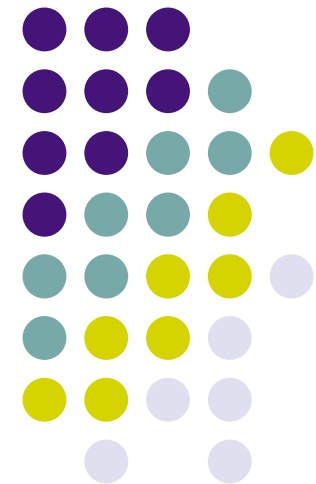


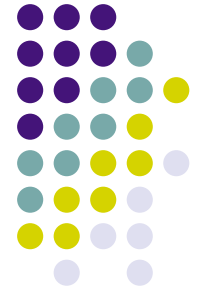
Influence of Selection and Replacement Strategies on Linkage Learning in BOA

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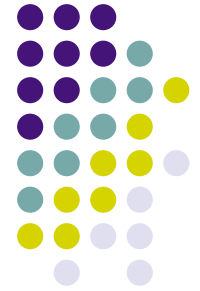


Outline



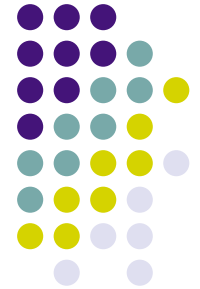
- BOA overview
- Motivation
- How to measure structural accuracy
- Influence of selection and replacement strategies
- Conclusions

BOA: Bayesian optimization algorithm (Pelikan *et al.*, 1999)



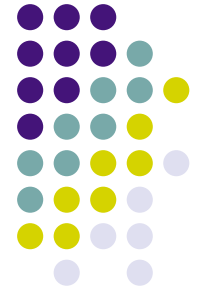
- EDA that builds and samples from a Bayesian network to guide search
- Similar to EBNA (Etzeberria *et al.*, 1999) and LFDA (Muhlenbein, 2000)
- BNs can model complex multivariate dependencies between variables
- Has been successfully applied in real-world problems (Pelikan; 2005, 2006)

A Closer Look at BOA (w/ DTs)



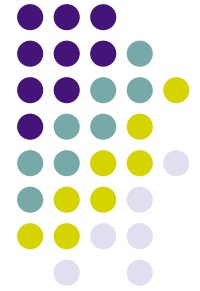
- Looking at the PMs in BOA, one can see:
 - Important dependencies are discovered
 - But with additional spurious linkages

When is Structural Accuracy Crucial?



- **Model-based** efficiency enhancement techniques
 - Evaluation relaxation (Sastry *et al.*, 2004, Pelikan *et al.*, 2004, Sastry *et al.*, 2006)
 - Local search (Lima *et al.*, 2005; Lima *et al.*, 2006)
- **Off-line** usage of linkage information
 - Learn about the problem while trying to solve it
 - Build problem-specific operators with gained knowledge

What can affect model accuracy in BOA?



- Parameters
 - Population Size (Pelikan, 2005)
 - Selection Strategy (Johnson *et al.*, 2001; Santana *et al.*, 2005)
 - Replacement Strategy
- Problem Structure
 - Subfunction Size
 - Subfunction Overlapping Degree
 - Subfunction Fitness Distribution
 - Hierarchy
- Bayesian Network Learning
 - Search Procedure (Wu *et al.*, 2006; Echevoyen *et al.*, 2007)
 - Score Metric (Pelikan, 2005; Correa *et al.*, 2006)

Experimental Setup for Measuring Structural Accuracy of PMs

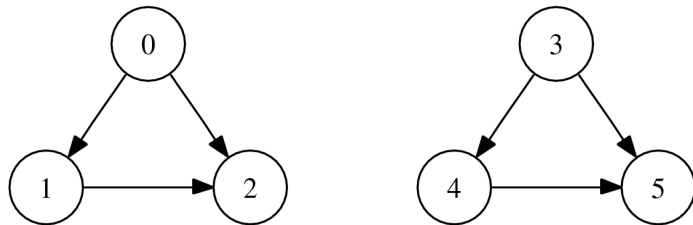


- Test problem(s) structure should be known
- Learning problem structure should be crucial to solve it efficiently
- Easy control of problem size and difficulty in order to test **scalability**
- *m-k* deceptive trap functions:
 - Concatenated **m** trap functions of size **k** each
 - Total problem size is $m \cdot k$ (in case of overlapping, $m(k-o)+o$)
 - **k**-order statistics are required to (efficient) success
 - Lower-order statistics lead the search away from the optimum

