OBSERVABILITY DETERMINATION IN POWER SYSTEM STATE ESTIMATION USING A NETWORK FLOW TECHNIQUE

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ABSTRACT

This paper addresses itself to the problem of topological observability in Power System State Estimation. After giving a brief theoretical background a new observability algorithm using a network flow technique is presented. The algorithm is explained with reference to the IEEE-5 bus system and its performance is evaluated using the IEEE-30 and IEEE-118 bus systems.

INTRODUCTION

State estimation is an essential element of a modern computer-assisted power system control package. It is required to provide a reliable estimate of the state vector of voltage magnitudes and angles at each node by processing telemetered measurements. Given the state vector all other variables of interest such as power flows or injections can be calculated straightforwardly.

During the operation of the power system the measurement set processed by the estimator varies as a result of telemetry or instrumentation malfunction. Furthermore the topology of the network can also be modified by the control action of the operator and by the operation of the automatic protection aquipment. Consequently, it is necessary to determine whether the available measurements are sufficient in number and location to enable proper estimation of the system state vector. If this is the case, the state estimation proceeds. Otherwise, the system is unobservable, and either the estimation is applied to the observable subsystems of the original system, or appropriate pseudo-measurements are added to the measurement set.

The importance of the observability problem has been recognised since the early stages of research on power system state estimation. However, the first algorithms [1], [2] making use of floating point computations were not well suited to on-line implementation and their applicability was limited to off-line meter placement studies. Other methods [3], [4] based on logical procedures were satisfactory from the computational point-of-view but proved to be conservative, labelling as unobservable some systems which in fact were observable.

An important contribution to the solution of the observability problem was made by Krumpholz, Clements and Davis [5] who formulated necessary and sufficient conditions for observability in terms of network topology. According to their analysis a network is observable if and only if it contains a spanning tree of full rank. Based on this result three different algorithms have been proposed in the literature. The algorithm described in reference [5] and further developed in [6] begins with an observable forest of the network graph and then attempts to link the forest components into an observable tree. Quintana, Simoes-Costa and Mandel [7] proposed a method which directly searches for an observable spanning tree in the measurement graph using an algorithm for matroid intersection. A third algorithm due to VanCutsem and Gailly [8] is an enumerative procedure which examines all possible measurement assignments. This algorithm is simple compared to the previous two but because of its computational requirements has limited applicability since the number of trees grows rapidly with network size.

85 SM 474-2 A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1985 Summer Meeting, Vancouver, B.C., Canada, July 14 - 19, 1985. Manuscript submitted August 15, 1984; made available for printing April 26, 1985.

In the present paper a new observability algorithm is proposed which is both simple and computationally efficient. The algorithm is based on an original formulation of observability determination as a maximum network flow problem. The method is general in that it always finds an observable spanning tree if one exists. If the system is found to be unobservable the algorithm returns the maximum observable forest, indicating which areas of the network require additional metering or pseudo-measurement.

THEORETICAL BACKGROUND

This section gives a brief outline of the observability theory developed in [5].

The nonlinear measurement model for the power system state estimation is given by

$$\underline{z} = \underline{h}(\underline{x}) + \underline{v} \tag{1}$$

where $\underline{x}$ is the state vector composed of n bus voltage magnitudes $\underline{x}_V$ and n-1 voltage angles $\underline{x}_\delta$, $\underline{z}$ is the mx1 measurement vector, $\underline{h}(\ )$ is a non-linear vector function and $\underline{v}$ is the measurement noise. The power system is said to be observable if the measurement set contains a subset of 2n-1 linearily independent measurements. Disregarding higher order terms in a Taylor series expansion of the function $\underline{h}(x)$ the above definition is equivalent to the requirement that the Jacobian matrix of $\underline{h}(x)$ is of full rank throughout the iterative solution of equation (1).

