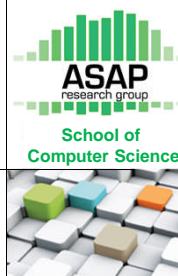


Fundamentals of Hyper-heuristics

Ender Özcan

AI-2014
Thirty-fourth SGAI International
Conference on Artificial Intelligence
Workshop Stream 2

<http://www.cs.nott.ac.uk/~exo/AI2014-HH-slides.pdf>



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Outline

- Basics – Heuristics
- Hyper-heuristics
 - Definition, Classification, Origins
- Generation Hyper-heuristics
 - Case Studies
- Selection Hyper-heuristics
 - HyFlex and Cross-domain Heuristic Search Challenge 2011
 - Case Studies

2

Need for Search Methodologies (Heuristics/Metaheuristics/ Hyper-heuristics, etc...) – Example

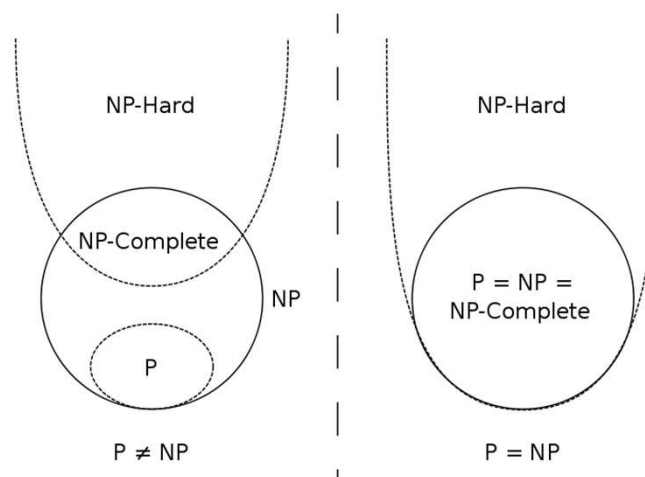


- Travelling salesman problem
- $N=3$, 6
- $N=5$, 120
- $N=7$, 5 040
- $N=10$, 3 628 800
- $N=81$, $\sim 5.8 \times 10^{120}$
- Number of particles in the universe is in between $10^{72} - 10^{87}$
- Tianhe-2: 30.65 PF (10^{15}), $\sim 6 \times 10^{96}$ years



3

Problem Classes



4

Heuristic Search



- Heuristics are rule of thumb methods
- They are informal, judgmental knowledge of area which can be used to arrive at "good" enough solutions to some "hard" problems.
- Good for solving
 - ill-structured problems, or
 - complex well-structured problems (large-scale combinatorial problems that have many potential solutions to explore)

5

Search Paradigms



- Single point based search vs. Multi-point (population) based search
- Constructive
 - partial candidate solutions
- Perturbative
 - complete solutions

6

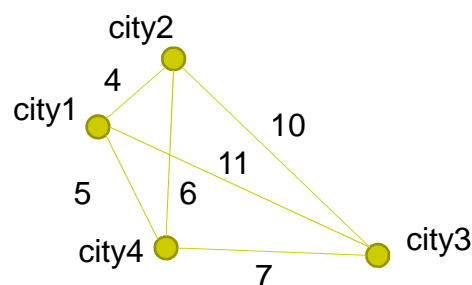
Examples – Heuristics for TSP



- **The nearest neighbour (NN) algorithm**
 - Constructive
- **Pairwise exchange (2-opt), or Lin–Kernighan heuristics**
 - Perturbative

7

The nearest neighbour (NN) algorithm



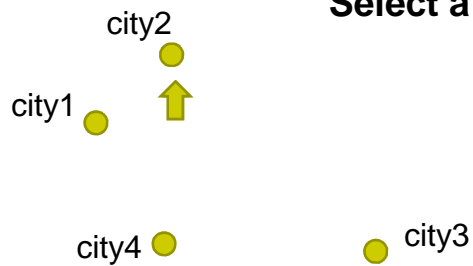
8

The nearest neighbour (NN) algorithm



<city2>

Select a starting city



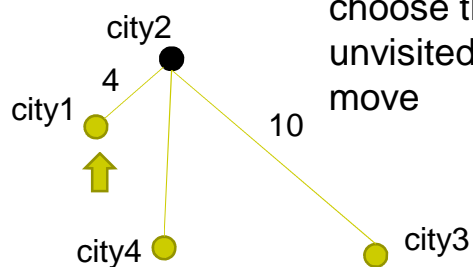
9

The nearest neighbour (NN) algorithm



<city2, >

choose the nearest unvisited city as the next move

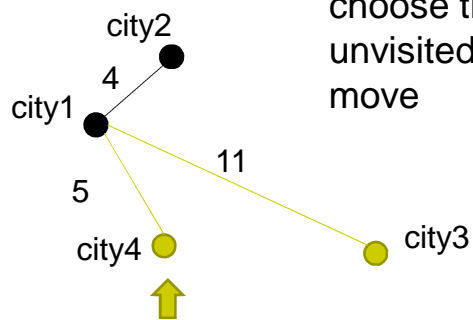


10

The nearest neighbour (NN) algorithm



$\langle \text{city2}, \text{city1}, \rangle$



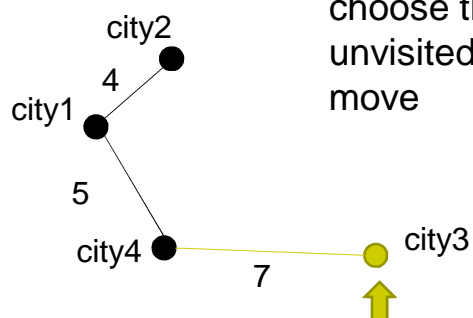
choose the nearest unvisited city as the next move

11

The nearest neighbour (NN) algorithm



$\langle \text{city2}, \text{city1}, \text{city4}, \rangle$



choose the nearest unvisited city as the next move

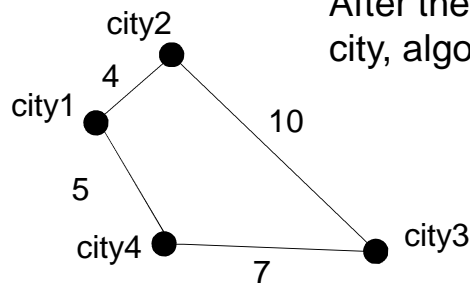
12

The nearest neighbour (NN) algorithm



$\langle \text{city2}, \text{city1}, \text{city4}, \text{city3} \rangle : 26$

After the choice of the last city, algorithm terminates



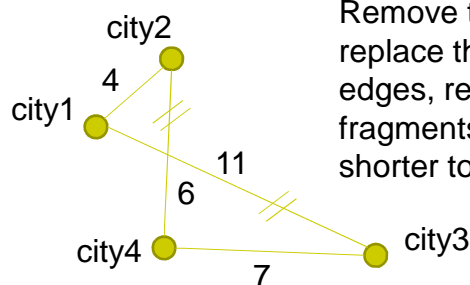
13

Pairwise exchange (2-opt)



$\langle \text{city2}, \text{city1}, \text{city3}, \text{city4} \rangle : 28$

Remove two edges and replace them with two different edges, reconnecting the fragments into a new and shorter tour.

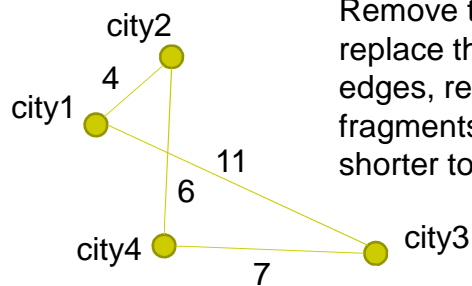


14

Pairwise exchange (2-opt)



<city2, city1, city3, city4> : 28



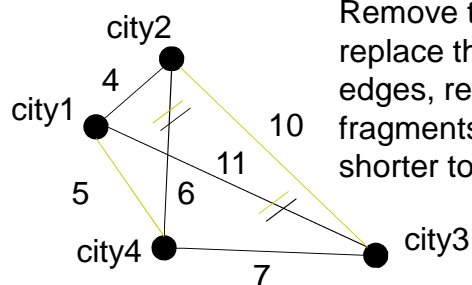
Remove two edges and replace them with two different edges, reconnecting the fragments into a new and shorter tour.

15

Pairwise exchange (2-opt)



<city2, city1, city3, city4> : 28

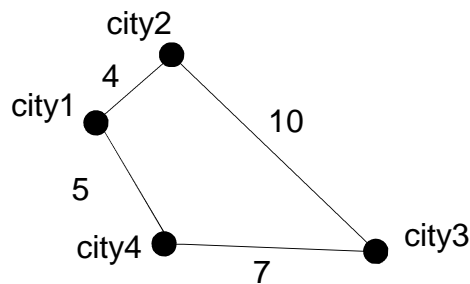


Remove two edges and replace them with two different edges, reconnecting the fragments into a new and shorter tour.