Analyzing Probabilistic Models in BOA: an example for Trap3



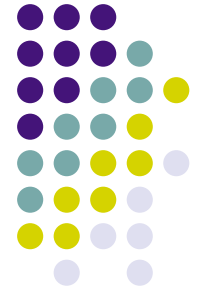
- Ideal BN for Trap3:



- Dependency relations

- $0 \leftarrow \{ \}$
- $1 \leftarrow \{ 0 \}$
- $2 \leftarrow \{ 0, 1 \}$
- $3 \leftarrow \{ \}$
- $4 \leftarrow \{ 3 \}$
- $5 \leftarrow \{ 2, 3 \}$

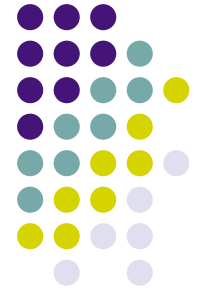
- Ideally, relations should respect a certain chain
- But the dependency that relates all k variables is crucial!
- And most difficult to learn
- In this way, k -order statistics are maintained



What are we going to measure?

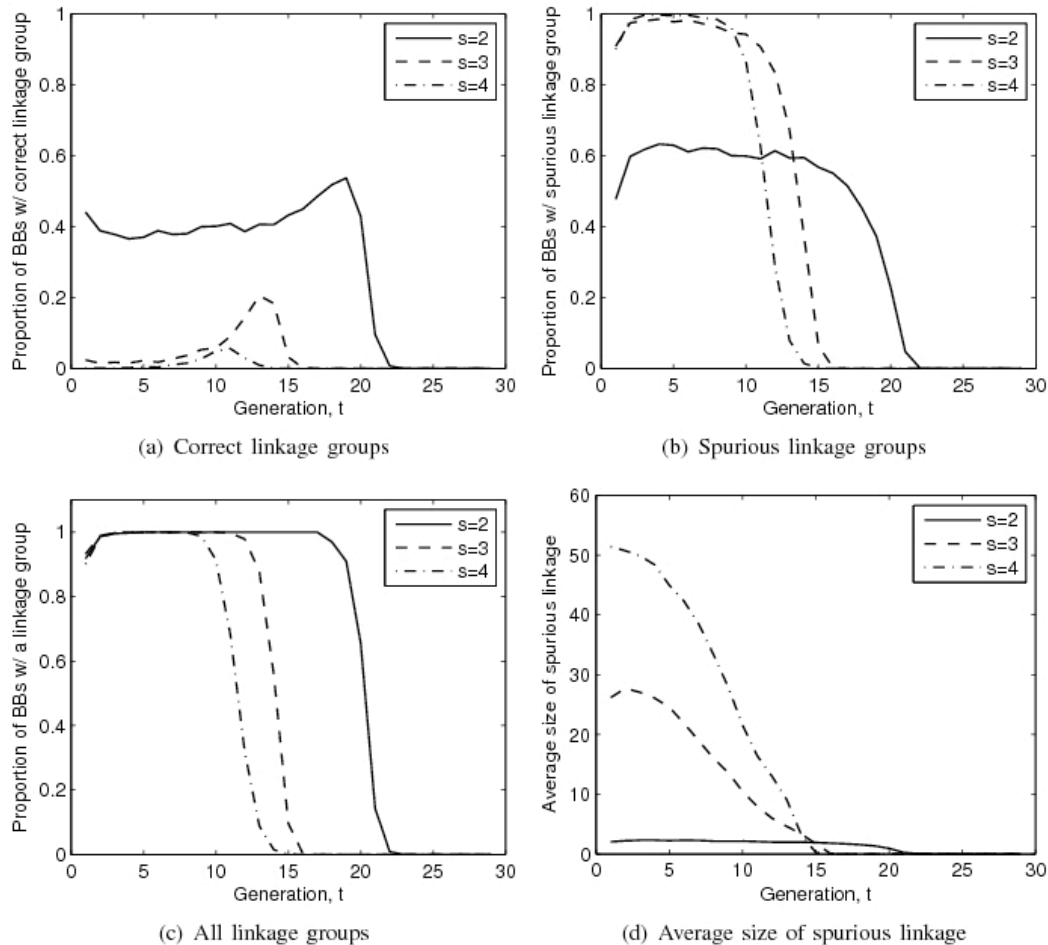
- Proportion of subfunctions with **correct** linkage group
 - Example: $2 \leftarrow \{ 0, 1 \}$
- Proportion of subfunctions with **spurious** linkage group
 - Example: $2 \leftarrow \{ 0, 1, 3, 4, 5 \}$
- Proportion of subfunctions with any of the above
 - Sum of the two previous proportions
- Average size of spurious linkage
 - Number of spurious variables

