent a, feature q is at least as important as feature p)[1], and $V : \mathsf{A} \to \wp(\mathsf{P})$ is a *feature* function ($p \in V(a)$ indicates that agent a has feature p). ◀

The social relation S is not required to satisfy any specific property (in particular, it is neither irreflexive nor symmetric); thus, it differs from the *friendship* relation of other approaches (e.g., Seligman et al. 2011, Liu et al. 2014, Baltag et al. 2016, Christoff et al. 2016).

Following Baltag et al. (2016), social network models can be described by a *propositional* language $\mathcal{L}$, with special atoms describing the agents' features, their social relationship and their feature-priority ordering (cf. Holliday 2009, Ghosh and Velázquez-Quesada 2015, Velázquez-Quesada 2017).

Definition 2.2 (Language $\mathcal{L}$) Formulas φ, ψ of the language $\mathcal{L}$ are given by

$$\varphi, \psi ::= p_a \mid \mathsf{S}_{ab} \mid p \sqsubseteq_a q \mid \neg\varphi \mid \varphi \wedge \psi$$

with $p, q \in \mathsf{P}$ and $a, b \in \mathsf{A}$. We read p_a as *"agent a has feature p"*, S_{ab} as *"agent a is socially connected to b"*, and $p \sqsubseteq_a q$ as *"for agent a, feature q is at least as important as feature p"* (with, recall, $p, q \in \mathsf{P}$). Boolean constants ($\top, \bot$) and other Boolean operators ($\vee, \to, \leftrightarrow$) are defined as usual. Given a SNM $M = \langle \mathsf{A}, S, \{\preccurlyeq_a\}_{a \in \mathsf{A}}, V \rangle$, formulas in $\mathcal{L}$ are semantically interpreted in the following way:

$$\begin{array}{lll@{\qquad}lll} M \Vdash p_a & \text{iff}_{\text{def}} & p \in V(a), & M \Vdash \neg\varphi & \text{iff}_{\text{def}} & M \nVdash \varphi, \\ M \Vdash \mathsf{S}_{ab} & \text{iff}_{\text{def}} & Sab, & M \Vdash \varphi \wedge \psi & \text{iff}_{\text{def}} & M \Vdash \varphi \text{ and } M \Vdash \psi. \\ M \Vdash p \sqsubseteq_a q & \text{iff}_{\text{def}} & p \preccurlyeq_a q, & & & \end{array}$$

A formula $\varphi \in \mathcal{L}$ is valid (notation: $\Vdash \varphi$) when $M \Vdash \varphi$ holds for all M. ◀

For an axiomatisation, any system for classical propositional logic is fit as long as it includes the following axioms, which characterise the properties of the feature-priority ordering (totality, reflexivity and transitivity, respectively):

$$\vdash p \sqsubseteq_a q \;\vee\; q \sqsubseteq_a p, \qquad \vdash p \sqsubseteq_a p, \qquad \vdash (p \sqsubseteq_a q \;\vee\; q \sqsubseteq_a r) \;\to\; p \sqsubseteq_a r.$$

3 Distance between agents

In a similarity-based context for social network creation, the crucial notion is that of *distance* between agents. Intuitively, this indicates 'how different' two agents are, thus providing a guideline for deciding whether an agent will add another to her social environment. Here is the basic definition on which this distance is defined: the set of features in which two given agents differ.

Definition 3.1 (Mismatch) Let $M = \langle \mathsf{A}, S, \{\preccurlyeq_a\}_{a \in \mathsf{A}}, V \rangle$ be a SNM. Define the set of features distinguishing agents $a, b \in \mathsf{A}$ in model M as

$$\textsc{msmtch}_M(a, b) := \mathsf{P} \setminus \{ p \in \mathsf{P} : \; p \in V(a) \text{ iff } p \in V(b) \}$$ ◀

[1] Thus, every two features are comparable, and there are maximum ones.

