Towards a High Level Quantum Programming Language

Thorsten Altenkirch

University of Nottingham

based on joint work with Jonathan Grattage

and discussions with V.P. Belavkin

Background

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Assumption: Nature is fair. . .

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- Richard Josza, QPL 2004: *We need to develop quantum thinking!*

QML

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  $\mathbf{FCC}$ Finite classical computations

  $\mathbf{FQC}$ Finite quantum computations

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- Compiler under construction (Jonathan)

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Overview

1. Semantics of finite classical and quantum computation
2. QML basics
3. Compiling QML
4. Conclusions and further work

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- **However:** Newtonian mechanics, Maxwellian electrodynamics are also time-reversible. . .
- . . . hence classical computation **should be** based on reversible operations.

Classical computation (FCC)

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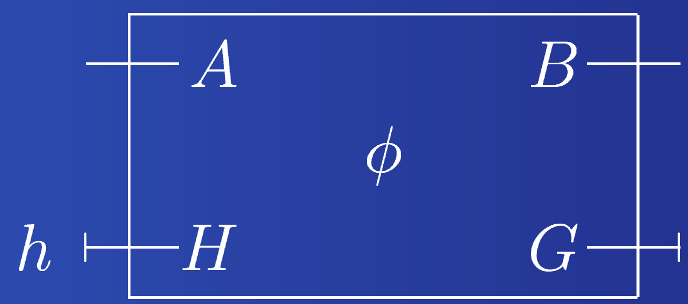
Given finite sets A (input) and B (output):



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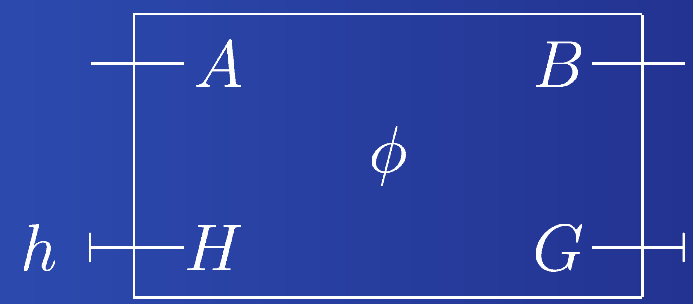

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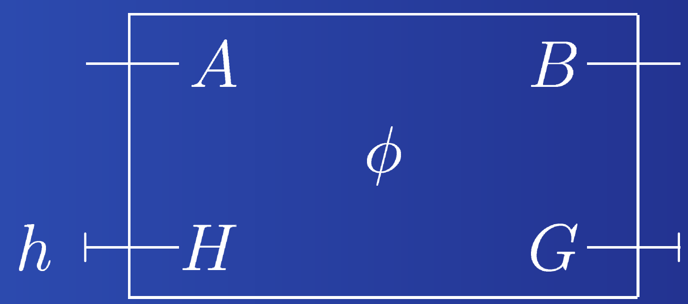

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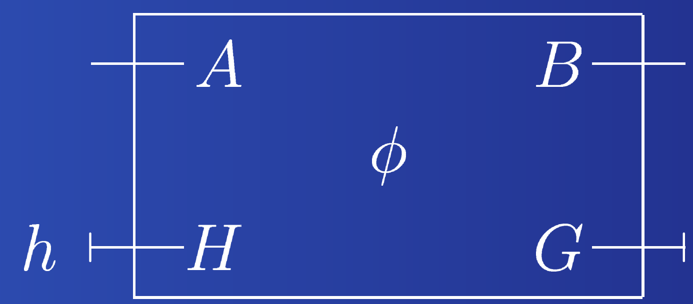

- a finite set of initial heaps H,
- an initial heap $h \in H$,
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- a bijection $\phi \in A \times H \simeq B \times G$,

Semantics

Semantics

- A classical computation $\alpha = (A, B, H, h \in H, G, \phi \in A \times H \simeq B \times G)$ induces a function $\mathsf{U}\alpha \in A \to B$ by

$$\mathsf{U}\alpha\, a = \pi_1\, \phi\, (h, a)$$

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- **Theorem** Any function $f \in A \to B$ (on finite sets A, B) can be realized by a quantum computation.