Influence of Selection Strategy



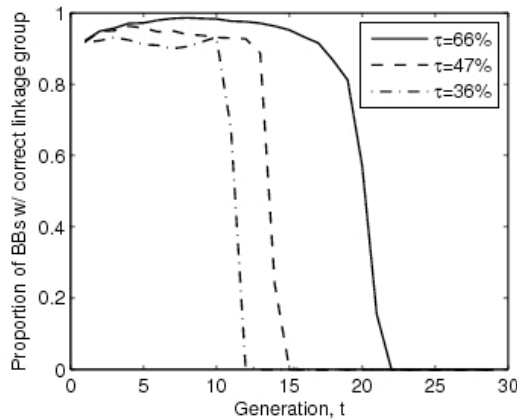
- Tournament Selection
 - Pick best solution from a tournament of s individuals
 - Repeat n times (w/ or wo/ replacement)
- Truncation Selection
 - Choose the best δ % of the population

Tournament Selection

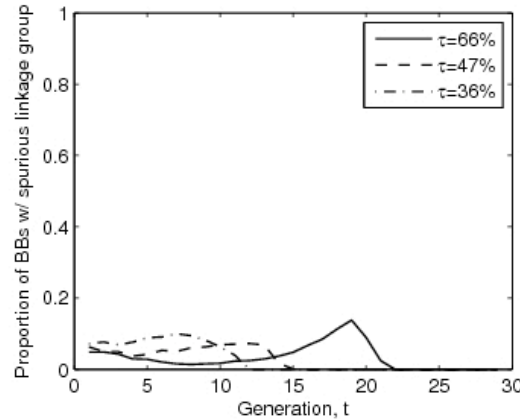


- $m = 24$ and $k = 5$ ($l = 120$)
- Tournament sizes $s = 2, 3, 4$
- Proportion of accurate structures is low (in particular for larger s)
- However, w/ or wo/ spurious linkage, all substructures are represented
- Higher selection intensity increases spurious linkage size