The concept of topological observability can be introduced by considering the meter configuration together with the approximate decoupled power system model (2).

$$\begin{bmatrix} \underline{z}_P \\ \underline{z}_Q \\ \underline{z}_V \end{bmatrix} = \begin{bmatrix} H_\delta & \\ & H_V \end{bmatrix} \begin{bmatrix} \underline{x}_\delta \\ \underline{x}_V \end{bmatrix} + \begin{bmatrix} \underline{v}_P \\ \underline{v}_Q \\ \underline{v}_V \end{bmatrix} \tag{2}$$

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where

$$H = \begin{bmatrix} H_\delta & \\ & H_V \end{bmatrix} \quad \text{– measurement matrix}$$

$\underline{z}_P, \underline{z}_Q, \underline{z}_V$ are the measurement vectors of active power, reactive power and voltage magnitude, respectively;

H_δ is the active power measurement matrix;

H_V is the reactive power and voltage magnitude measurement matrix;

$\underline{x}_\delta, \underline{x}_V$ is the (n−1)x1 vector of bus angles and nx1 vector of bus voltage magnitudes respectively;

$\underline{v}_P, \underline{v}_Q, \underline{v}_V$ are the vectors of measurement noise of active and reactive power and voltage magnitude, respectively.

The power system is said to be topologically observable if the matrix H has rank equal to 2n−1 which is the dimension of the state vector $\underline{x}$.

Since the matrix H is block diagonal the question of topological observability can be divided into two separate problems of P−δ and Q−V observability. The power system is P−δ (Q−V) observable if the H_δ (H_V) matrix is of rank n−1 (n). If the measurement set consists of one voltage magnitude measurement at the reference node and pairs of active and reactive power measurements the P−δ and Q−V observability problems are equivalent and the topological observability of the overall system can be decided by solving either the P−δ or the Q−V problem.

By transforming equation (2) from the nodal voltage to the branch current frame of reference and assuming that the line impedances are such that they do not reduce the rank of the measurement matrix, it can be shown [5] that the topological observability of the power system is equivalent to the existence of a spanning tree of full rank. This result can be stated in the form of a theorem.

Theorem 1

If a power system is topologically observable with respect to the measurements $\underline{z}_P$ ($\underline{z}_Q$), then there exists a spanning tree of the network graph whose branches are associated one-to-one with the measurements $\underline{z}_P$ ($\underline{z}_Q$).

Such a tree is called an observable spanning tree. Because of the simplifying assumption about the line impedances the reciprocal of Theorem 1 can be stated only in a weaker form.

Theorem 2

If there exists an observable spanning tree of the power network graph and the vector formed by the impedances of the transmission lines does not lie on a certain (n−1) dimensional surface then the power system is topologically observable with respect to the measurements $\underline{z}_P$ ($\underline{z}_Q$).

It must be emphasised however that in a real power system it is unlikely that the line impedances would combine in such a way so as to reduce the rank of the measurement matrices H_δ or H_V. Therefore, topological observability may be investigated by seeking an observable spanning tree of the network graph.

In the more general case where the active and reactive power measurements are not taken in pairs and there are several voltage magnitude measurements both the P−δ and the Q−V observability problems must be studied. By analysing the nonzero elements of the measurement matrix H_V it can be concluded that the voltage measurements result in the same nonzero locations as those produced by measurements of the reactive power flow in hypothetical lines connecting voltage measured nodes with the reference node. The network graph extended by such lines is called an augmented network graph [8].

Theorem 1 can now be rephrased in a slightly more general form.

Theorem 3

If a power system is topologically observable with respect to the measurements $\underline{z}_P$ ($\underline{z}_Q$, $\underline{z}_V$), then there exists a spanning tree of the network graph (augmented network graph) whose branches are associated one-to-one with the measurements $\underline{z}_P$ ($\underline{z}_Q$, $\underline{z}_V$).

Note, that if there is only one voltage measurement at the reference node the augmented network graph is identical with network graph and Theorems 1 and 3 are equivalent.

The sufficient condition for topological observability can be rephrased in a similar way.

Theorem 4

If there exists an observable spanning tree of the power network graph (augmented network graph) and the vector formed by the impedances of the transmission lines does not lie on a certain n−1 dimensional surface, then the power system is topologically observable with respect to the measurements $\underline{z}_P$ ($\underline{z}_Q$, $\underline{z}_V$).

THE OBSERVABILITY ALGORITHM

The theoretical results of the previous section indicate that the P−δ and Q−V observability problems are identical in principle, the only difference being the topology of the network studied. In order to keep the description of the algorithm in general terms both the network graph and the augmented network graph will be called the network graph. The set of branches of this graph will be denoted by B and the set of its nodes by N. The active and reactive power flow measurements and the voltage measurements will be collectively called the flow measurements and the active and reactive power injection measurements will be called the injection measurements. The set of available measurements will be denoted by M.