16

Pairwise exchange (2-opt)

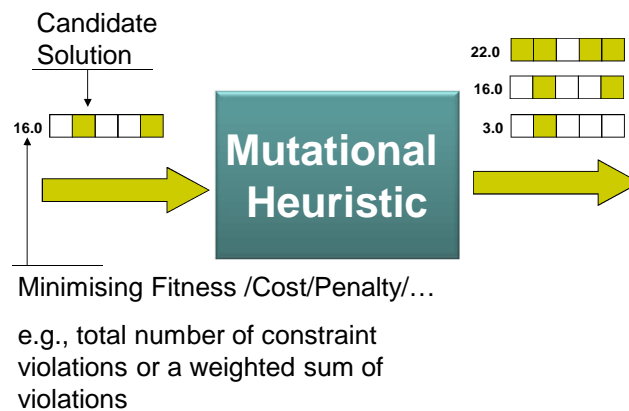
<city2, city1, city3, city4> : 26



17

Mutational Heuristic

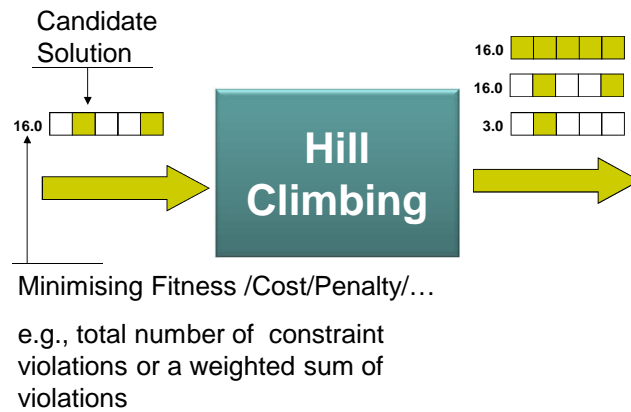
Processes a given candidate solution and generates a solution which is not guaranteed to be better than the input



18

Hill Climbing Heuristic

Processes a given candidate solution and generates a better or equal quality solution



19

Hyper-heuristics



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Problem with Heuristics? – Bin Packing



- Place a set of N items with given sizes {e.g., $N=33$ items: 1x85, 1x442, 6x10, 7x252, 2x9, 5x127, 4x106, 3x12, 1x84, 1x46, 2x37} into minimal number of bins, each having a fixed capacity of C (e.g., $C=524$)

How would you do it?

21

Sort Items → First Fit Heuristic							
	Bin1	Bin2	Bin3	Bin4	Bin5	Bin6	Bin7
442	442						
252		252					
252		252					
252			252				
252			252				
252				252			
252				252			
127					127		
127					127		
127						127	
127						127	
106						106	
106							106
106							106
85							85
84							84
46 – removed							
37						37	
37							37
12	12						
12	12						
12	12						
10		10					
10		10					
10			10				
10			10				
10				10			
10				10			
9					9		
9					9		
524 524 524 524 524 524 524							
Instance#1							

	Bin1	Bin2	Bin3	Bin4	Bin5	Bin6	Bin7	Bin8
442	442							
252		252						
252		252						
252			252					
252			252					
252				252				
252				252				
127					252			
127					127			
127					127			
127						127		
106						106		
106							106	
106							106	
85							85	
84							84	
37								37
37								37
12		12						
12			12					
12				12				
10					10			
10						10		
10							10	
10							10	
10								10
9								9
9								9
516 516 516 516 516 517 516 9								
Instance#2								

Problem with Heuristics? – Examination Timetabling



Problem	[24]	[19]	[21]	[26]	[20]	[12]	[27]	[28]	ALC
car91	7.1	4.97	5.03	<i>4.97</i>	5.17	6.6	<u>4.6</u>	4.8	5.12
car92	6.2	4.32	4.22	<i>4.28</i>	4.32	6.0	<u>3.9</u>	4.1	4.41
ears83 I	36.4	36.16	36.06	35.86	<i>35.70</i>	<u>29.3</u>	32.8	34.92	36.91
hec92 I	10.8	11.61	11.71	11.85	11.93	<u>9.2</u>	10.0	10.73	<i>11.31</i>
kfu93	14.0	15.02	16.02	<i>14.62</i>	15.30	13.8	<u>13.0</u>	<u>13.0</u>	14.75
lse91	10.5	10.96	11.15	<i>11.14</i>	11.45	<u>9.6</u>	10.0	10.01	11.41
pur93 I	3.9	-	-	<i>4.73</i>	-	<u>3.7</u>	-	4.73	5.87
rye92	7.3	-	9.42	9.65	-	<u>6.8</u>	-	9.65	<i>9.61</i>
sta83 I	161.5	161.90	158.86	158.33	159.05	158.2	<u>156.9</u>	158.26	157.52
tre92	9.6	8.38	8.37	<i>8.48</i>	8.68	9.4	7.9	<u>7.88</u>	8.76
uta92 I	3.5	3.36	3.37	3.40	<i>3.30</i>	3.5	<u>3.2</u>	<u>3.2</u>	3.54
ute92	25.8	27.41	27.99	28.88	28.00	<u>24.4</u>	24.8	26.11	<i>26.25</i>
vor83 I	41.7	40.77	39.53	40.74	40.79	36.2	<u>34.9</u>	36.22	<i>39.67</i>

S. Abdul-Rahman, A. Bargiela, E. K. Burke, E. Özcan, B. McCollum and P. McMullan, Adaptive Linear Combination of Heuristic Orderings in Constructing Examination Timetable, European Journal of Operational Research, 232 (2), pp. 287-297, 2014

23

Metaheuristic – Definition



A high-level problem-independent algorithmic framework that provides a set of guidelines or strategies to develop heuristic optimisation algorithms

Source: Glover, F. And Sorensen, K. In: Encyclopaedia of OR/MS, Springer Verlag, Berlin (to appear)

24

Sophisticated Metaheuristics



- Simulated annealing
 - Tabu search
 - Iterated Local Search
 - GRASP
 - Evolutionary computation
 - Evolutionary strategies, Genetic algorithms, Memetic algorithms, Genetic programming
 - Ant colony optimization
- and more...

25

Random Mutation Hill Climbing vs. Iterated Local Search



Algorithm 1: Pseudocode of the iterated local search method

```

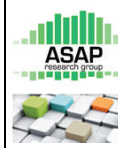
Let  $S$  represent the candidate solution;
 $S_{initial} \leftarrow \text{CreateInitialSolution}();$ 
→  $S \leftarrow \text{LocalSearch}(S_{initial});$ 
repeat
   $S' \leftarrow \text{Perturbation}(S);$ 
  →  $S'' \leftarrow \text{LocalSearch}(S');$ 
  if  $\text{Accept}(S, S'')$  then
     $S \leftarrow S'';$ 
  end
until  $\text{TerminationCriteriaSatisfied}();$ 

```

H. R. Lourenco, O. C. Martin, and T. Stutzle. Iterated local search: framework and applications. In M. Gendreau and J.-Y. Potvin (eds), *Handbook of Metaheuristics*, vol. 146 of *International Series in Operations Research and Management Science*, pp. 363–397, 2010.

26

Genetic Algorithm vs. Memetic Algorithm



Algorithm 2: Pseudocode of memetic algorithm

```

CreateInitialSolutions(); // create initial population of solutions
→ LocalSearch();
repeat
    Evaluate(); // calculate fitness of each solution in the population
    SelectParents(); // select solutions from the population to breed
    Crossover(); // apply crossover operator with a given probability
    Mutate(); // apply mutation operator with a given probability
→ LocalSearch();
→ Evaluate();
    ReplaceSolutions(); // generate new population of solutions
until TerminationCriteriaSatisfied();
  
```

Moscato, P.: On evolution, search, optimization, genetic algorithms and martial arts: Towards memetic algorithms, Caltech Concurrent Computation Program Report 826, California Institute of Technology (1989)

27

OBSERVATIONS



- Most of the real-world problems are proven to be NP-hard
- The current state of the art in search methodologies (i.e., metaheuristics) tend to focus on bespoke systems
 - In general, these systems are expensive to build, but provide successful results
 - Unfortunately, their application to new problem domains or even new problem instances from a known domain or a slight change in the problem definition could still require expert involvement.