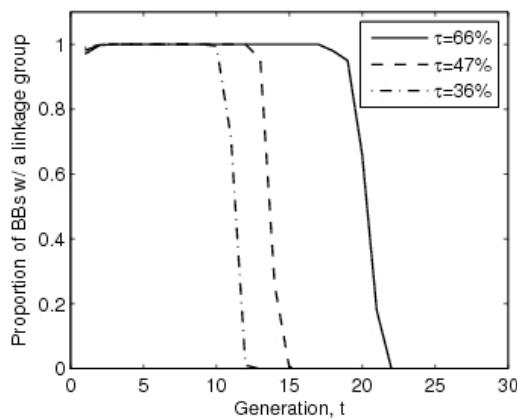
Truncation Selection



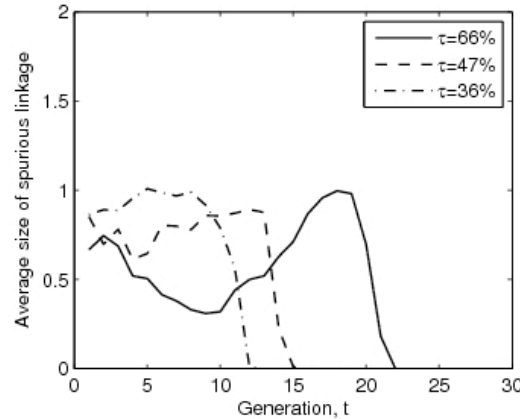
(a) Correct linkage groups



(b) Spurious linkage groups



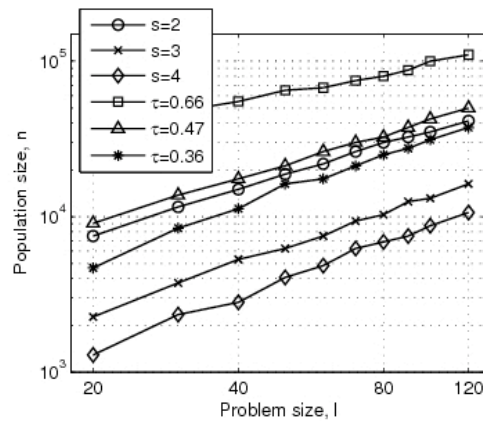
(c) All linkage groups



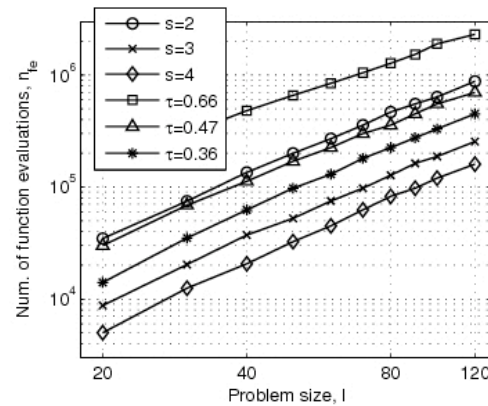
(d) Average size of spurious linkage

- $m = 24$ and $k = 5$ ($l = 120$)
- Truncation thresholds $\delta = 66\%, 47\%, 36\%$
- Same selection intensity as $s = 2, 3, 4$
- Proportion of accurate structures is now close to 100%
- Number of spurious variables is low
- Higher selection intensity has small impact on model structural accuracy

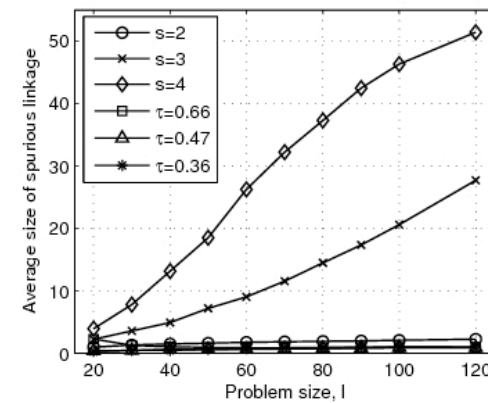
Scalability



(a) Population size



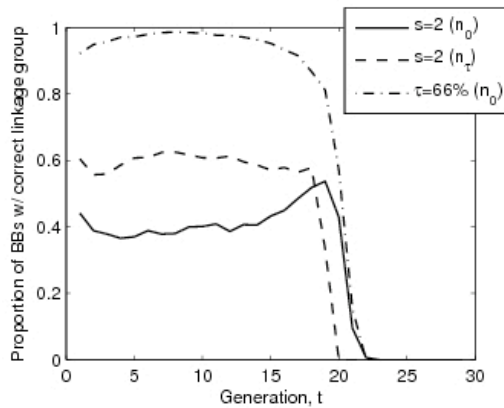
(b) Number of function evaluations



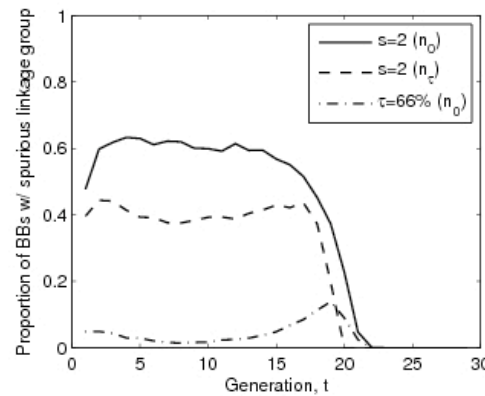
(c) Average size of spurious linkage

- n and n_{fe} required for truncation is higher than for tournament by a significant however constant factor (2-3 times more)

