Coordination without communication[1]

Valentin Goranko[2]
Stockholm University
Sweden

Antti Kuusisto[3]
University of Bremen
Germany

Raine Rönnholm[4]
University of Tampere
Finland

Abstract

We study pure coordination games where in every outcome, all players have identical payoffs, ‘win’ or ‘lose’. We identify and discuss a range of ’purely rational principles’ guiding the reasoning of rational players in such games and analyze which classes of coordination games can be solved by such players with no preplay communication or conventions.

1 Introduction

Pure coordination games ([7]), aka *games of common payoffs* ([8]), are strategic form games in which all the players receive the same payoffs and thus all payers have fully aligned preferences to coordinate in order to reach the best possible outcome for everyone. Here we study one-step *pure win-lose coordination games* (WLC games) in which all payoffs are either 1 (i.e., *win*) or 0 (*lose*).

Clearly, if players can discuss before playing such a pure coordination game with at least one winning outcome, then they can simply agree on a winning strategy profile, and the game is thus trivialised. What makes such games non-trivial is the limited possibility of preplay communication *before playing the game.* In this paper we assume *no preplay communication* at all, and thus players must decide on their choices of strategy only by reasoning individually.

Even if no preplay communication is possible, players may still share some *conventions* ([7], [9], [4]) which they believe everyone to follow. In this paper we assume that the players share no conventions either. Thus, in our setting, players play independently of each other and they can be considered to come from, e.g., completely different cultures—or different galaxies for that matter. However we assume all players know the structure of the game and that it is common knowledge that all the players share the same winning profile. Furthermore, we sometimes also assume that it is common belief that *every player is rational.*

Therefore our main objective is to analyse what kinds of reasoning can be accepted as “purely rational” and what kinds of WLC games can be solved by such reasoning. Thus we try identify *purely rational principles* that every purely

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[2] `valentin.goranko@philosophy.su.se`

[3] `kuusisto@uni-bremen.de`

[4] `raine.ronnholm@uta.fi`

rational player ought to follow in any WLC game. We also study the hierarchy of such principles based on the classes of WLC games that can be won by following different purely rational principles. One of the principal outcomes of our study is that it is highly nontrivial to demarcate which principles are "purely rational".

In addition to the theoretical work presented here, we have also run some empirical experiments on people's behaviour in certain WLC games. One of our tests can be accessed from the link given in [5]. For a other works related to coordination and rationality see e.g., [3], [4], [10], [1].

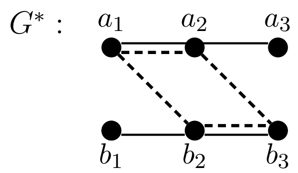
2 WLC games and structural protocols

We study one-step *pure win-lose coordination games* G, which are strategic form games where all players collectively either win or lose. G has n players $(1, \dots, n)$ whose available *choices* are given by sets $\{C_i\}_{i \leq n}$. The set of winning *choice profiles* is presented by an n-ary *winning relation* W_G. A formal definition follows.

Definition 2.1. An n-player **win-lose coordination game (WLC game)** is a relational structure (see, e.g., [2]) $G = (A, C_1, \dots, C_n, W_G)$ where A is a finite domain of choices, $C_i \neq \emptyset$ are unary predicates such that $C_1 \cup \dots \cup C_n = A$ and W_G is an n-ary relation such that $W_G \subseteq C_1 \times \dots \times C_n$. We also assume that $C_i \cap C_j = \emptyset$ for every $i, j \leq n$ s.t. $i \neq j$.

A **protocol** is a mapping Σ that assigns to every pair (G, i), where G is a WLC game and i a player in G, a nonempty set $\Sigma(G, i) \subseteq C_i$ of choices. Thus a protocol gives global nondeterministic strategy for playing any WLC game in the role of any player. Intuitively, a protocol thus identifies a global way to act in any situation that involves playing WLC games, and thus protocols can be informally regarded as global "reasoning styles" or "behaviour modes."