28

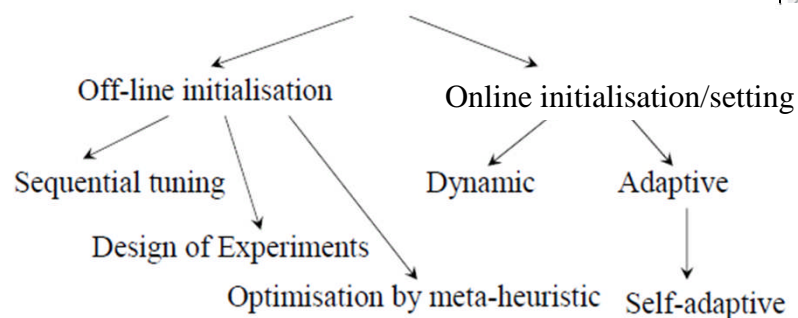
Drawbacks of (meta)heuristic search



- There is no guarantee for the optimality of the obtained solutions.
 - May give a poor solution.
- Usually can be used only for the specific situation for which they are designed.
- Often, (meta)heuristics have some parameters
 - Performance of a heuristic could be sensitive to the setting of those parameters

29

Parameter Tuning Parameter Control



- **ParamILS**: F. Hutter, D. Babic, H. H. Hoos, and A. J. Hu, "Boosting verification by automatic tuning of decision procedures," in Proc. of the Formal Methods in Computer Aided Design, ser. FMCAD '07. IEEE Computer Society, 2007, pp. 27–34.
- **iRace**: M. Lopez-Ibanez, J. Dubois-Lacoste, T. Stutzle, and M. Birattari, "The irace package, iterated race for automatic algorithm configuration," IRIDIA, Universite Libre de Bruxelles, Belgium, Tech. Rep. TR/IRIDIA/2011-004, 2011

Hyper-heuristics



A hyper-heuristic is a search method or learning mechanism for *selecting* or *generating* heuristics to solve computationally difficult problems

- A class of general purpose search methodologies with the common goal of automating the design and tuning of heuristic methods

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Motivation – Grand Challenge

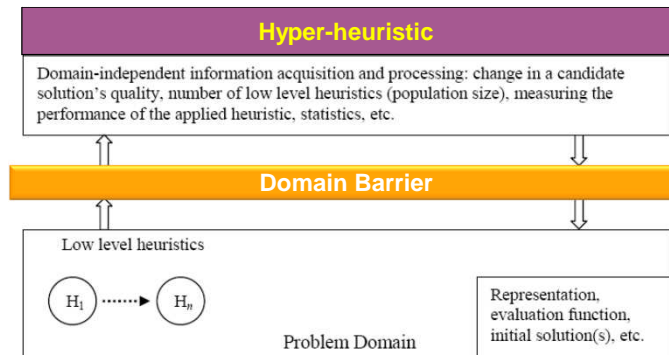


- Automating the search/heuristic design process
 - ▀ Motivated by raising the level of generality.



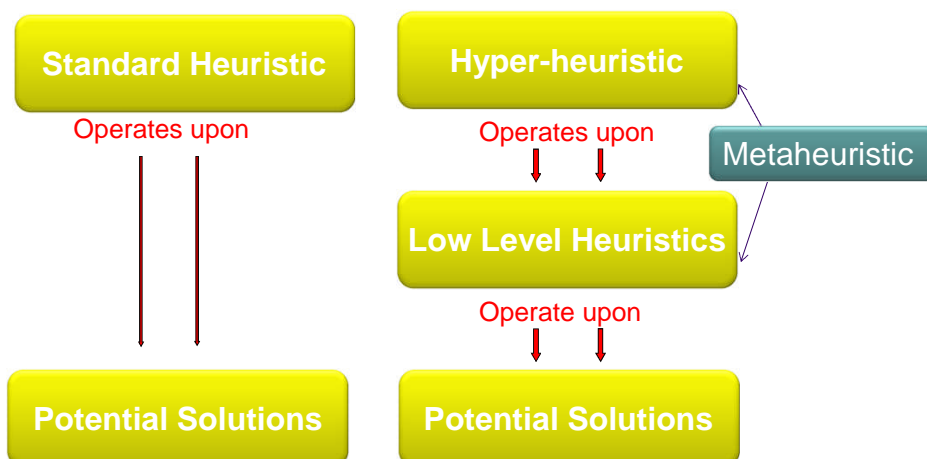
32

A Hyper-heuristic Framework



33

Different Search Spaces



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Characteristics of Hyper-heuristics



- Operate on a **search space of heuristics** rather than directly on a search space of solutions
- Existing (or computer generated) heuristics can be used within hyper-heuristics
- Aims to take advantage of strengths and avoid weaknesses of each heuristic
- No problem specific knowledge is required during the search over the heuristics space (and so hyper-heuristic components are reusable)
- Easy to implement/deploy/use (easy, cheap, fast)
- Applicable to a range of real-world problems
- Extremely desirable: Employs data science (i.e., machine learning) techniques

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Related Areas



- Reactive search
- Algorithm portfolios
- Co-evolution, multimeme memetic algorithms
- Adaptive operator selection
- Parameter tuning
- Parameter control in EAs
- Variable Neighbourhood Search
- Meta-learning
- Algorithm configuration
- Cooperative Search

...

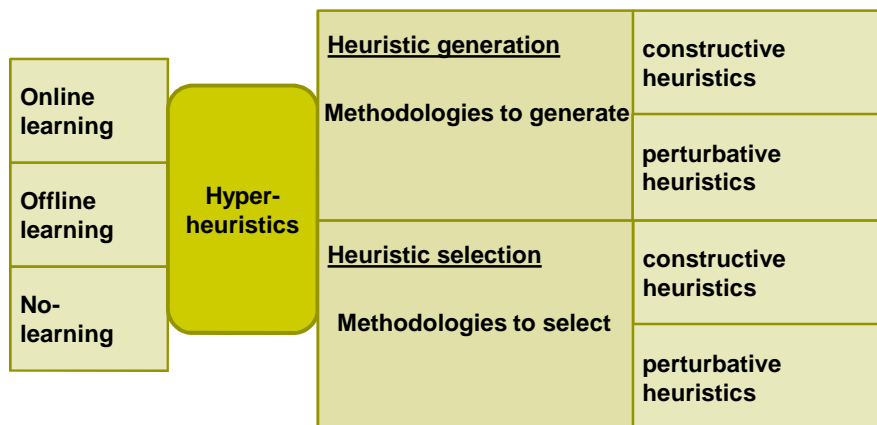
36

A Classification of Hyper-heuristics



Feedback

Nature of the heuristic search space



37

Hyper-heuristics: Origins

Cowling P.I., Kendall G. and Soubeiga E. (2001): "A Hyperheuristic Approach to Scheduling a Sales Summit", selected papers from PATAT 2000, Springer, LNCS 2079, 176-190.



Fisher H. and Thompson G.L. Probabilistic Learning Combinations of Local Job-shop Scheduling Rules. Ch 15,:225-251, Prentice Hall, New Jersey, 1963

Crowston W.B., Glover F., Thompson G.L. and Trawick J.D. Probabilistic and Parameter Learning Combinations of Local Job Shop Scheduling Rules. ONR Research Memorandum, GSIA,CMU, Pittsburgh, (117), 1963

High Performance ATP Systems by Combining Several AI Methods

Jörg Denzinger, Matthias Fuchs

reproduce this run. So \bar{C} can be seen as the description of a "hyper-heuristic" and is used instead of a single heuristic H when storing data regarding distributed proofs.

MANAGEMENT SCIENCE
Vol. 36, No. 10, October 1990
Printed in U.S.A.

NEW SEARCH SPACES FOR SEQUENCING PROBLEMS WITH APPLICATION TO JOB SHOP SCHEDULING*
ROBERT H. STORER, S. DAVID WU and RENZO VACCARI
Department of Industrial Engineering, Lehigh University, Bethlehem, Pennsylvania 18013

A Promising Hybrid GA/Heuristic Approach for Open-Shop Scheduling Problems

Heinrich Pang¹ and Peter Ross¹ and David Corne²
In Proceedings of the 11th European Conference on Artificial Intelligence, John Wiley and Sons, 1994, pages 590-594.