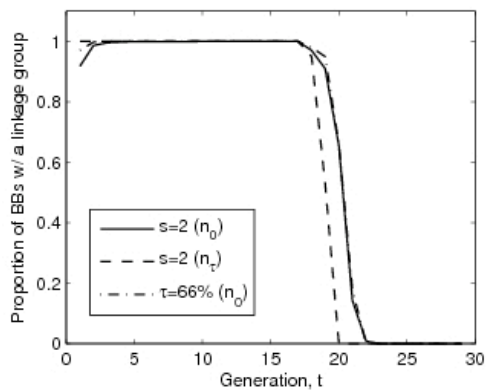
Tournament vs. Truncation: Is it a matter of population size?



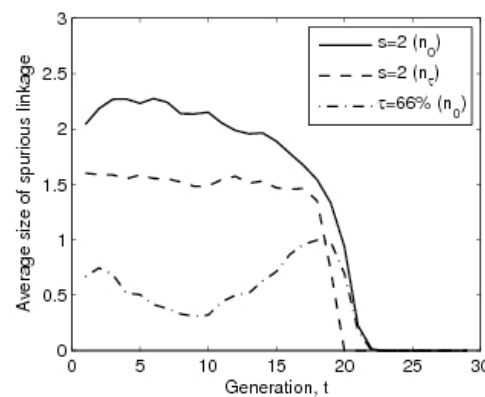
(a) Correct linkage groups



(b) Spurious linkage groups



(c) All linkage groups



(d) Average size of spurious linkage

- $m = 24$ and $k = 5$ ($l = 120$)

- Comparing tournament ($s = 2$) and truncation ($\delta = 66\%$) with the same:

- population size
- selection intensity

- Tournament selection improves accuracy but is still not close to truncation

What is it then?



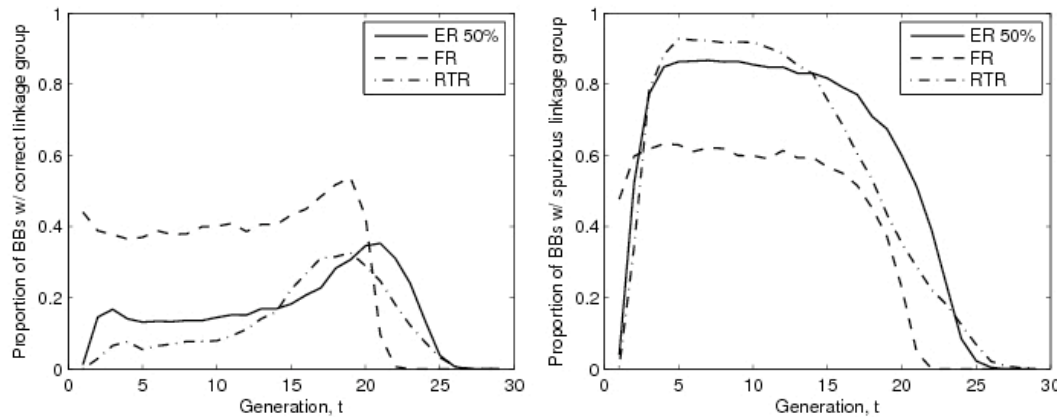
- For the same selection intensity:
 - Truncation has a higher loss of diversity...
 - ...and a lower selection variance (Blickle & Thiele, 1997)
- **But...** distribution of the number of copies in the selected population affects BN learning
 - In tournament the number of copies is somewhat proportional to its rank (best guy gets exactly s copies)
 - In truncation no particular relevance is given to top solutions
- Model overfitting of top individuals takes place (in particular for increasing s)