Example 2.2. Let $G^* = (\{a_1, b_1, a_2, b_2, a_3, b_3\}, C_1, C_2, C_3, W_{G^*})$, where for each player i, $C_i = \{a_i, b_i\}$ and $W_{G^*} = \{(a_1, a_2, a_3), (a_1, a_2, b_3), (a_1, b_2, b_3), (b_1, b_2, b_3)\}$. The 3-player WLC game G^* can be naturally presented as the following *hypergraph*:



When assuming a setting based on pure rationality with no special conventions or preplay communication, a protocol should only take into account the *structural properties* of the game and its winning relation. Thus the names of the choices and the names (or ordering) of the players should make no difference. In order to make this issue precise, we give the following definitions.

Definition 2.3. An isomorphism(see, e.g., [2]) between WLC games G and G' is a **choice-renaming**. An automorphism of G is called a **choice-renaming of** G.

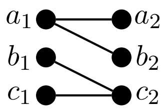
Definition 2.4. Consider n-player WLC games $G = (A, C_1, \dots, C_n, W_G)$ and $G' = (A, C'_1, \dots, C'_n, W'_G)$. A permutation $\beta : \{1, ..., n\} \to \{1, ..., n\}$ is called a **player-renaming** between G and G' if the following conditions hold.

(1) $C_{\beta(i)} = C'_i$ for each $i \leq n$.
(2) $W'_G = \{\, (c_{\beta(1)}, \ldots, c_{\beta(n)}) \mid (c_1, \ldots, c_n) \in W_G \,\}$.

Definition 2.5. Consider WLC games G and G'. A pair (β, π) is a **full renaming** between G and G' if there is a WLC game G'' such that β is a player-renaming from G to G'' and π is a choice-renaming between G'' and G'. If $G = G'$, we say that (β, π) is a **full renaming of** G. We say that choices $c \in C_i$ and $c' \in C_j$ in the same game are **structurally equivalent**, denoted by $c \sim c'$, if there is a full renaming (β, π) of G such that $\beta(i) = j$ and $\pi(c) = c'$.

Example 2.6. Consider the following 2-player WLC game.

$G(1 \times 2 + 2 \times 1)$**:**



It is easy to see that $\sim$ has the equivalence classes $\{a_1, c_2\}$ and $\{b_1, c_1, a_2, b_2\}$.

We say that a protocol Σ is **structural** if it is "indifferent with respect to full renamings," which means that, given any WLC games G, G' for which there exists a full renaming (β, π) from G to G', for any i and any choice $c \in C_i$, it must hold that $c \in \Sigma(G, i)$ iff $\pi(c) \in \Sigma(G', \beta(i))$. Intuitively, this reflects the idea that when following a structural protocol, one acts independently of the names of choices and the ordering of player roles.

3 Hierarchy of purely rational principles

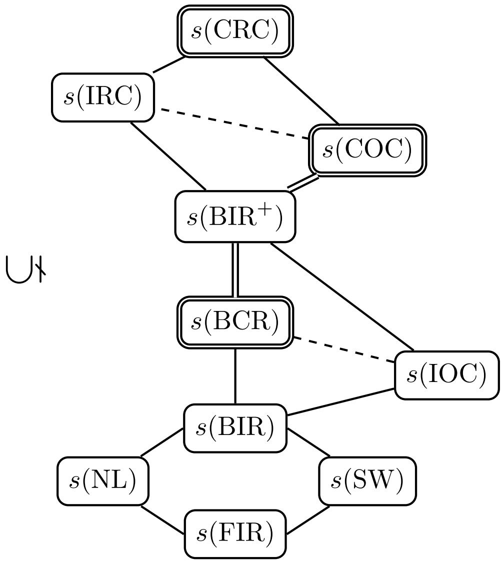
By a **principle** we mean any nonempty class of protocols. Intuitively, these are the protocols "complying" with that principle. If protocols are regarded as "reasoning styles", then principles are *properties* of such reasoning styles. Principles that contain only structural protocols are called **structural principles.**

A player i **follows a principle P** in a WLC game G if she plays according to some protocol in P. We are mainly interested in structural principles that describe "purely rational" reasoning that involves neither preplay communication nor conventions. Such principles will be called **purely rational principles**. Intuitively, purely rational principles should be followed by rational players in *every* game. We can formulate various structural principles which have a strong justification for being purely rational (for the formal definitions for these principles, see [6]).

