Relating proof standards and abstract argumentation

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Outline

Motivation and background

Carneades

From Carneades to abstract argumentation

   The translation

   Properties and results
My contribution schematically

Three models for argumentation:

- Carneades
- ASPIC$^+$
- Dung
My contribution schematically

Three models for argumentation:

Structured  
\[ \text{Carneades} \]

Structured  
\[ \text{ASPIC}^+ \]

Abstract  
\[ \text{Dung} \]
My contribution schematically

Three models for argumentation:

- **Structured**
  - *Carneades*

- **Structured**
  - *ASPIC*⁺

- **Abstract**
  - *Dung*  
    - *Dung (1995)*
My contribution schematically

Three models for argumentation:

**Structured**
- Carneades

**Structured**
- ASPIC$^+$

**Abstract**
- Dung

Prakken (2010)
My contribution schematically

Three models for argumentation:

Structured  
- Carneades

Structured  
- ASPIC

Abstract  
- Dung

Gordon, Prakken and Walton (2007)  
Gordon and Walton (2009)
My contribution schematically

Three models for argumentation:

Structured  
Carneades

Structured  
ASPIC+
translates to  
Dung

Abstract
My contribution schematically

Three models for argumentation:

Structured | Structured | Abstract

Carneades | \textit{ASPIC}^+ | Dung

Prakken (2010)
My contribution schematically

Three models for argumentation:

Structured \[\rightarrow\] \textit{Carneades} \[\rightarrow\] \textit{ASPIC}^+ \[\rightarrow\] \textit{Dung} \[\rightarrow\] Abstract
My contribution schematically

Three models for argumentation:

Structured \(\rightarrow\) \text{Abstract}

\begin{align*}
\text{Carneades} & \rightarrow \text{ASPIC}^+ \rightarrow \text{Dung} \\
\text{My thesis} & \\
\text{van Gijzel and Prakken (2011)}
\end{align*}
Contributions conceptually

- Local proof standards, in Dung,
- Carneades modelled as knowledge-based argumentation,
- Rationality postulates for Carneades,
- Generalised Carneades to cycle-containing structures,
- Full correspondence/acyclicity proof.
- Improving insight in (implemented) Carneades.
  - Used by Gordon (2011)
Outline

Motivation and background

Carneades

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Properties and results
Argumentation theory

Interdisciplinary area with various applications:
Argumentation theory

Interdisciplinary area with various applications:

▶ **Law:**

Systems **modelling** legal problems/cases,
Argumentation theory

Interdisciplinary area with various applications:

- **Law:**
  Systems *modelling* legal problems/cases,

- **Decision making:**
  Organising information and source of *efficiency* in decision theory,
Argumentation theory

Interdisciplinary area with various applications:

▶ **Law:**
  Systems *modelling* legal problems/cases,

▶ **Decision making:**
  Organising information and source of *efficiency* in decision theory,

▶ **Communication theory:**
  Making argumentation in existing texts *precise.*
Argumentation

What is argumentation?
Argumentation

In its basic setting:

1. Construct arguments in favour and against a certain statement,
Argumentation

In its basic setting:

1. Construct arguments in favour and against a certain statement,

2. Select the acceptable arguments,
Argumentation

In its basic setting:

1. Construct arguments in favour and against a certain statement,
2. Select the acceptable arguments,
3. Determine whether the statement holds.
Abstract argumentation

Structured

\textbf{Carneades} \quad \text{translates to} \quad \textbf{ASPIC}^+ \quad \text{translates to} \quad \textbf{Dung}

Abstract
In 1995, Dung gave an *abstract* account of argumentation.

- Was able to model several contemporary approaches to non-monotonic logic,
Dung’s argumentation frameworks (AFs)

In 1995, Dung gave an **abstract** account of argumentation.

- Was able to model several contemporary approaches to non-monotonic logic,
- Some scholars believe it to be too abstract,
In 1995, Dung gave an abstract account of argumentation.

- Was able to model several contemporary approaches to non-monotonic logic,
- Some scholars believe it to be too abstract,
- However the model can be instantiated with more structure
  - For instance: ASPIC+. 
Two parts: a set of abstract arguments and an (abstract) notion of defeat between arguments.