The algorithm proposed in this paper is based on the observation that the search for an observable spanning tree of the network graph is equivalent to the maximization of the number of network nodes which are incident to branches with measurement assignment. This may be visualised by considering the observability problem as two interacting subproblems. The objective of the first subproblem is to find a maximal subset B_o of the set of branches B whose elements can be associated with elements of the measurement set M in a one-to-one manner. This one-to-one correspondence is refered as the measurement assignment (M_o, B_o). The second subproblem is concerned with the search for an observable spanning tree using only the branches B_o which have measurement assignment. However, since the node-to-branch connectivity of the network graph is not directly taken into account during solution of the first subproblem it is not usually possible to find a spanning tree for every assignment (M_o, B_o) in an observable network. Consequently, in general, the measurement assignment and the tree search procedure must be executed alternately. If a spanning tree is found the network is observable. Otherwise, an attempt must be made to modify the measurement assignment by breaking loops formed by the branches of B_o and adding new, previously unassigned, branches so as to enable construction of a tree of full rank. If this proves to be impossible, the network is declared unobservable and the algorithm returns a maximal observable forest of the network graph.

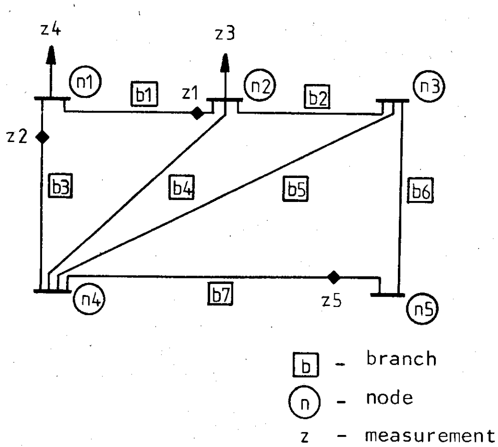
By proposing a novel formulation of the subproblems as network flow problems it has become possible to achieve coordination of the measurement assignment and tree search procedure so that the observability question amounts to solving a maximum flow problem on an appropriately defined network.

Measurement-to-branch assignment

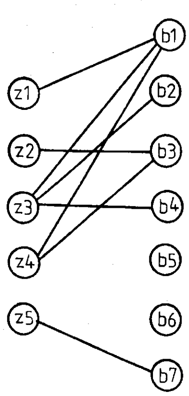
To associate the elements of the measurement set M with the branch set B of the network graph a bipartite graph (M,B) is constructed. The edges of this graph correspond to the nonzero elements of the measurement matrix H and can be determined by the following rules:

a) If measurement z_i is a flow measurement in a line represented by branch $b_i \epsilon B$, then the corresponding edge of the bipartite graph connects z_i and b_i;

b) If measurement z_i is an injection measurement at node i, then the vertex $z_i \epsilon M$ is connected to all vertices $b_i \epsilon B$ which are incident to node i.



(a)



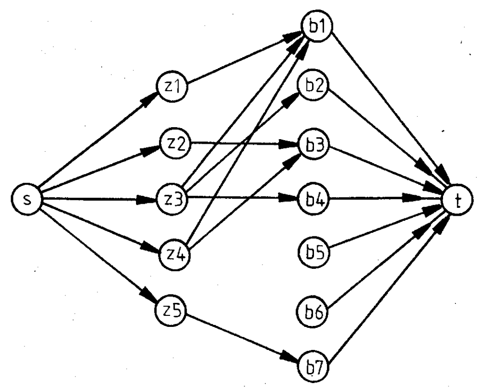
(b)

Figure 1 (a) Network graph;

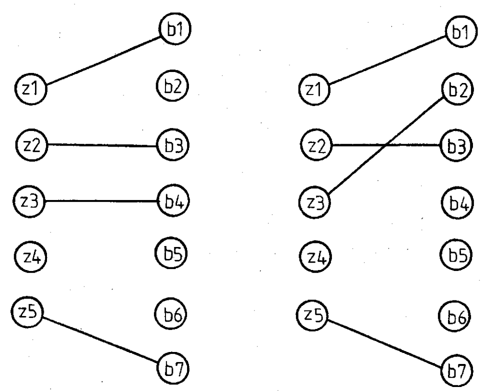
(b) Bipartite (M,B) graph.