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October 29, 1996

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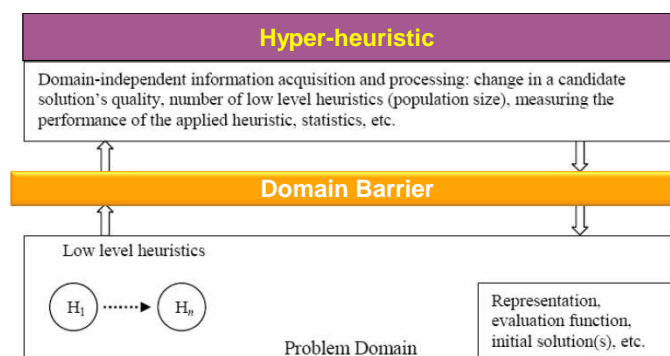
Generation Hyper-heuristics



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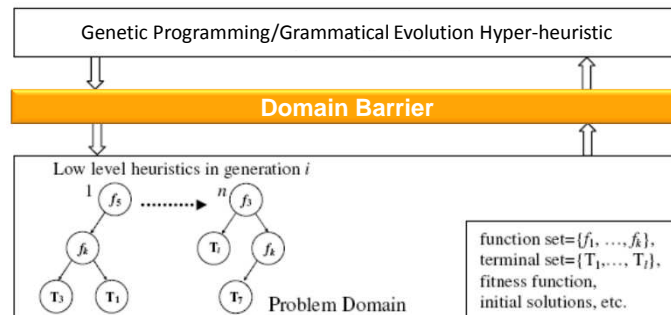
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A Hyper-heuristic Framework – revisited



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A Generation Hyper-heuristic Framework



45

Some Java based Software Libraries



- ECJ: <http://cs.gmu.edu/~eclab/projects/ecj/>
- TinyGP: <http://cswww.essex.ac.uk/staff/rpoli/TinyGP/>
- GEVA (grammatical evolution): <http://ncra.ucd.ie/Site/GEVA.html>
- Cartesian GP resources: <http://www.cartesiangp.co.uk/resources.html>

46

Case Study: Genetic Programming Hyper-heuristic for Packing

from the PhD Thesis (2010) of
Matthew Hyde



The University of
Nottingham

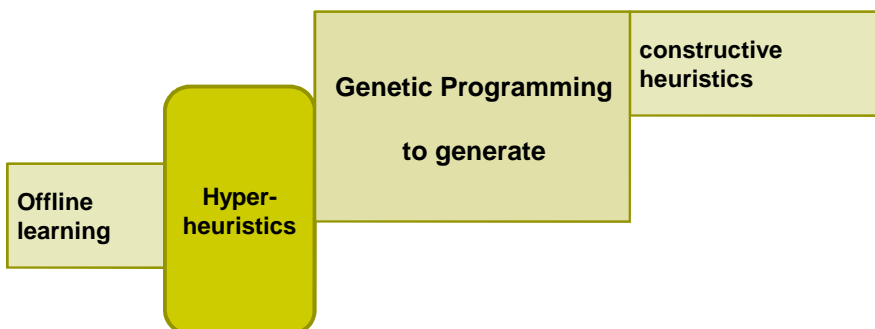
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Classification of the Approach



Feedback

Nature of the heuristic search space



1D Offline Bin Packing



Pack a **set** of items of sizes s_i for $i = 1, \dots, n$

- Sizes are integer values and $s_i \in [1, C]$
- C is the fixed capacity of each bin

in such a way that

- Never exceed bin capacity
- Minimise number of bins used

Standard NP-hard problem

49

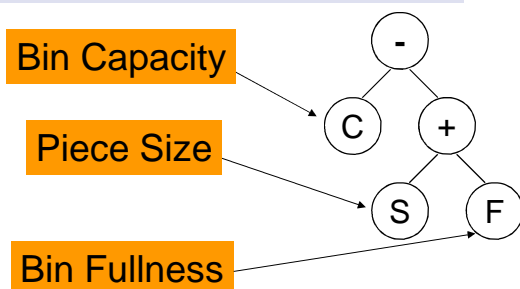
Genetic Programming 101



- Evolves trees representing a program
- Following tree is a program that calculates the space left at the top of the bin
- Train and test

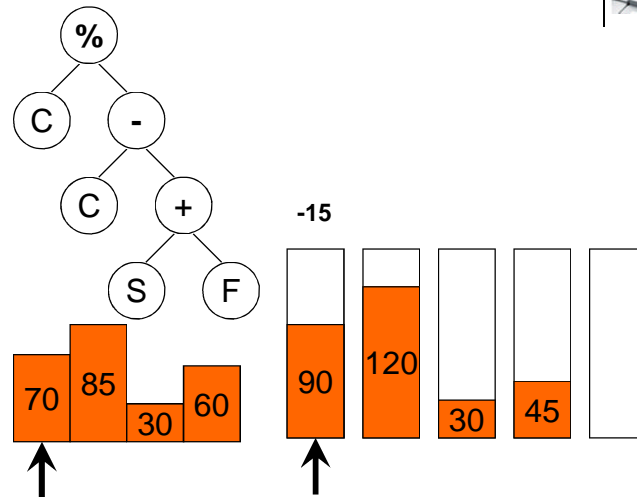
Terminals: {C, S, F}

Non terminals: {%, +, *, -}



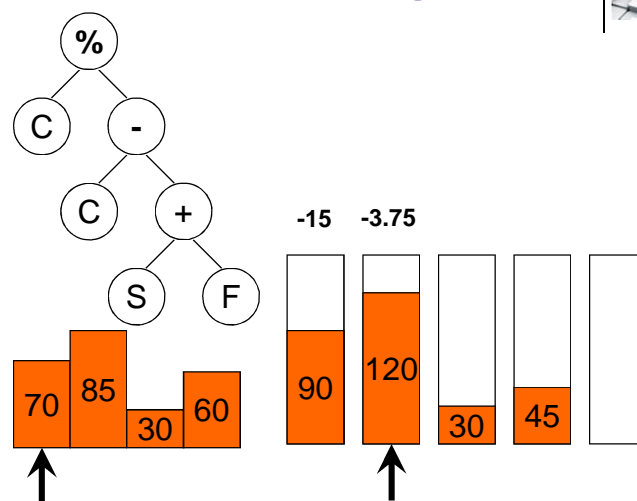
50

Genetic Programming Heuristics – Bin Packing



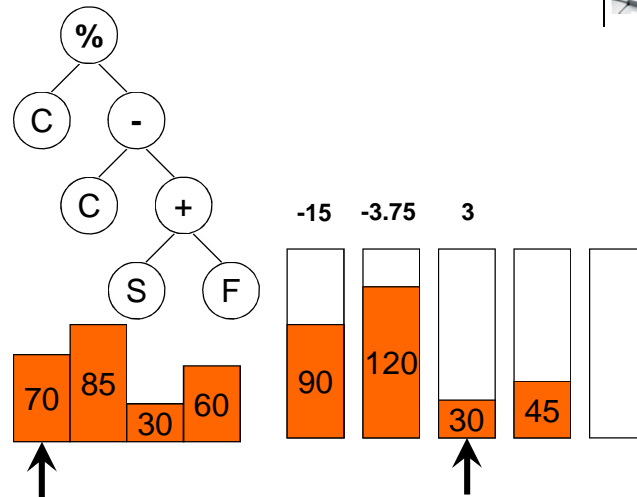
51

Genetic Programming Heuristics – Bin Packing



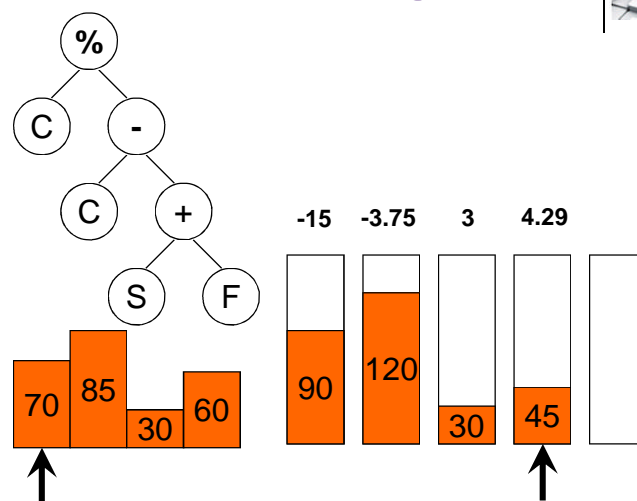
52

Genetic Programming Heuristics – Bin Packing



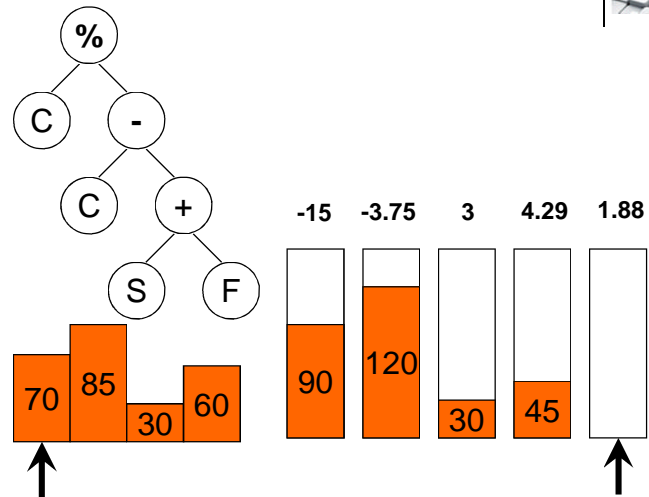
53

Genetic Programming Heuristics – Bin Packing



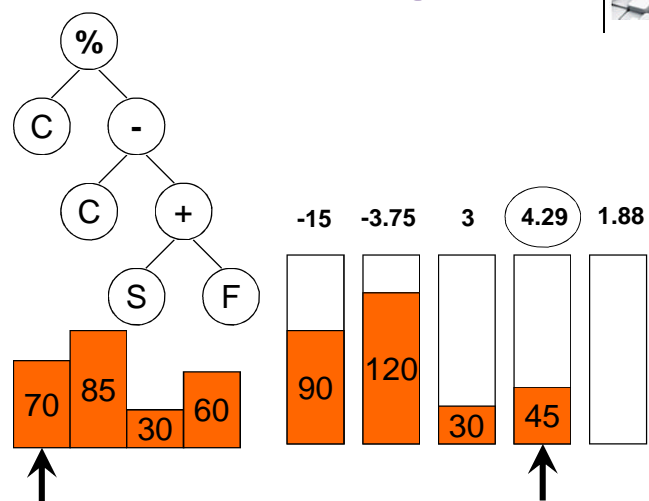
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Genetic Programming Heuristics – Bin Packing