C  B  A
Definition

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\[ C \rightarrow B \rightarrow A \]
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\[ C \rightarrow B \rightarrow A \]
Cycles

\[ B \leftrightarrow A \]

Dung defined different semantics for dealing with cycles using extensions.

- Complete extension
- Grounded extension
- Preferred extension
- Stable extension
Dung defined different semantics for dealing with cycles using extensions.

- Complete extension
- Grounded extension
- Preferred extension
- Stable extension

Where extension =

“set of arguments that are acceptable when taken together”
Structured argumentation: $ASPIC^+$

Structured Abstract

Carneades translates to $ASPIC^+$ translates to Dung
Structured argumentation: \textit{ASPIC}^+

\textbf{ASPIC}+ by Prakken (2010) starts with:

- \textbf{Knowledge:}
  indisputable facts, assumptions, normal premises, issue premises.

- \textbf{Inference rules:}
  strict or defeasible.

Arguments are given \textit{structure}. 
Structured argumentation: \( \text{ASPIC}^+ \)

\( \text{ASPIC}^+ \) by Prakken (2010) starts with:

- **Knowledge:**
  indisputable facts, assumptions, normal premises, issue premises.

- **Inference rules:**
  strict or defeasible.

Arguments are given **structure**.

The **defeat relation** is given structure as following:

- Contrariness function: \( \neg \)

- Preferences on rules/knowledge (not used).
Arguments in ASPIC$^+$ (1)

Argument structure:

- Trees where:

  - Nodes are terms of a logical language $L$,
  - Links are applications of inference rules:
    - $R_s$, strict rules of the form: $\phi_1, \ldots, \phi_n \rightarrow \phi$,
    - $R_d$, defeasible rules of the form: $\phi_1, \ldots, \phi_n \Rightarrow \phi$,
  - Smallest argument is a fact from knowledge base $K \subseteq L$.

Acceptability of arguments determined by corresponding Dung framework.
Arguments in ASPIC$^+$ (1)

Argument structure:

- **Trees** where:
  - **Nodes** are wff of a logical language $\mathcal{L}$,
  - **Links** are applications of inference rules:
    - $\mathcal{R}_s$, strict rules of the form: $\phi_1, \ldots, \phi_n \rightarrow \phi$,
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- **Acceptability of arguments** determined by corresponding Dung framework.
Structured evaluation (1)

C: \[
\begin{array}{c}
talk\_and\_drinks \\
\rightarrow \ \\
-\normal\_talk
\end{array}
\]

B: \[
\begin{array}{c}
argumentation\_talk \\
\rightarrow \ \\
bored
\end{array}
\]

A: \[
\begin{array}{c}
friends \\
\rightarrow \ \\
fun
\end{array}
\]

Evaluated through Dung:
Structured evaluation (1)

C: \( \frac{\text{talk_and_drinks}}{\neg \text{normal_talk}} \)  
B: \( \frac{\text{argumentation_talk}}{\text{bored}} \frac{\text{normal_talk}}{\text{fun}} \)  
A: \( \frac{\text{friends}}{\text{fun}} \)

Evaluated through Dung:

C → B → A
Rationality postulates

Proved by Caminada and Amgoud (2007) for ASPIC, generalized by Prakken (2010) to ASPIC⁺:
Rationality postulates

Proved by Caminada and Amgoud (2007) for ASPIC, generalized by Prakken (2010) to ASPIC⁺:

- Closure under subarguments
- Closure under strict rules
- Direct consistency
- Indirect consistency
Outline

Motivation and background

Carneades

From Carneades to abstract argumentation

The translation

Properties and results
Structured argumentation: Carneades

Structured

Carneades  translates to  ASPIC$^+$

Abstract

translates to  Dung
Carneades


- Structured argumentation (like ASPIC$^+$)
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▶ Structured argumentation (like ASPIC$^+$)
▶ Distinguishing feature: proof standards on a local level.
  - Model is put forward as different than Dung’s approach and argumentation based on a knowledge-base.
  - Brewka and Gordon (2010) claim that Dung’s frameworks are not able to model this cycle-free.
Pro and con arguments

Two types of arguments regarding a conclusion $c$:

- An argument with conclusion $c$ is called $\text{pro } c$,
- An argument for an opposite conclusion, $\overline{c}$, is called $\text{con } c$. 
Pro and con arguments

Two types of arguments regarding a conclusion $c$:

- An argument with conclusion $c$ is called pro $c$,
- An argument for an opposite conclusion, $\overline{c}$, is called con $c$.