To illustrate the concept of the (M,B) bipartite graph, consider the network graph with some injection and flow measurements as indicated in Figure 1a. The corresponding (M,B) bipartite graph is presented in Figure 1b.

We are now interested in finding a measurement assignment which has a maximum number of elements. It can be easily shown [9] that this is equivalent to maximizing the flow on the directed network given in Figure 2a for which all the branches have lower capacity equal to 0 and upper capacity equal to 1. The vertices of the bipartite graph (M,B) which are connected by saturated edges (flow equal to 1) form the (M_o, B_o) assignment. It must be noted that in general there are sevaral possible measurement assignments; each of them however has the same number of components [9]. Figure 2b gives an example of two equivalent measurement assignments. The first one contains a loop formed by the branches b1, b3 and b4 of the network graph, at the same time none of its branches is incident to node 3 of the network. The second assignment contains a loop-free set of branches which, as can be seen from Figure 1a, contains a spanning tree of the network graph.



(a)



(b)

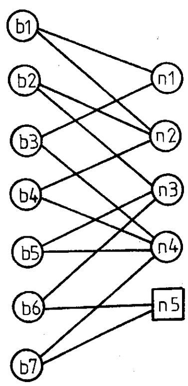
Figure 2 (a) Measurement assignment as a network flow problem;

(b) (M_o, B_o) assignments.

Tree search procedure

Any tree search procedure implies a two-stage process. Firstly an initial vertex of the network graph, called the root of the tree, is selected and then new edges are added sequentially. Each new tree-edge has exactly one of its vertices incident to the tree from the previous iteration. A classical method of selecting these new tree-edges is to employ a breadth-first-search (BFS) or a depth-first-search (DFS) as described in [10] and [9]. However, if the network topology changes, the whole tree search procedure using BFS or DFS must be repeated and no advantage can be taken from knowledge of the previously identified network tree. For the purpose of observability determination, where the network formed by the branches B_o of the measurement assignment (M_o, B_o) is being succesively modified, a new method is proposed which enables modification of the tree in accordance with changes in network topology.

Consider the bipartite branch-node graph (B,N) given in Figure 3, which corresponds to the network graph of Figure 1a. Additionally, introduce a definition of an alternating path as a path formed by the edges of the bipartite graph (B,N) whose vertices are alternately in N and B.



n_r – tree root

Figure 3 Bipartite (B,N) graph.

In terms of the bipartite graph a DFS and BFS tree search procedure can be seen as a search for a set of alternating paths in the (B_o,N) graph starting at the initial vertex $n_r \epsilon N$ (tree root) and terminating at the vertices of $(N-n_r)$. It is apparent that the construction of the alternating path is equivalent to the identification of one-to-one correspondence between the vertices of B_o and $(N-n_r)$, which can be expressed as a maximum flow problem. The only constraint imposed on such a maximum flow problem is that at least one of the vertices $b_r \epsilon B_o$, which are incident to n_r must be included in the resulting (B_1,N_1) matching. This ensures that every network tree build on the branches of B_1 contains the tree root n_r.

Figure 4 gives an example of two (B_1,N_1) matchings which correspond to (M_o,B_o) assignments of Figure 2b.

The tree search procedure can be summarised as follows:

a) Select an initial vertex (tree root) $n_r \epsilon N$;

b) Find an edge connecting n_r to one of the vertices $b_r \epsilon B_o$. If none can be found the network is unobservable since the reference node is isolated and consequently no observable spanning tree can be found;

c) Solve a maximum flow problem on the network defined by $(B_o,N-n_r)$ graph with the constraint that at least one of the vertices $b_r \epsilon B_o$ is saturated.