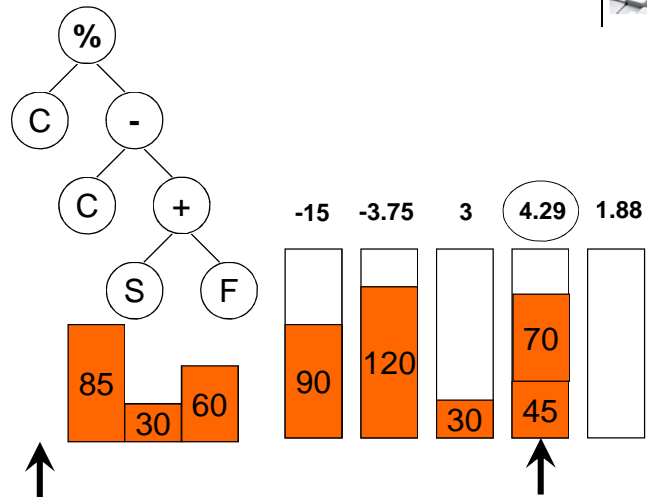
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Genetic Programming Heuristics – Bin Packing



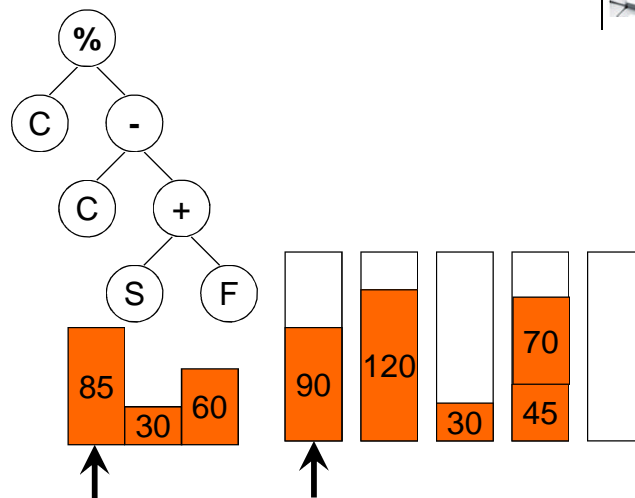
56

Genetic Programming Heuristics – Bin Packing



57

Genetic Programming Heuristics – Bin Packing



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GP Hyper-heuristic for packing – Conclusions



- A **more general** packing methodology for 1D, 2D and 3D bin packing and knapsack problems
- **Achieved generality without the loss of solution quality**

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Policy Matrix Evolution for Generation of Heuristics

Ender Özcan
Joint work with
Andrew J. Parkes

Best paper Award:

E. Özcan, and A. J. Parkes, Policy Matrix Evolution for Generation of Heuristics, Proceedings of the 13th Annual Conference on Genetic and Evolutionary Computation (GECCO '11), Natalio Krasnogor (Ed.). ACM, New York, NY, USA, pp. 2011-2018, 2011



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Nottingham

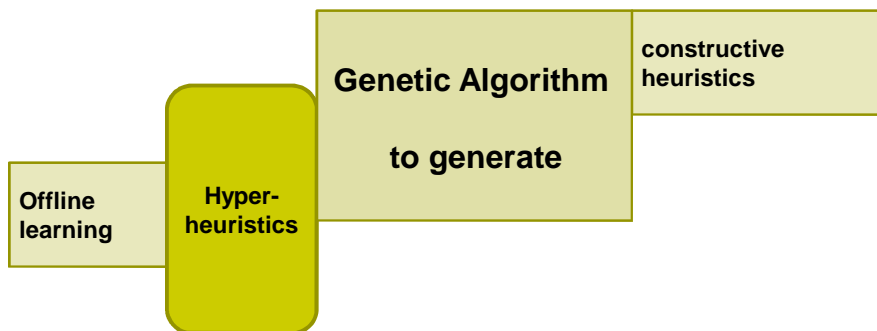
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Classification of the Approach



Feedback

Nature of the heuristic search space



61

Policy Generation



- Vast O.R. literature on finding policies for **stochastic processes**. Potential usages
 - Customer service centres
 - Call centres
 - Health services
 - Distribution centres
 - items onto trucks
 - etc
- In some cases analytical solutions are possible
- Generally, will need “numerical” methods for complex situations

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1D Offline Bin Packing



Pack a **set** of items of sizes s_i for $i = 1, \dots, n$

- Sizes are integer values and $s_i \in [1, C]$
- C is the fixed capacity of each bin

in such a way that

- Never exceed bin capacity
- Minimise number of bins used

Standard NP-hard problem

63

1D Online Bin Packing



Pack a **stream** of items of sizes s_i for $i = 1, \dots$

- Sizes are integer values and $s_i \in [1, C]$
- C is the fixed capacity of each bin

upon their arrival (one item at a time)

in such a way that

- Never exceed bin capacity
- Minimise number of bins used (Maximise the average bin-fullness) **at the end**

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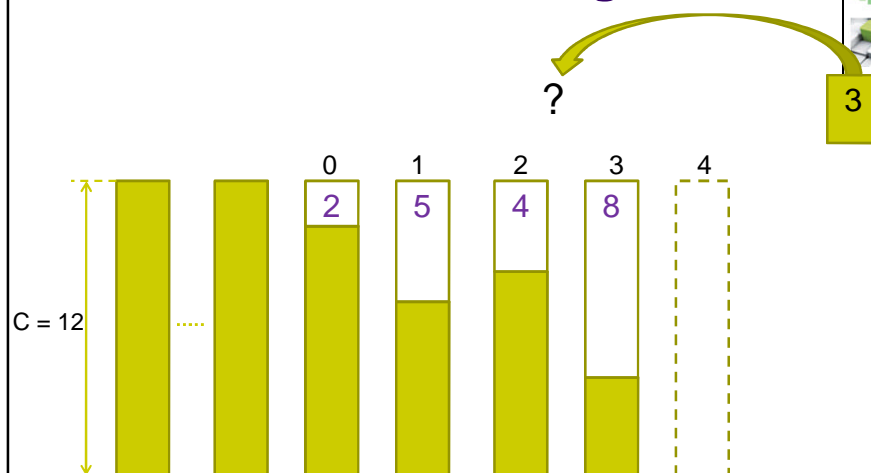
1D Online Bin Packing



- A new empty bin is always available (**open**)
- A bin is **closed** if it can take no more items
 - e.g. if **residual space** is smaller than size of any item
- We need a *good* “policy”, i.e. a method to assign a new item upon its arrival to one of the *open* bins