Aggregation of pro and con is done through proof standards.
Arguments in Carneades consist of a two step inference:

- Applicability of an argument.
- Acceptability of the conclusion \( c \).
Applicability

- Witness
- Liar
- Selfdefense
- Intent
- Kill

$a_1$
a2
$a_3$
Arguments in Carneades

- \( a_1 \) from witness to intent, weight 0.4
- \( a_2 \) from liar to \( \neg \text{intent} \), weight 0.6
- \( a_3 \) from intent to kill, weight 0.8
Arguments in Carneades (formally)

A propositional language $\mathcal{L}$.
An argument $\langle P, E, c \rangle$ has 3 parts:
Arguments in Carneades (formally)

A propositional language $\mathcal{L}$. An argument $\langle P, E, c \rangle$ has 3 parts:

- premises, $P \subseteq \mathcal{L}$,
- exceptions, $E \subseteq \mathcal{L}$,
- conclusion, $c \in \mathcal{L}$. 
A propositional language $\mathcal{L}$. 
An argument $\langle P, E, c \rangle$ has 3 parts:

- premises, $P \subset \mathcal{L}$,
- exceptions, $E \subset \mathcal{L}$,
- conclusion, $c \in \mathcal{L}$.

With $P \cap E = \emptyset$, and all premises, exceptions and conclusions literals.
A Carneades argument evaluation structure (CAES) is a tuple \((\text{arguments}, \text{audience}, \text{standard})\) where:

- **arguments** is an acyclic dependency graph of arguments,
- **audience** consists of two parts:
  - assumptions: a set of propositions similar to axioms,
  - weight: function mapping arguments to range 0 to 1,
- **standard** is a total function mapping literals in \(L\) to their applicable proof standards.
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A Carneades argument evaluation structure (CAES) is a tuple \( \langle \text{arguments}, \text{audience}, \text{standard} \rangle \) where:

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- \textit{standard} is a total function mapping literals in \( \mathcal{L} \) to their applicable proof standards.
Applicability of arguments

An argument $\langle P, E, c \rangle$ is **applicable** in a CAES iff:

- $p \in P$ implies $p \in \text{assumptions or } [p < \text{assumptions} \text{ and } p \text{ acceptable}]$.
- $e \in E$ implies $e < \text{assumptions and } [e \in \text{assumptions or } e \text{ not acceptable}]$. 
Applicability of arguments

An argument $\langle P, E, c \rangle$ is applicable in a CAES iff:

- $p \in P$ implies $p \in \text{assumptions}$ or $[\neg p \notin \text{assumptions} \text{ and } p \text{ acceptable}]$.
- $e \in E$ implies $e \notin \text{assumptions}$ and $[\neg e \in \text{assumptions} \text{ or } e \text{ not acceptable}]$. 
Acceptability of propositions

Given a CAES $C = \langle \text{arguments}, \text{audience}, \text{standard} \rangle$.
A literal $p$ is acceptable in $C$ iff its proof standard returns $true$. 
Proof standards

Five proof standards:
Proof standards

Five proof standards:

▶ Scintilla of evidence,
▶ Preponderance of the evidence,
▶ Clear and convincing evidence,
▶ Beyond reasonable doubt,
▶ Dialectical validity.
Beyond reasonable doubt

Given a CAES $C = \langle \text{arguments}, \text{audience}, \text{standard} \rangle$ and $p \in \mathcal{L}$. 

