Formal Proof of Petri Net Refinement using Coq

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Motivations

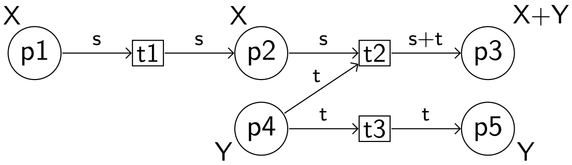
- Petri Nets are a formalism for modeling and validating critical systems: protocols, state systems, ...
- Refinement techniques to obtain a more concrete specification from an abstract view: to deal with the number of states.
- To prove formally and automatically the refinement relation between two nets: to keep interesting properties.
- Using formal proofs in the verification domain.
- First experiment.

Definition of Colored Petri Net

Tuple $N = (P, T, A, C, E, M, Y, M_0)$ where:

1. P is a set of places
2. T is a set of transitions
3. A is a set of arcs
4. C determines the colors of places and transitions
5. E gives the arc inscriptions
6. M is the set of markings
7. Y is the set of steps
8. M_0 is the initial marking

Example



Kind of refinement

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- type refinement (colors).

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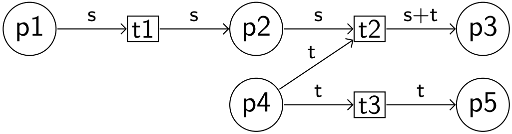

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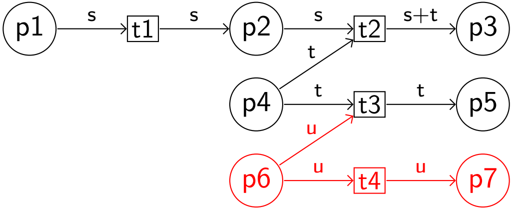

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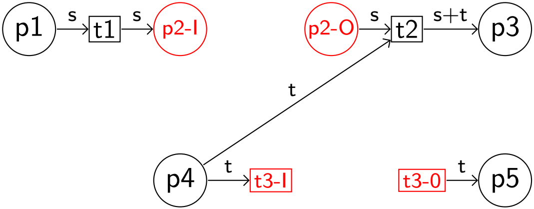

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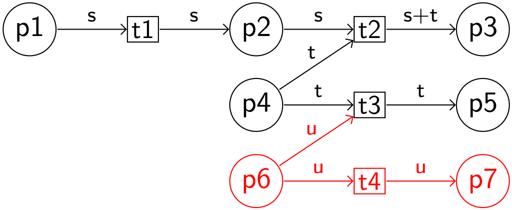

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Definition Aip1t1 := arc_pt Pi1 Ti1 1.
Definition Ait2p3 := arc_tp Ti2 Pi3 2.
```

Coq formalization (refined net)

```
Definition P :=
  Add places (Add places (Add places (Add places (Add places
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     P1) P2) P3) P4) P5) P6) P7.

Definition T :=
  Add transitions (Add transitions (Add transitions
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```

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Using subnet refinement properties

```
Lemma subnet_refined:

      Included places Pi P /\
      Included transitions Ti T /\
 (Included  (prodT (prodT places transitions) nat) Aipt Apt /\

   (forall (pe:places) (te:transitions) (n:nat),
      (In places Pi pe) -> (In transitions T te) ->
      ~In (prodT (prodT places transitions) nat)
   (Setminus (prodT (prodT places transitions) nat) Aipt Apt)
  (pairT (pairT pe te) n))) /\
```

similar for Aitp Atp...

Manual proof and automation

Simple: `unfold, auto with set,...`

```
Ltac is_subnet_refinement :=
  try
   match goal with
   | |- ?X1 /\ ?X2 =>
     split;[try (match goal with | |- (Included _ ?X3 ?X4) =>
          unfold X3, X4; auto with sets end) | intros;
     (match goal with | |- ~(In _ (Setminus _ ?X3 ?X4) _ )=>
          unfold X3, X4 end);unfold Setminus;intro; ... ]
   end.
```

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- First experiment using formal proofs in the Petri Net domain.

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- Full automation.
- Real case study.

Future work (2): missile simulator