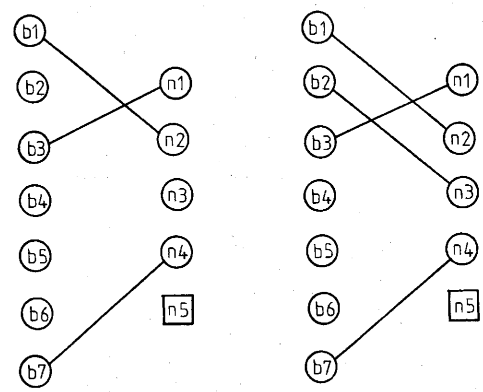
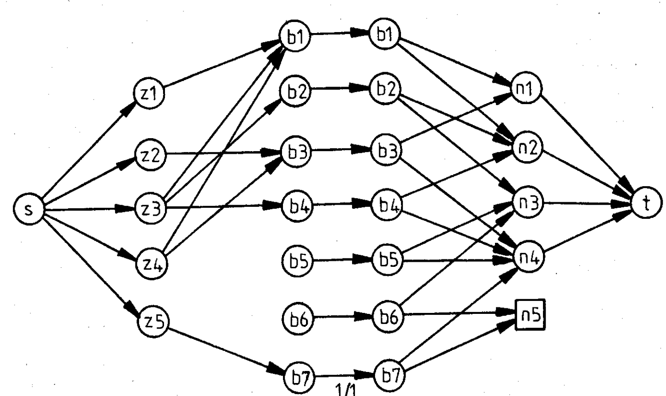


Figure 4 (B_1,N_1) matchings.

Observability network

Formulation of the observability problem as network flow problem is now a logical consequence of the description of the measurement assignment problem and tree search procedure. The observability network, corresponding to the system given in Figure 1a, is shown in Figure 5. All branches of this network are directed and all, except one, have lower capacity equal to 0 and upper capacity equal to 1. The only branch which has lower and upper capacity equal to 1 is found separately during the initalisation stage which consists of backtracking from the n_r vertex to any one of the measurement vertices using, for example, a depth-first-search.



n5 – tree root

b7 - b7 – saturated branch

Figure 5 Observability network.

NUMERICAL RESULTS

The observability algorithm has been implemented using initially a general purpose network flow algorithm [11] and then a specialised routine taking advantage of the layered structure of the observability network. Both programs were coded in FORTRAN 77 and run on a Perkin Elmer 3220 minicomputer. In order to asses computational efficiency of the programs several measurement configurations have been studied. A representative sample of the results obtained is given in Table 1. Examples 1 to 4 refer to the IEEE-30 bus system and Examples 5 and 6 refer to the IEEE-118 bus network. The measurement sets have been devised as follows:

Example 1 (base case for IEEE-30 bus system) - The network has a maximal set of measurements. Each line has a flow measurement and each bus has an injection measurement associated with it. The network is observable.

Example 2 - The measurement set of the base case is reduced by four flow measurements in lines 1, 3, 5 and 6 and five injection measurements at buses 1, 2, 4, 5 and 6. As a result of the modifications bus 2 becomes an unobservable island and consequently the system is declared unobservable.

Example 3 - A minimal set of measurements is examined for which the network is still observable. The measurement set consists of 10 flow measurements and 19 injection measurements.

Example 4 - The measurement set of the Example 3 is modified by replacing one of the injection measurements by a flow measurement. Because of the meter positioning the network becomes unobservable with respect to the modified measurement set.

Example 5 (base case for IEEE-118 bus system) - The network has a maximal set of measurements; 179 flow measurements and 118 injection measurements. The network is observable.

Example 6 - A minimal observable set of measurements is examined which consists of 38 flow measurements and 79 injection measurements.

Computational times obtained with two network flow algorithms indicate that by taking advantage of the layered structure of the observability network the maximum flow can be calculated in a time proportional to the network size. Moreover, an increase in measurement redundancy actualy decreases the computational time since the maximum flow between the layers can be found more easily.

Table1

Example / Measurements		General purpose network-flow alg. [s]	Specialised network-flow alg. [s]
1.	MF=41 MI=30	0.742	0.035
2.	MF=37 MI=25	0.720	0.049
3.	MF=10 MI=19	0.585	0.112
4.	MF=11 MI=18	0.549	0.109
5.	MF=179 MI=118	10.117	0.120
6.	MF=38 MI=79	8.257	0.222

MF - number of flow measurements
MI - number of injection measurements

CONCLUSIONS

This paper has presented a new algorithm for determination of topological observability in Power System State Estimation. The observability problem has been decomposed into two subproblems: measurement-to-branch assignment and a restricted tree search. Both subproblems have been shown to be equivalent to the search for a maximum flow in the corresponding bipartite graphs. This implies that the observability question can be decided by calculating a maximum flow in an appropriately defined network. Computational efficiency of general purpose and specialised network flow algorithms has been compared on the IEEE-30 and IEEE-118 bus systems with various measurement configurations.

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