$\textit{beyond-reasonable-doubt}(p, \text{arguments}, \text{audience}) = \text{true}$ iff

- There is an applicable $a \in \text{arguments}$ with weight $(a) > \alpha$,
- weight $(a)$ exceeds the weight of the applicable con arguments by $\beta$,
- the weight of all applicable con arguments is less than $\gamma$. 
Beyond reasonable doubt

Given a CAES $C = \langle \text{arguments}, \text{audience}, \text{standard} \rangle$ and $p \in \mathcal{L}$. 
\textit{beyond-reasonable-doubt}(p, arguments, audience) = true iff

- There is an \textbf{applicable} $a \in$ arguments with $\text{weight}(a) > \alpha$, 

\textbf{Universiteit Utrecht}
Beyond reasonable doubt

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The translation

\[ \text{Structured} \quad \text{Structures} \quad \text{Abstract} \]

\[ \text{Carneades} \quad \text{translates to} \quad \text{ASPIC}^+ \quad \text{translates to} \quad \text{Dung} \]
Translating an argument

Applicability:
Translating an argument

Applicability:

\[ \frac{p_1}{\text{arg}_{a_1}} \]

\[ \frac{p_2}{\text{app}_{a_1}} \]

\[ \frac{\neg \text{app}_{a_1}}{e_1} \]
Translating an argument

Applicability and acceptability:

\[
\begin{align*}
p_1 & \quad p_2 & \quad e_1 \\
\text{arg}_{a_1} & \quad \text{app}_{a_1} & \quad \neg \text{app}_{a_1}
\end{align*}
\]

\[
\begin{align*}
p_1 & \quad p_2 & \quad \text{arg}_{a_1} & \quad \text{app}_{a_1} & \quad \neg \text{app}_{a_1} \\
\hline
0.4 & \quad c & \quad \text{acc}_{a_1}
\end{align*}
\]
Translating arguments

- **Witness** and **liar** lead to **intent** with a probability of 0.4.
- **Selfdefense** leads to **intent** with a probability of 0.6.
- **Intent** leads to **kill** with a probability of 0.8.
Translating arguments

\[
\begin{align*}
\text{witness} & \quad \frac{\text{app}_{a_1}}{\text{arg}_{a_1}} \\
\text{intent} & \quad \frac{\text{kill}}{\text{arg}_{a_3}} \\
\text{murder} & \quad \frac{\text{self defense}}{\text{arg}_{a_2}} \\
\text{liar} & \quad \frac{\text{\neg intent}}{\text{\neg app}_{a_1}}
\end{align*}
\]
Argumentation system corresponding to a CAES (1)

Given a CAES:
For every argument $a = \langle P, E, c \rangle$ in arguments:

\[
\mathcal{R}_{da} = \{ P \Rightarrow_{app_a} arg_a; \ arg_a \Rightarrow_{acc_a} c \} \cup \\
\{ e_i \Rightarrow \neg app_a \mid e_i \in E \}
\]
Argumentation system corresponding to a CAES (2)

\[ \mathcal{L}_{AS} = \mathcal{L}_{CAES} \cup \text{argument nodes} \cup \text{rule names}, \]
Argumentation system corresponding to a CAES (2)

- $\mathcal{L}_{AS} = \mathcal{L}_{CAES} \cup \text{argument nodes} \cup \text{rule names}$,
- $\mathcal{K}_n = \text{assumptions}$.
Argumentation system corresponding to a CAES (2)

- $\mathcal{L}_{AS} = \mathcal{L}_{CAES} \cup \text{argument nodes} \cup \text{rule names},$
- $\mathcal{K}_n = \text{assumptions},$
- $\mathcal{K}_i = \mathcal{L}_{CAES} \setminus (\text{assumptions} \cup \{c \mid \langle P, E, c \rangle \in \text{arguments}\}).$
Beyond reasonable doubt

Given a CAES $C = \langle \text{arguments}, \text{audience}, \text{standard} \rangle$ and $p \in \mathcal{L}$. 