Composing classical computations

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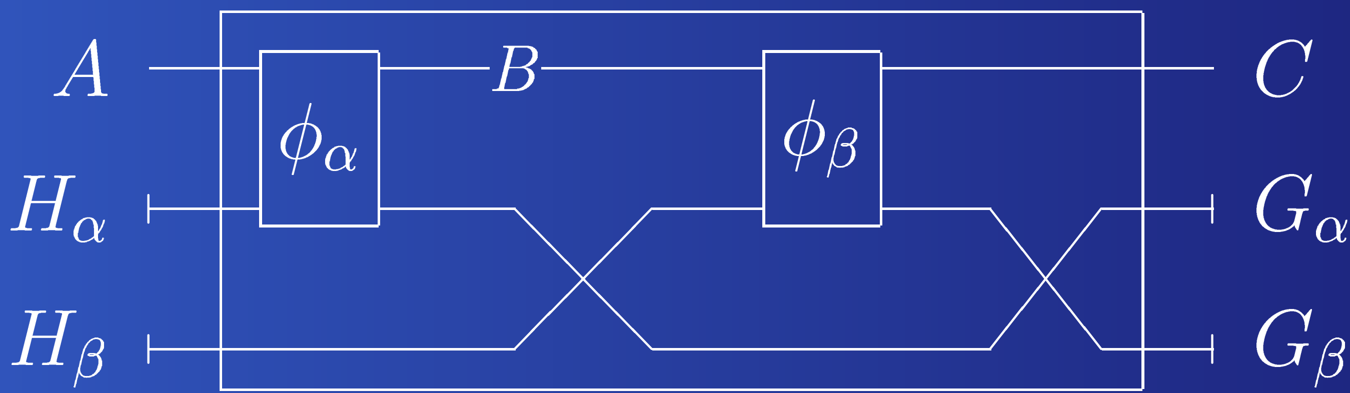

$\phi_{\beta\circ\alpha}$

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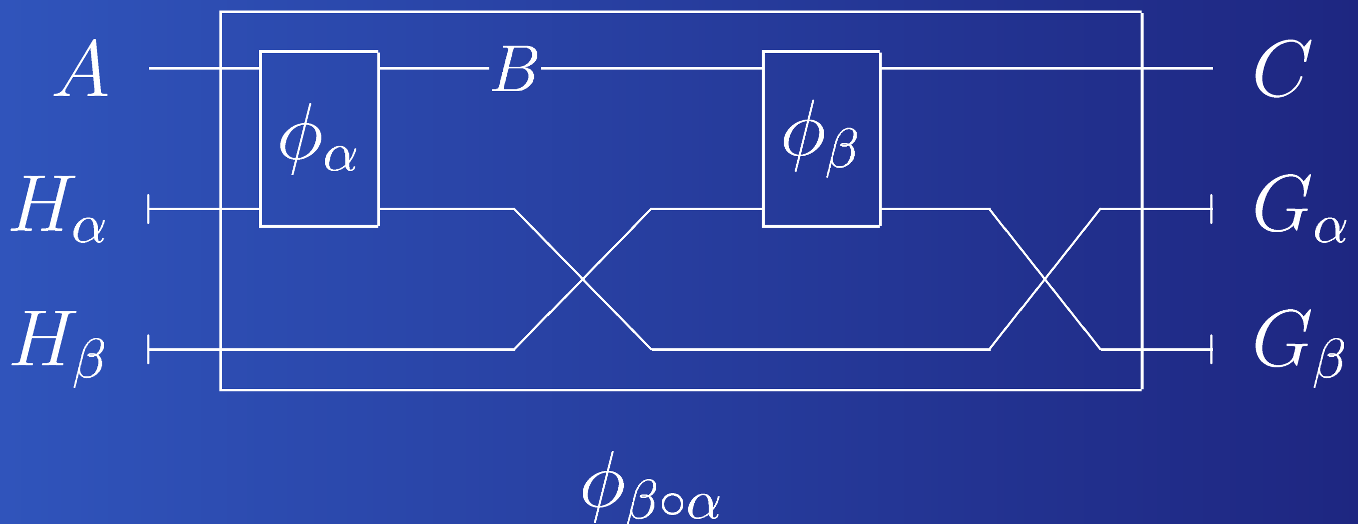

Theorem:

$$\mathbf{U}\,(\beta \circ \alpha) = (\mathbf{U}\,\beta) \circ (\mathbf{U}\,\alpha)$$

Coming next: Quantum computations

Develop $\mathbf{FQC}$ analogously to $\mathbf{FCC}$...