Influence of Replacement Strategy



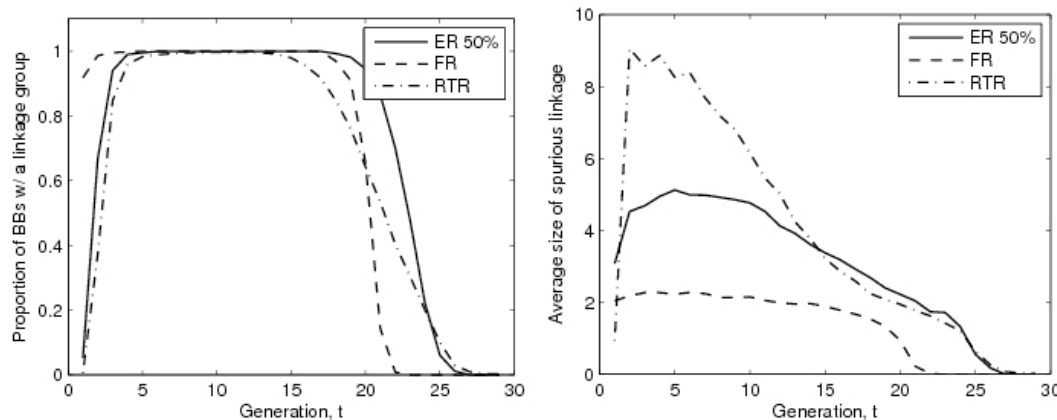
- Full replacement (FR)
 - Offspring population fully replaces the parents
- Elitist replacement (ER)
 - Worst portion of the parents is replaced by offspring
- Restricted tournament replacement (RTR)
 - Offspring directly competes with similar parent for a place in the next population (diversity preservation)

Influence of Replacement (Tournament Selection, $s=2$)



(a) Correct linkage groups

(b) Spurious linkage groups

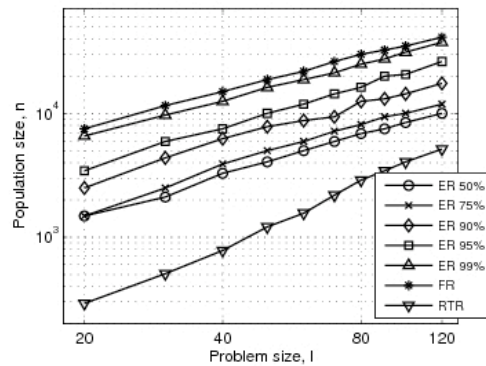
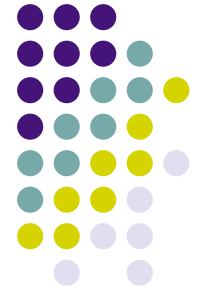


(c) All linkage groups

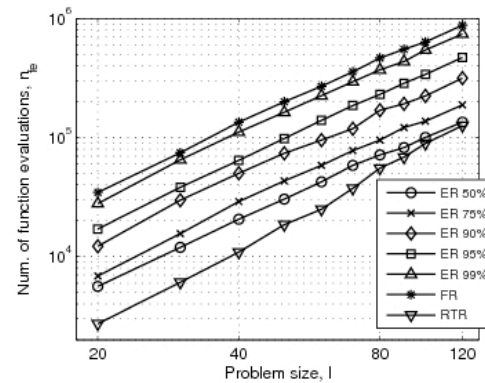
(d) Average size of spurious linkage

- $m = 24$ and $k = 5$ ($l = 120$)
- Comparing ER50%, FR, and RTR with tournament selection ($s = 2$)
- Structural accuracy is higher with FR
- Replacement strategy does not have the same impact as selection on the spurious linkage size
- Spurious linkage is more frequent with RTR because of the smaller pop. size