\textit{beyond-reasonable-doubt}(p, arguments, audience) = true iff

- There is an \textbf{applicable} $a \in \text{arguments}$ with $\text{weight}(a) > \alpha$,
- $\text{weight}(a)$ exceeds the weight of the \textbf{applicable} con arguments by $\beta$,
- the weight of all \textbf{applicable} con arguments is less than $\gamma$. 
Argumentation system corresponding to a CAES (3)

For every argument \( a = \langle P, E, c \rangle \) in arguments with \( \text{standard}(a) = \text{beyond-reasonable-doubt} \):

\[
\mathcal{R}_{s_a} = \{ \rightarrow \neg \text{acc}_a \mid \text{weight}(a) \leq \alpha \} \\
\neg a = \{ (\text{acc}_a, \text{arg}_b) \mid b = \langle P', E', \overline{c} \rangle \in \text{arguments}, \\
\quad \text{weight}(a) \leq \text{weight}(b) + \beta \\
\quad \lor \text{weight}(b) \geq \gamma \} \\
\cup \{ (\text{acc}_a, \neg \text{acc}_a) \}
\]
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Carneades

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Two part translation

Carneades was translated in two steps:
1. To an ASPIC$^+$ argumentation system,
2. To a Dung framework.
Interesting results about both!
A special case of Dung frameworks: well-founded argumentation frameworks

An argumentation framework is **well-founded** iff there does not exist an infinite sequence of arguments: $A_0, A_1, \ldots, A_n, \ldots$ such that for each $i$, $\text{defeats}(A_{i+1}, A_i)$ holds.
A special case of Dung frameworks: well-founded argumentation frameworks

An argumentation framework is well-founded iff there does not exist an infinite sequence of arguments: \( A_0, A_1, \ldots, A_n, \ldots \) such that for each \( i \), \( \text{defeats}(A_{i+1}, A_i) \) holds.

\[
A_0 \leftarrow A_1 \leftarrow \ldots \leftarrow A_n \leftarrow \ldots
\]

nor

\[
B \leftrightarrow A
\]
Theorem 30 of Dung (1995):

A well-founded argumentation framework has exactly one complete extension which is grounded, preferred and stable.
Proposition:

*Every argumentation framework corresponding to a CAES is well-founded.*
Uniqueness of extension of AF corresponding to CAES

Proposition:

Every argumentation framework corresponding to a CAES is well-founded.

Corollary:

Every argumentation framework corresponding to a CAES has exactly one complete extension which is grounded, preferred and stable.
Correspondence results

Theorem:

Given a CAES $C$ and corresponding argumentation framework, $AF$. Then:

1. An argument $a \in \text{arguments}$ is **applicable** in $C$ iff there is an argument contained in the complete extension of $AF$ with the corresponding conclusion $\text{arg}_a$.

2. A propositional literal $c \in \mathcal{L}_{CAES}$ is **acceptable** in $C$ or $c \in \text{assumptions}$ iff there is an argument contained in the complete extension of $AF$ with the corresponding conclusion $c$. 
Generalisation of a CAES

Given a cyclic CAES $C$. Then for $s \in \{complete, preferred, grounded, stable\}$:

- An argument $a \in arguments$ is **applicable** in $C$ under sceptical (credulous) $s$ semantics iff all (some) $s$ extensions of $AF$ contain an argument with conclusion $arg_a$.

- A propositional literal $c \in L_{CAES}$ is **acceptable** in $C$ or $c \in assumptions$ under sceptical (credulous) $s$ semantics iff all (some) $s$ extensions of $AF$ contain an argument with conclusion $c$. 
 Corresponding argumentation system

Advantages of using ASPIC$^+$ as an intermediate step:

▶ Carneades modelled as knowledge-based argumentation,
▶ Lift the language of Carneades to predicate logic or further,
▶ Argument schemes and ASPIC$^+$ knowledge and rules can be used,
▶ Rationality postulates.
Future work

Multiple avenues for future work:

- **Argument generation** of ASPIC$^+$ lacks detail,
  - Translation and work as done by Besnard and Hunter can be **integrated**,
  - Initial results on **computational complexity**. Should be formalised!
- **Uses of ASPIC$^+$ argumentation system** in translated Carneades should be worked out further,
- **Dialogical** component of Carneades should be related.