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Norm of a vector:
$\|v\| = \Sigma_{a \in A}(va)^*(va) \in \mathbb{R}^+$,
Unitary operators:
A unitary operator $\phi \in A \multimap_{\text{unitary}} B$ is a linear isomorphism that preserves the norm.

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- A **pure state** over A is a vector $v \in \mathbb{C}\,A$ with unit norm $\|v\| = 1$.

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- a unitary operator $\phi \in A \otimes H \multimap_{\text{unitary}} B \otimes G$.

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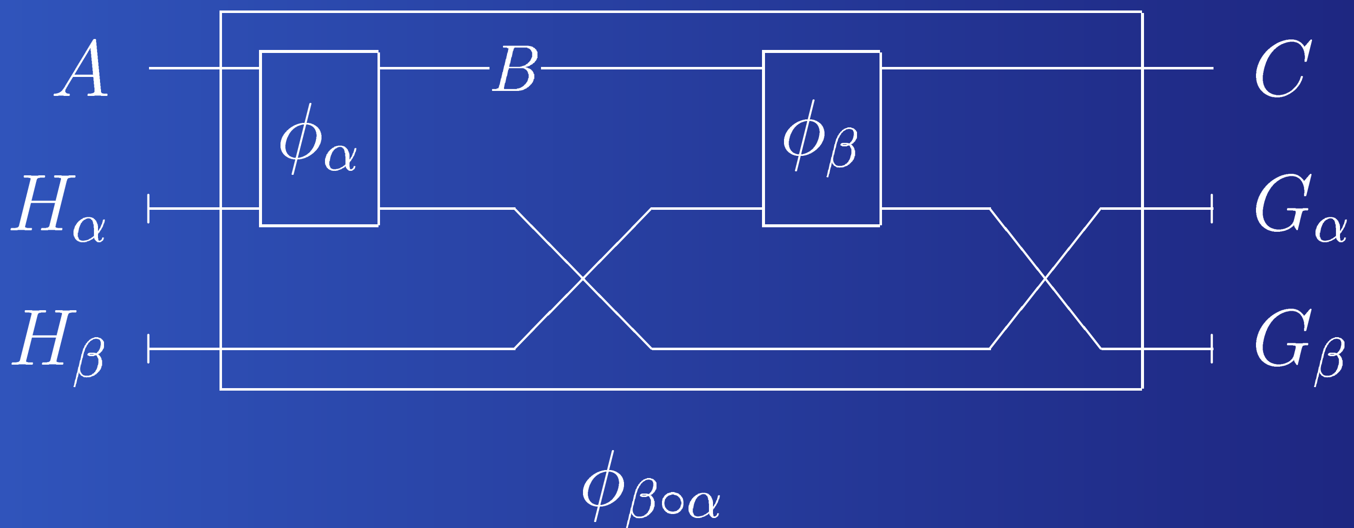

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- . . . is a bit more subtle.
- There is no (sensible) operator on vector spaces, replacing $\pi_1 \in B \times G \to B$.
- **Indeed:** Forgetting part of a **pure state** results in a **mixed state**.

Density matrices and superoperators

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- Every unitary operator ϕ gives rise to a superoperator $\widehat{\phi}$.
- There is an operator

$$\mathrm{tr}_{B,G} \in B \otimes G \multimap_{\text{super}} B$$

called *partial trace*.

Semantics

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Every quantum computation α gives rise to a superoperator $\mathsf{U}\,\alpha \in A \multimap_{\text{super}} B$

$$\begin{tikzcd} A \otimes H \arrow[r, "\widehat{\phi}"'] & B \otimes G \arrow[d, "\mathrm{tr}_G"] \\ A \arrow[u, "- \otimes \widetilde{h}"'] \arrow[r, "\mathsf{U}\,\alpha"'] & B \end{tikzcd}$$

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Theorem: Every superoperator $F \in A \multimap_{\text{super}} B$ (on finite Hilbert spaces) comes from a quantum computation.