The similarity-based approaches of Smets and Velázquez-Quesada (2017a,b) for creating social networks use an *objective* distance, given by the number of features in which the given agents differ (the cardinality of MSMTCH). The concept of distance that will be used here differs in an important aspect: it is a *subjective* notion of distance, as the features distinguishing two agents might create a wider gap from the perspective of, say, a, than from the perspective of, say, b.[2] As a consequence, the distance used here will be given by the *weighted sum* of the features in which the agents differ.

In order to make this definition precise, note first how each priority relation on features $\preccurlyeq_a$ induces a sequence of layers on P, each one containing features that are, from a's perspective, equally important.

Definition 3.2 Let $M = \langle \mathsf{A}, S, \{\preccurlyeq_a\}_{a\in\mathsf{A}}, V\rangle$ be a SNM, and take any $a \in \mathsf{A}$. By defining the notion of $\leq_a$-maximum in the standard way (for every $\mathsf{Q} \subseteq \mathsf{P}$, take $\max_a(\mathsf{Q}) := \{p \in \mathsf{Q} \mid q \leq_a p \text{ for all } q \in \mathsf{Q}\}$), each $\preccurlyeq_a$ induces the following sequence of layers on P (for $k \geq 0$):

$$\mathrm{P}^a_0 := \max_a(\mathsf{P}), \qquad \mathrm{P}^a_{k+1} := \max_a(\mathsf{P} \setminus \bigcup_{i=0}^{k} \mathrm{P}^a_i)$$

◀

Thus, while P^a_0 contains the features that are, from a's perspective, the most important, P^a_1 contains her next-to most important features, and so on. In fact, the layers define a quasi-partition: some of them might be empty, but nevertheless they are pairwise disjoint and collectively exhaustive. Moreover, since P is finite, there will be a first empty layer, and from that moment on all layers will be empty.[3]

As sketched above, this proposal's strategy is to assign a *weight* to each feature, and then define the distance between two agents as the weighted sum of the features distinguishing them. There are two natural requirements for such assignment. First, features in the same layer should receive the same weight (so equally important features have the same 'distance' effect); second, the layers' order should be respected (so a difference in one of the most important features is more significant than a difference in one of the least important ones).[4]

Here WEIGHT will be defined as a function from a *layer's number* to a natural number. In order to simplify its definition, it will be assumed that the 'distance' between layers is simply 1.

Definition 3.3 Let $M = \langle \mathsf{A}, S, \{\preccurlyeq_a\}_{a\in\mathsf{A}}, V\rangle$ be a SNM, and take any $a \in \mathsf{A}$. Let ℓ_a be the index of the first empty layer generated by $\preccurlyeq_a$ (i.e., $\mathrm{P}^a_k \neq \varnothing$ for $k \in \{0, \ldots, \ell_a - 1\}$, and $\mathrm{P}^a_k = \varnothing$ for $k \in \{\ell_a, \ldots\}$). The function $\textsc{weight}_a : \{0, \ldots, \ell_a - 1\} \to \mathbb{N}$ is defined as

$$\textsc{weight}_a(k) := \ell_a - k$$

◀

[2] Smets and Velázquez-Quesada (2017a) also explores an epistemic setting in which what matters is not only the agents' differences but also what they know about them. This makes the notion of distance *subjective*, as in general the knowledge of the agents is dissimilar. Still, the proposal here is different, as the notion of distance is subjective without considering epistemic aspects.

[3] More precisely, there is a $\ell_a > 0$ such that $\mathrm{P}^a_k \neq \varnothing$ for $k \in \{0, \ldots, \ell_a - 1\}$, and $\mathrm{P}^a_k = \varnothing$ for $k \in \{\ell_a, \ldots\}$.

[4] More precisely, ***(i)*** if $p_1, p_2 \in \mathrm{P}^a_k$ for some $k \geq 0$, then $\textsc{weight}(p_1) = \textsc{weight}(p_2)$; ***(ii)*** if $p_1 \in \mathrm{P}^a_{k_1}$ and $p_2 \in \mathrm{P}^a_{k_2}$ for $0 \leq k_1 \leq k_2$, then $\textsc{weight}(p_2) \leq \textsc{weight}(p_1)$.

