Mathematics for Computer Scientists 2 (G52MC2)

L05 : General Predicate Logic

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Which of the following are true?

1. $\forall x\, y : \text{bool}, x = y \rightarrow \text{negb}\, x = \text{negb}\, y$

2. $\forall x\, y : \text{bool}, \text{negb}\, x = \text{negb}\, y \rightarrow x = y$

3. $\forall x : \text{bool}, \exists y : \text{bool}, x \neq y$

4. $\exists y : \text{bool}, \forall x : \text{bool}, x \neq y$

5. $(\forall x : \text{bool}, x = \text{true} \lor x = \text{false})$
   $\rightarrow (\forall x : \text{bool}, x = \text{true}) \lor (\forall x : \text{bool}, x = \text{false})$

6. $(\forall x : \text{bool}, x = \text{true}) \lor (\forall x : \text{bool}, x = \text{false})$
   $\rightarrow (\forall x : \text{bool}, x = \text{true} \lor x = \text{false})$

7. $(\exists x : \text{bool}, x = \text{true} \lor x = \text{false})$
   $\rightarrow (\exists x : \text{bool}, x = \text{true}) \lor (\exists x : \text{bool}, x = \text{false})$

8. $(\exists x : \text{bool}, x = \text{true}) \lor (\exists x : \text{bool}, x = \text{false})$
   $\rightarrow (\exists x : \text{bool}, x = \text{true} \lor x = \text{false})$

9. $\exists x\, y : \text{bool}.x \neq y$

10. $\exists x\, y\, z : \text{bool}.x \neq y \land x \neq z \land y \neq z$

The drinkers paradox

In every non-empty pub there is sombeody, if he (or she) drinks then everybody drinks.

- Is this true?
- Or more precisely: Is this a tautology in classical predicate logic?
- I.e. is it true independent of the domain (here *pubs*, *people*) and the meanings of *pub* and *to drink*?