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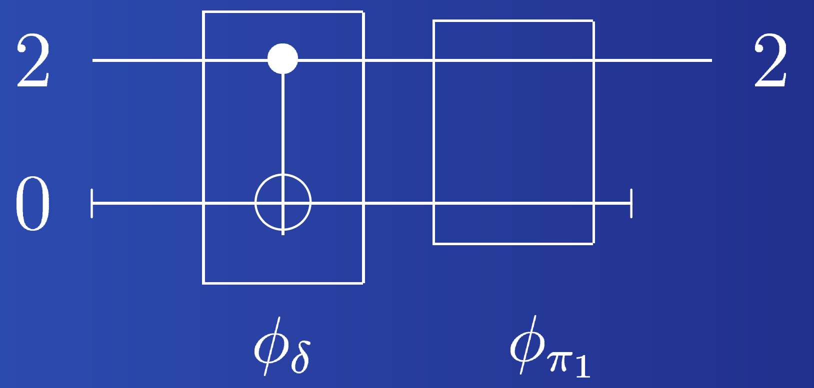

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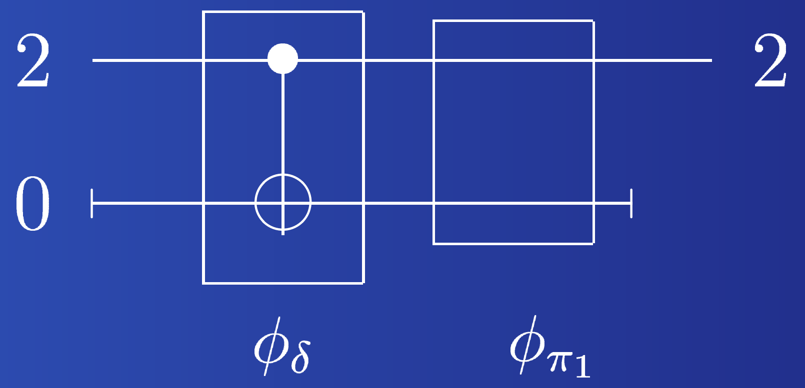

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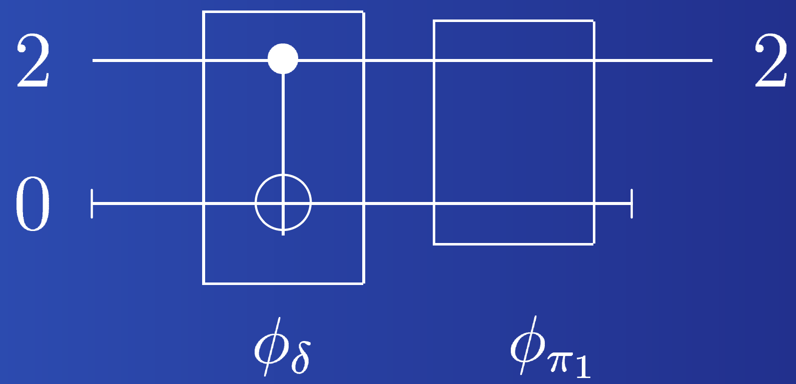

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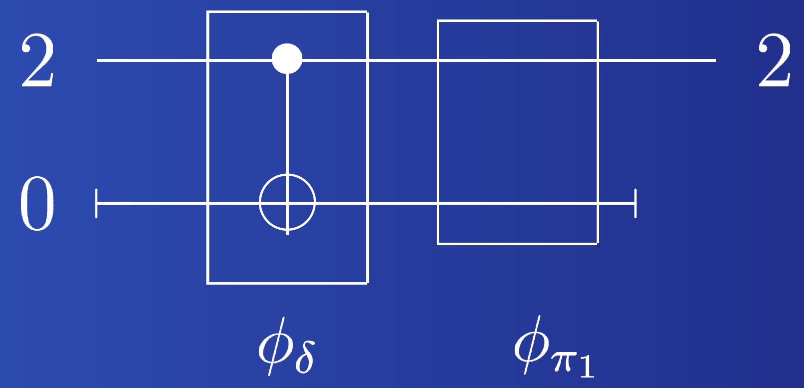