It is easy to see that this functions satisfies the two natural requirements described above. Note also how, with this definition, while the weight of the most important features from a's perspective (those in P_0^a) is ℓ_a, the weight of the least important ones (those in $\mathrm{P}_{\ell_a-1}^a$) is 1, so they still count. Of course, one can use more general weight assignments in which the distance between layers is not uniform. Moreover, one could also use an even more 'personal' $\textsc{weight}_a$, with the 'gap' between layers differing from agent to agent. For simplicity, here the same weight assignment will be used for all agents. Still, as it is defined as a function from a *layer* to a natural number, and as the layers are potentially different from agent to agent, this yields an objective distance.

With these tools, here is finally a distance between two agents, *from a given agent's point of view*.

Definition 3.4 Let $M = \langle \mathsf{A}, S, \{\preccurlyeq_a\}_{a\in\mathsf{A}}, V\rangle$ be a SNM, and take $a, b_1, b_2 \in \mathsf{A}$. Then, the distance between b_1 and b_2 in M according to a is given by

$$\textsc{dist}_a^M(b_1, b_2) := \sum_{k=0}^{\ell_a-1} \left(\left| \textsc{msmtch}_M(b_1, b_2) \cap \mathrm{P}_k^a \right| \cdot \textsc{weight}_a(k) \right)$$

with $\mathrm{P}_0^a, \ldots, \mathrm{P}_{\ell_a-1}^a$ the layers induced on P by her priority ordering $\leq_a \subseteq (\mathsf{P} \times \mathsf{P})$, and $\textsc{weight}_a$ the weight function defined above. ◄

In words, the distance between b_1 and b_2 according to a is given by the weighted sum of the features in which b_1 and b_2 differ, with the weight given by the importance of the features from a's point of view. It is not hard to see how $\textsc{dist}$ is indeed a mathematical distance, as it satisfies non-negativity ($\textsc{dist}_a^M(b_1, b_2) \geq 0$), symmetry ($\textsc{dist}_a^M(b_1, b_2) = \textsc{dist}_a^M(b_2, b_1)$) and reflexivity ($\textsc{dist}_a^M(b, b) = 0$). Moreover, $\textsc{dist}$ is a *semi-metric*, as it also satisfies subadditivity ($\textsc{dist}_a^M(b_1, b_3) \leq \textsc{dist}_a^M(b_1, b_2) + \textsc{dist}_a^M(b_2, b_3)$). Still, it is not a *metric*, as it does not satisfy *identity of indiscernibles*: $\textsc{dist}_a^M(b_1, b_2) = 0$ does not imply that b_1 and b_2 are the same, as two different agents may have exactly the same features.[5]

4 Ongoing work

Having defined a subjective notion of distance between agents, it is now possible to create social networks.[6] The approaches mentioned in the introduction (*threshold*: Smets and Velázquez-Quesada 2017a; *group-size*: Smets and Velázquez-Quesada 2017b) are two possibilities, but there might be others.

Then, the definition of a new social network can be based not only on the agents' distances, but also on other factors. Smets and Velázquez-Quesada (2017a) already considers the original network (so agents will become related if and only if they are close enough *and a middleman can 'introduce' them*) and an epistemic factor (so agents will become related if and only if *they know* they are close enough). The latter requires an epistemic setting, but one can even consider an doxastic/epistemic setting (e.g., Baltag and Smets 2008), and also ask for individual and/or group epistemic/doxastic requirements.

[5]See Deza and Deza (2009, Chapter 1) for more on mathematical distances.

[6]This notion of distance can be further generalised if it is defined relative to a given set of features, as done in Smets and Velázquez-Quesada (2017b).

Still, in this setting one can also represent actions of feature-priority change. Such actions, combined with the network-changing discussed above, would allow the study of interaction between different dynamics.

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