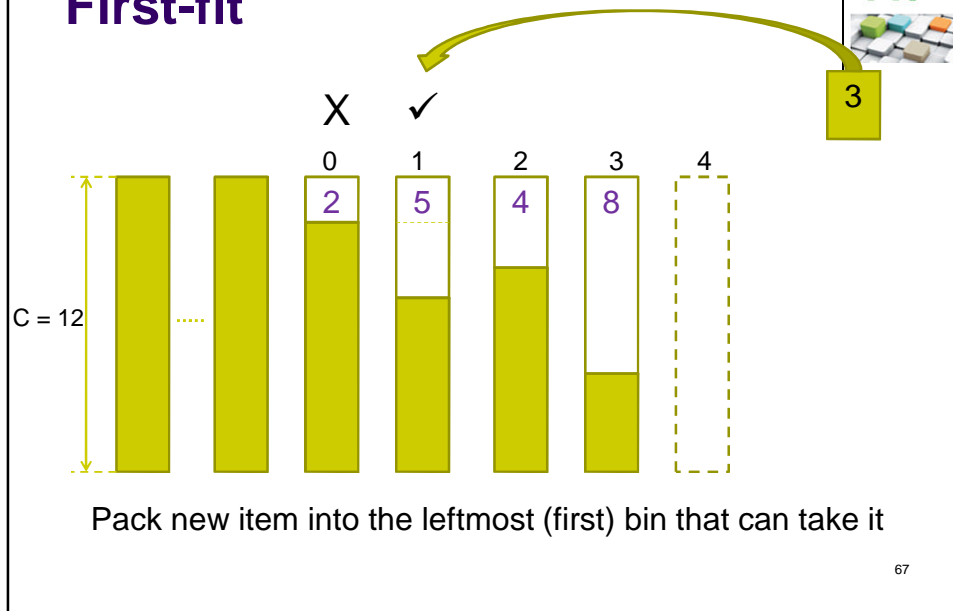
65

1D Online Bin Packing

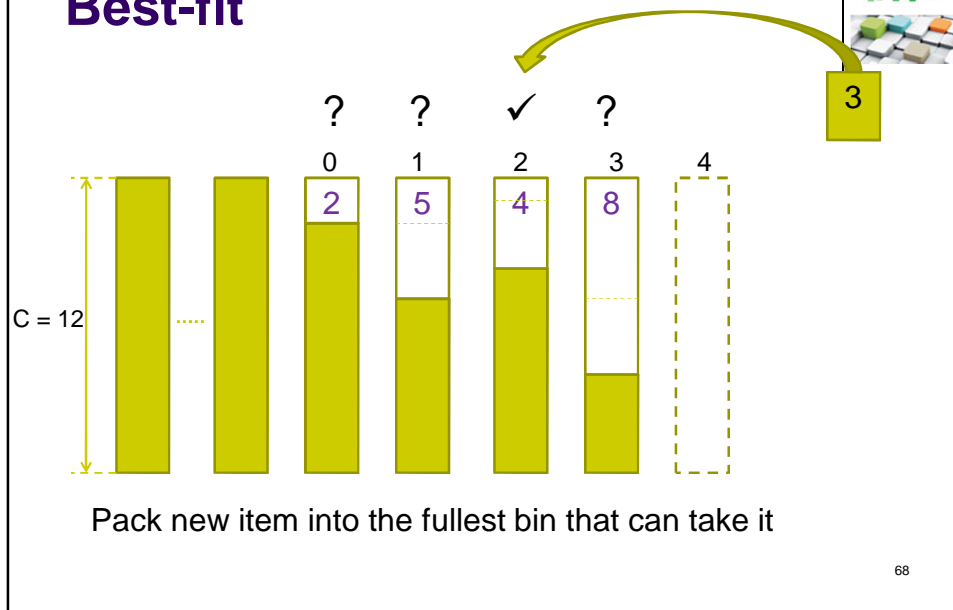


66

Standard Heuristic Policies: First-fit



Standard Heuristic Policies: Best-fit



Potential General Method for 1D Online Bin Packing



- On arrival of new item of size s_i
 - Inspect the current set of open bins
 - Simultaneously use the entire set of residual spaces in the open bins to pick where to place the new item
- This is difficult and expensive (in general)

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“Index Policies”



- “index policy”: each choice option is given a score, or “index value” independently of the other options
 - The option with the highest index value is taken
 - Also need a rule to break ties
- Although index policies are a special case, in some situations, they can be optimal, or at least very good
- Index policies occur in bandit problems, with use in search control
 - OR has lots of work in this area, e.g. Gittins/Whittle indices

70

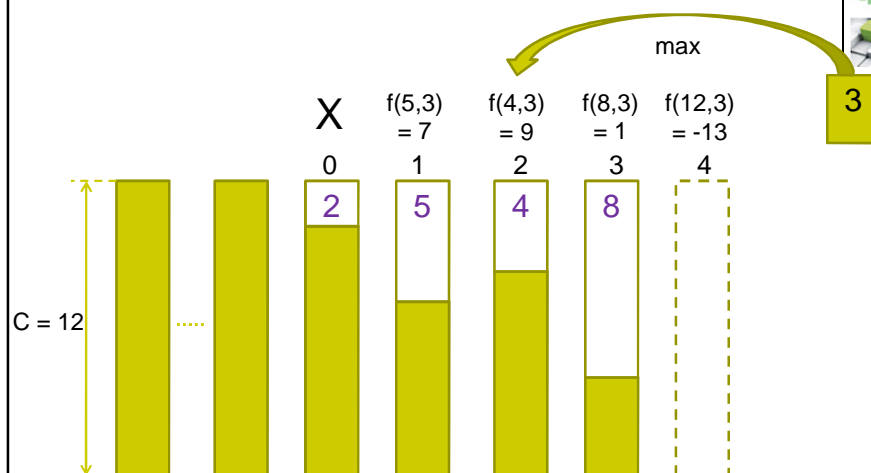
Index Policies for 1D Online Bin Packing



- Given
 - r : remaining capacity of bin (residual space)
 - s : item size
- score of bin is $f(r,s)$ for some function f
- Given a new item of size then place into bin with largest value of $f(r,s)$
- We will break any ties using FF:
 - **place item in earliest bin with the best available score**

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1D Online Bin Packing



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Generating Heuristics



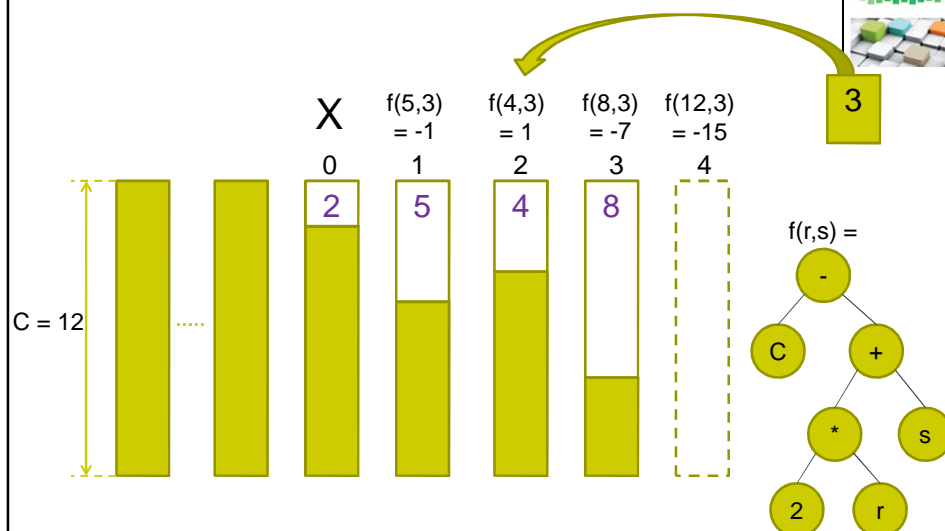
- Within search methods, often have score functions, “index functions” to help make some choice
 - difficult to invent successful ones; want to automate this
- GP approach: evolve arithmetic score functions
- Burke, Hyde, Kendall, Woodward (GECCO 2007)

(and other papers, also on other problem domains, please see <http://www.cs.nott.ac.uk/~mvh/>)

 - Use Genetic Programming to learn $f(r,s)$
 - $f(r,s)$ is represented as arithmetic function tree
 - Automatically creates functions that at least match FF, BF

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GP – 1D Online Bin Packing



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Generating Heuristics



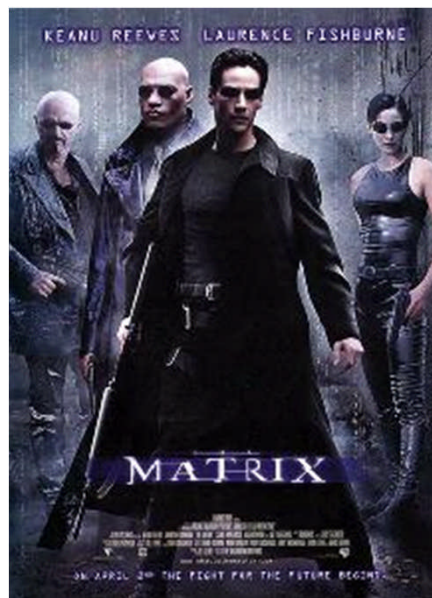
Challenge:

- Space of functions, as used in GP,
 - is hard to understand
 - potentially biased because of the choice of representation
 - some perfectly good functions might have “bloated” representations

Can one do even better?

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Is there
another way
to find
policies?