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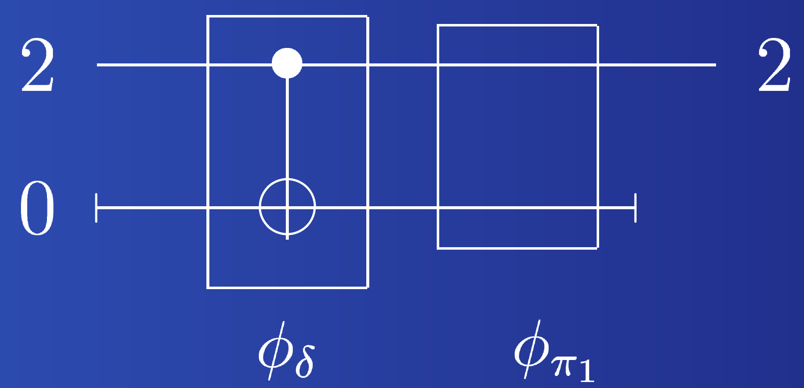

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2. QML basics

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- Qbytes $\mathcal{Q}_2^8 = \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2 \otimes \mathcal{Q}_2$.

QML basics . . .

- A QML program is an expression in a context of typed variables, e.g.

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- We can compile QML programs into quantum computations (i.e. quantum circuits).

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  E.g.
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  is illegal,
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  $$qfst\ (x, y) = x \uparrow \{\, y \,\}$$
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introduces a measurement (end hence decoherence).

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  However, the terms in the superposition have to be orthogonal.

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- Correct QML programs are defined by typing rules, e.g.

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For each rule we can construct a quantum computation, i.e. a circuit.

$\otimes$-elim

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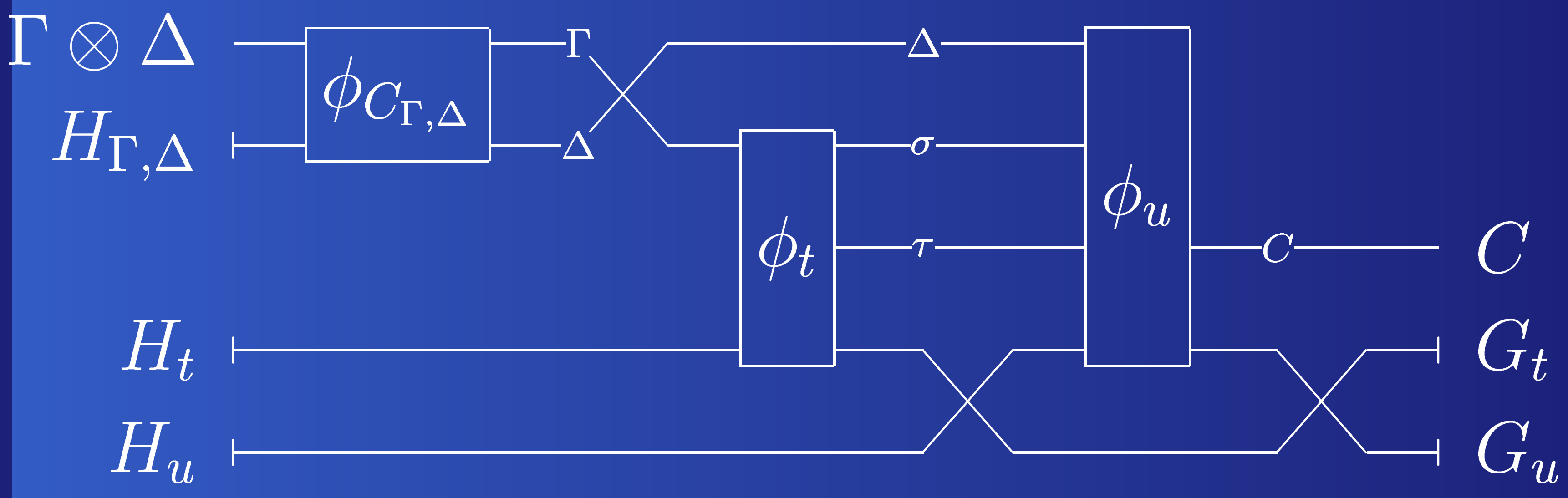

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- The output of the compiler are quantum circuits which can be simulated by a quantum circuit simulator.
- Amr Sabry and Juliana Vizotti (Indiana University) embarked on an independent implementation of QML based on our paper.

4. Conclusions

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4. Conclusions and further work

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- Quantum programming introduces the problem of *control of decoherence*, which we address by making forgetting variables explicit and by having different if-then-else constructs.

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- Investigate the similarities/differences between FCC and FQC from a categorical point of view.

The end

Thank you for your attention.