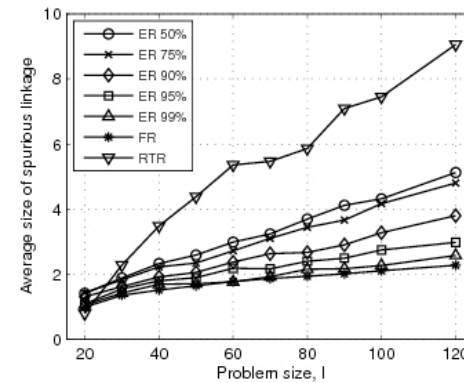
Scalability



(a) Population size



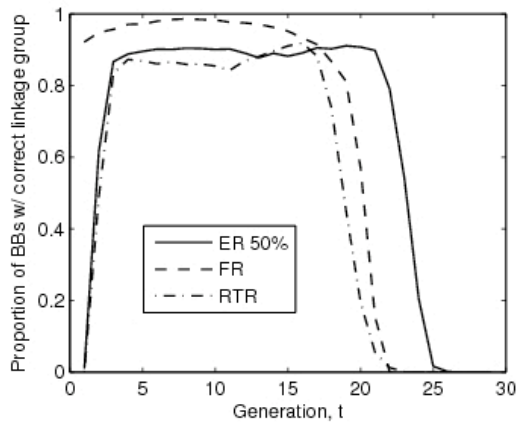
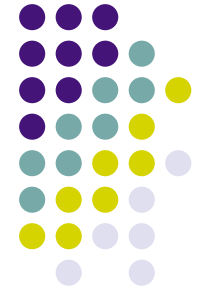
(b) Number of function evaluations



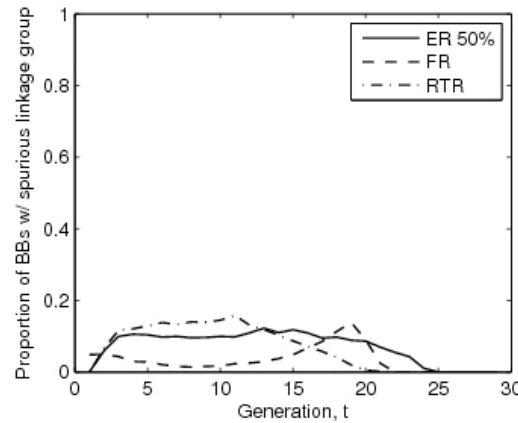
(c) Average size of spurious linkage

- Additional elitist proportions are shown (1-50%)
- RTR clearly requires less n and n_{fe}
- However, linkage information is not so accurate

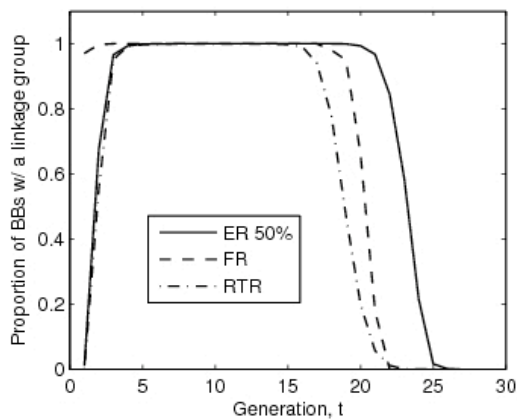
Influence of Replacement (Truncation selection)



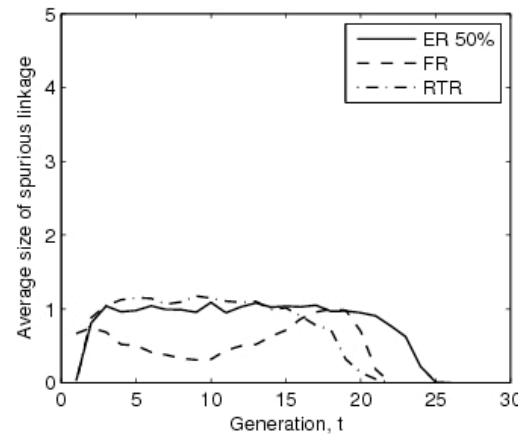
(a) Correct linkage groups



(b) Spurious linkage groups



(c) All linkage groups



(d) Average size of spurious linkage

- $m = 24$ and $k = 5$ ($l = 120$)
- Same comparison with truncation selection ($\delta = 66\%$)
- Structural accuracy is close to 100% for all replacement strategies
- FR is still slightly better because it needs larger n
- Size of spurious linkage is quite low

Conclusions



- Trade-off between model accuracy and overall performance in BOA
- Truncation is better than tournament selection for accurate modelling
- For the same purpose, the replacement method is more relevant if tournament selection is used (in which case full replacement is better)
- If overall performance (num. of evals) is our main concern, tournament and RTR are the best options