Source: http://en.wikipedia.org/wiki/The_Matrix

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Matrix View of Policies/Heuristics



- Since all item sizes (s) and residual capacities (r) are integer, then $f(r,s)$ is simply a large ($C \times C$) matrix $M(r,s)$ of **parameter values**

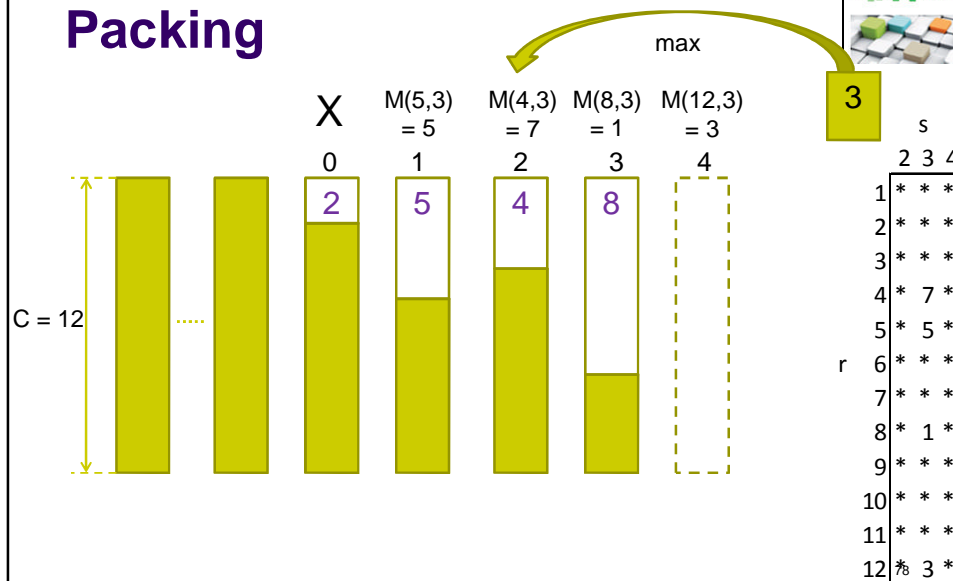
$M(r,s)$

$r \setminus s$	1	2	3	4	5	6
1
2	.	2
3	.	1	2	.	.	.
4	.	2	1	.	.	.
5
6	.	2	2	.	.	.

Diagram illustrating the matrix $M(r,s)$ and its relationship to residual capacity C . The matrix is shown with a diagonal line separating the region $r \geq s$ (below the diagonal) from the region $r < s$ (above the diagonal). The region $r < s$ is labeled "NOT USABLE". Arrows indicate the mapping from the matrix to the residual capacity C .

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Policy Matrix – 1D Online Bin Packing



Uniform (random) Instances



We empirically studied matrix policies on Uniform Bin Packing problems

$$\text{UBP}(C, s_{\min}, s_{\max}, N)$$

- Bin capacity C
- N items are generated with integer sizes independently taken uniformly at random from the range $[s_{\min}, s_{\max}]$
 - N is usually taken to be large: 100k

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UBP(6,2,3)



- (Bin capacity 6, items are size 2 or 3 only.)
- The only perfect packings are
 - 2+2+2
 - 3+3
- Hence the 'obvious' policy is ...
- ... "never mix even and odd sizes"

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UBP(6,2,3)



- ... “never mix even and odd sizes”
- Index policy as a matrix:
 - rows: residual capacity of the bin
 - columns: item size

resid \ item	1	2	3	4	5	6
1
2	.	2
3	.	1	2	.	.	.
4	.	2	1	.	.	.
5
6	.	2	2	.	.	.

- Ties are broken using First-Fit (FF)
- Grey entries “.” are never usable

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Creating Heuristics via Many Parameters - CHAMP



- Basic idea:
 - Take values in matrix $M(r,s)$ to be integers
 - Do (meta-)heuristic search to find good choices for $M(r,s)$: Evaluation is by simulation
- Our Original Expectation:
 - the matrix will tweak the functions from GP and might slightly improve performance
- Potential expected disadvantages:
 - matrices can be much more verbose than functions
 - they fail to take into account of the good structure captured by functions

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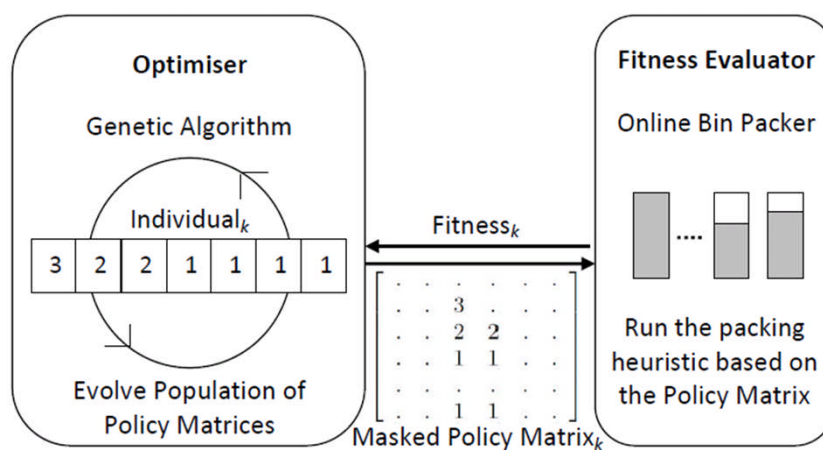
Implementation Details



- Apply a standard Genetic Algorithm
 - Trans-generational (with weak elitism), Uniform Crossover, standard mutation
- Only the active members of the matrix are stored as integer values in the chromosome
- Evaluation:
 - write matrix to a file
 - use matrix as input for a program that packs many items

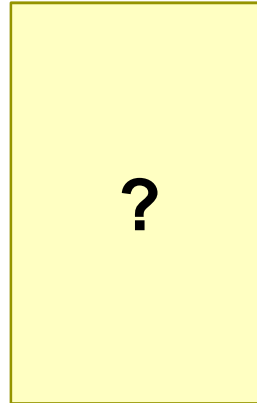
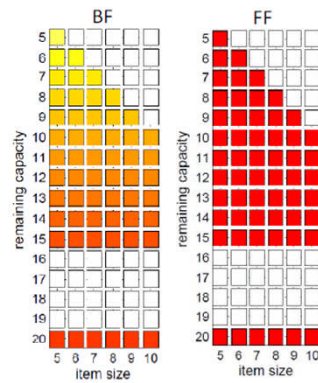
83

CHAMP-GA Architecture



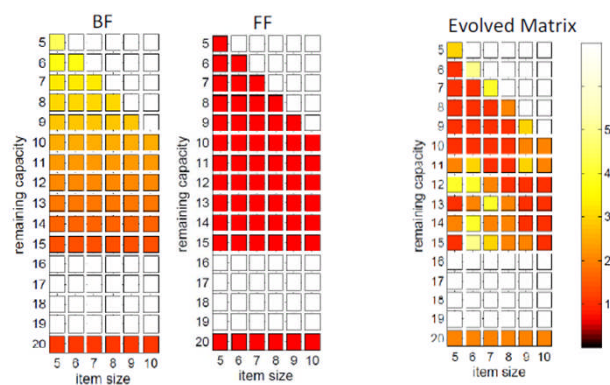
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UBP(20,5,10) – Example of a good evolved matrix



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UBP(20,5,10) – Example of a good evolved matrix



- Does not look like a smooth function
 - “Weird”
 - Seems to have spikes

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UBP(20,5,10)



• Empirical results

Heuristic	%-Avg. Fullness
First-Fit	91.55
Best Fit	91.54
"Best run" Evolved Matrix	98.18
"Worst run" Evolved Matrix	97.00

- Even the worst run of the GA outperforms FF
- The gap is quite large – the wasted space is reduced by a factor of ~7

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Results – Best of runs for GA



Alg.	UBP(6.2.3)	UBP(15.5.10)	UBP(20.5.10)	UBP(30.4.20)	UBP(30.4.25)	UBP(40.10.20)	UBP(60.15.25)	UBP(75.10.50)	UBP(80.10.50)	UBP(150.20.200)
BF	92.30	99.62	91.55	96.84	98.38	90.23	92.55	96.08	96.39	95.82
FF	92.30	99.55	91.54	96.68	97.93	90.22	92.55	95.91	96.29	95.64
GA1	99.99	99.63	98.18	99.41	98.39	96.99	99.68	98.22	98.54	97.88
GA2	99.99	99.61	98.42	99.58	99.55	96.75	96.96	98.45	98.46	97.63

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Conclusions

- Can use standard metaheuristics to create policies expressed in matrix representation
 - Policies exist that out-perform standard heuristics
 - Finding the policies is easier than expected
 - There are many different policies with similar performance
 - The policies are “weirder” than expected, even after smoothing.
 - The good policies could have “random” structures
 - Not necessarily easy to capture with an algebraic function of GP
 - The results can be “analysed” (inspected) to produce simple policies that out-perform standard ones
 - and that then scale to larger problems



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Recent Work: Genetic Programming Hyper-heuristics for Scheduling



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Job Shop Scheduling



- Single objective: Rachel Hunt, Mark Johnston, Mengjie Zhang, **Evolving "less-myopic" scheduling rules for dynamic job shop scheduling with genetic programming**, Proc. of the 2014 conference on Genetic and evolutionary computation, pp. 927-934, 2014
- Multi-objective: Su Nguyen, Mengjie Zhang, Johnston, M., Kay Chen Tan, **Automatic Design of Scheduling Policies for Dynamic Multi-objective Job Shop Scheduling via Cooperative Coevolution Genetic Programming**, *Evolutionary Computation, IEEE Transactions on*, vol.18, no.2, pp.193,208, 2014