- **Fundamental individual rationality (FIR):** Never play a *strictly dominated choice.*
- **Non-losing principle (NL):** Never play a *surely losing choice*, if possible.
- **Sure winning principle (SW):** Always play a *surely winning choice*, if possible.
- **Basic individual rationality (BIR):** Follow NL and SW.
- **Basic collective rationality (BCR)**: Follow BIR assuming that everyone follows BIR.
- **Individual optimal choices (IOC):** Always play an *optimal choice*, if possible (an optimal choice weakly dominates all the other choices).
- **Improved basic individual rationality (BIR$^+$):** Follow NL and IOC.

- **Collective optimal choices (COC):** Follow IOC assuming that everyone follows IOC.
- **Individually rational choices (IRC)**: Never play a *weakly dominated choice*.
- **Collective rational choices (CRC)**: Follow IRC assuming that everyone follows IRC.

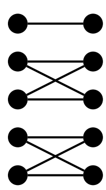
We say that a **principle P solves** a WLC game G (or G **is P-solvable**), if G is *always* won when every player follows any protocol that belongs to P. Formally, this means that $\Sigma_1(G,1) \times \cdots \times \Sigma_n(G,n) \subseteq W_G$ for all protocols $\Sigma_1, \ldots, \Sigma_n \in$ P. The class of all P-solvable games is denoted by $s(\text{P})$. The partially ordered diagram below presents the hierarchy of solvable games with the principles above (see [6] for the related proofs). The principles that only use *individual reasoning* have normal frames and the ones that use *collective reasoning* have double frames.



- Normal lines present proper inclusions in both the general *and* 2-player case.
- *Double* lines present proper inclusions in the general case. In 2-player case there is an identity.
- *Dashed* lines present proper inclusions in 2-player case. In the general case the two sets are not comparable.

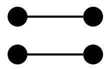
There are also principles that are based on *symmetries* in WLC games and that can be regarded purely rational (see [6]). They can solve certain games which are unsolvable by the principles presented above. Consider the following WLC game.

$G(1 \times 1 + 2(2 \times 2))$**:**

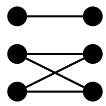


Here, if a player selects a choice within one of the "(2×2)-components", then the other player should select a move from within the same component. But since these two components are structurally identical, it is impossible that players could guarantee coordination by using any structural protocol. Hence we can argue that both players should avoid selecting choices within (2×2)-components. But then the only choice left is the choice within the (1×1)-component, which leads to win.

By using symmetries in WLC games we can also characterize the class of games[1] which cannot be solved by any structural principle and which we call **structurally unsolvable**. Assuming that purely rational principles must be structural, these games are unsolvable by any purely rational principle. The simplest non-trivial example of such a game is the $G(2(1 \times 1))$ below. Also the game $G(1 \times 2 + 2 \times 1)$ in Example 2.6 is unsolvable by any structural principle.

$G(2(1 \times 1))$**:** 

There are many games that are not structurally unsolvable, but in order to solve them, the players need to follow structural principles that seem arbitrary and certainly cannot be considered purely rational. We call such principles *structural conventions*. However, sometimes it is very hard to separate purely rational principles from structural conventions. Consider the following WLC game.

$G(1 \times 1 + 2 \times 2)$**:** 

This game can be solved by following **principle of probabilistically optimal reasoning (PR)** which instructs players to choose whithin (2×2)-component. But on the other hand this can be solved with so-called **Occam razor principle (OR)** which instructs players to choose whithin (1×1)-component. These two principles are not compatible and it is very controversial whether either of them can be called purely rational (see [6] for more discussion). To sum it up, it appears highly nontrivial to pinpoint a boundary between purely rational principles and other decision methods, such as structural conventions, for solving WLC games.

References

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[1] See [6] for the formal characterization of this class.