91

How does an evolved rule look like?



```

Rule #1 - Objectives(757.16,3520.19,0.17,164.52,1811.77)
(((IF(SJ,RJ,max(PR,WT)))+(max(RO,RT)+(RJ/IF(SJ,PR,rJ))))-WINQ)+(((max(RO,RT)+
IF(SJ,IF(SJ,PR,rJ)))+(-1*(IF(SJ,PR,rJ))+IF(SJ,DD/PR,rJ)))-min(SJ,(WINQ*min(PR,WINQ))))
-Abs((rJ-RT)+min(min(SJ,IF(SJ,PR,rJ)),(rJ-RT)))
Rule #2 - Objectives(828.45,2322.88,0.19,165.04,1931.40)
(-rJ-SJ+max(RO,RT)+(((R/PR)+max(RO,RT))+max(PR,max(RO,RT)))+(-PR-RT))-0.8968051)
-Abs(IF(min(SJ,WINQ),WINQ,DD/PR)+Abs(min(SJ,WINQ)))
Rule #3 - Objectives(720.28,4383.52,0.09,105.67,2401.59)
((max(RM,(Abs(min(WT,SJ))*(RT*PR)))/PR)/Abs(PR+RO))/Abs(max(((PR*max(RT,max(APR,SJ)))*(PR
+WINQ)),((Abs(PR)*(RT*PR))*DD)/Abs(min(WT,(SJ/APR))))))
Rule #4 - Objectives(716.52,3842.36,0.11,82.67,1714.88)
((max((PR*APR),(Abs(min(WT,SJ))*WINQ))/PR)/%WINQ)/Abs(max(((PR*PR)*(max(Abs(RT),max(APR,SJ))
+min(WT,(SJ/APR))))),((PR*WINQ)*DD)/Abs(min(WT,(SJ/APR))))))
Rule #5 - Objectives(708.05,4141.63,0.13,109.08,1977.63)
((((RT/rJ)+rJ)/max(min(DD,SJ),RT))-min(-(IF(SJ,RJ,NPR)/(SJ+WINQ)),DD))+(-WINQ+(-RO-
min(min(SJ,WINQ),rJ))))+((max(SJ,rJ)+((IF(SJ,RJ,-RO)/PR)-(rJ+max((WINQ+PR),0.371))))-NPR)
Rule #6 - Objectives(687.85,5708.02,0.16,134.13,4046.06)
Abs((((R/SJ)/PR)/PR)/max(APR,WINQ))*Abs((((S/APR)-SJ)/min(RT,SJ))*min(RT,SJ))
/min(((R/SJ)*(R/SJ)),RT))

```

S. Nguyen, M. Zhang, M. Johnston, and K-C. Tan, Dynamic Multi-objective Job Shop Scheduling: A Genetic Programming Approach, Automated Scheduling and Planning, Studies in Comp. Intelligence vol. 505, 2013, pp 251-282

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Flexible Job Shop Scheduling



- Single objective: Beham, A; Winkler, S.; Wagner, S.; Affenzeller, M., **A genetic programming approach to solve scheduling problems with parallel simulation**, *Parallel and Distributed Processing, 2008. IPDPS 2008. IEEE International Symposium on*, pp.1-5, 2008
- Multi-objective: Joc Cing Tay, Nhu Binh Ho, **Evolving dispatching rules using genetic programming for solving multi-objective flexible job-shop problems**, *Computers & Industrial Engineering*, Vol. 54, Issue 3, 2008, pp. 453-473

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Single Machine Scheduling



- Single objective: C. Dimopoulos, A.M.S. Zalzala, **Investigating the use of genetic programming for a classic one-machine scheduling problem**, *Advances in Engineering Software*, Volume 32, Issue 6, June 2001, Pages 489-498
- Multi-objective: S. Nguyen, M.Zhang, M. Johnston, K. C. Tan, **Learning Reusable Initial Solutions for Multi-Objective Order Acceptance and Scheduling Problems with Genetic Programming**, *Proc. of the 16th European Conference on Genetic Programming, EuroGP 2013*, pp 157-168, LNCS 7881, 2013

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Others

- Parallel Machine: Domagoj Jakobović, Leonardo Jelenković, Leo Budin, **Genetic Programming Heuristics for Multiple Machine Scheduling**, LNCS 4445, 2007, pp 321-330
- Flow shop scheduling: Franco Mascia, Manuel López-Ibáñez, Jérémie Dubois-Lacoste, Thomas Stützle, **From Grammars to Parameters: Automatic Iterated Greedy Design for the Permutation Flow-Shop Problem with Weighted Tardiness**, LNCS 7997, pp 321-334, 2013



95

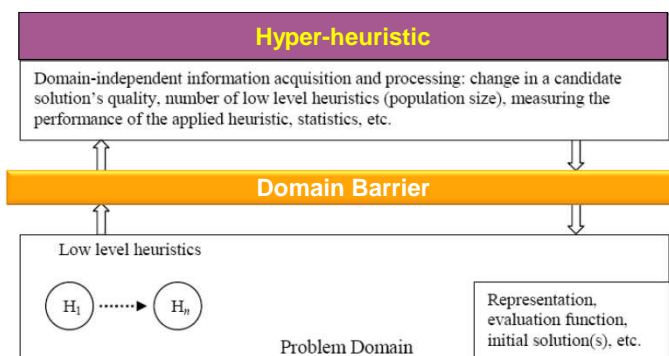
Selection Hyper-heuristics



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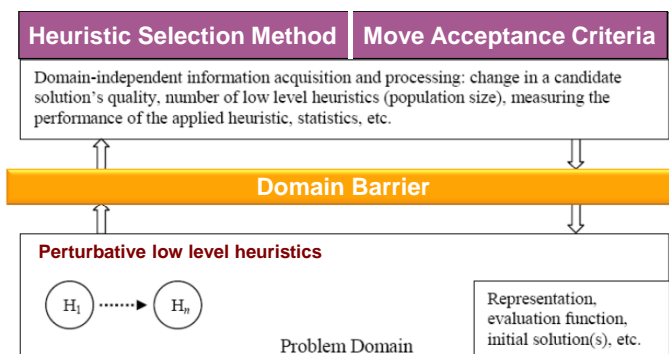
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A Hyper-heuristic Framework – revisited



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A Selection Hyper-heuristic Framework – Single Point Search



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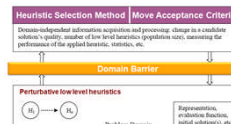
A Selection Hyper-heuristic Framework – Single Point Search



1. generate initial candidate solution p
2. while (termination criteria not satisfied){
3. select a heuristic (or subset of
 heuristics) h from $\{H_1, \dots, H_n\}$
4. generate a new solution (or solutions) s
 by applying h to p
5. decide whether to accept s or not
6. if (s is accepted) then
7. $p=s$
8. return p ;

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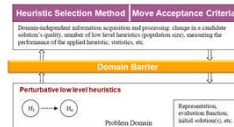
Heuristic Selection



Component name	Reference(s)
Heuristic selection with no learning	
Simple Random	Cowling et al (2000, 2002b)
Random Permutation	Cowling et al (2000, 2002b)
Heuristic selection with learning	
Peckish	Cowling and Chakhlevitch (2003)
Greedy	Cowling et al (2000, 2002b); Cowling and Chakhlevitch (2003)
Random Gradient	Cowling et al (2000, 2002b)
Random Permutation Gradient	Cowling et al (2000, 2002b)
Choice Function	Cowling et al (2000, 2002b)
Reinforcement Learning	Nareyek (2003); Pisinger and Ropke (2007); Bai et al (2007a)
Reinforcement Learning with Tabu Search	Burke et al (2003b); Dowsland et al (2007)
Learning Automata	Misir et al. (2009)
Quality Index and Tabu based Learning Heuristic Selection	Misir et al. (2009)

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Move Acceptance



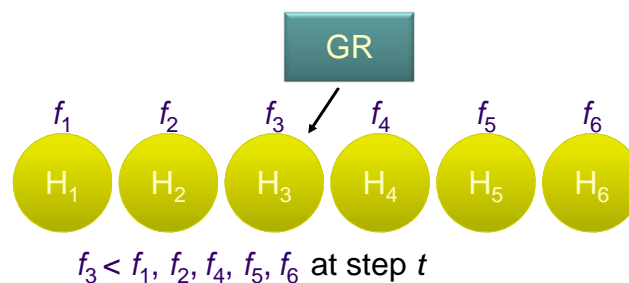
Component name	Reference(s)
Deterministic move acceptance	
All Moves	Cowling et al (2000, 2002b)
Only Improvements	Cowling et al (2000, 2002b)
Improving and Equal	Cowling et al (2000, 2002b)
Non-deterministic move acceptance	
Monte Carlo	Ayob and Kendall (2003)
Great Deluge	Kendall and Mohamad (2004a); Bilgin et al (2006)
Record to Record Travel	Kendall and Mohamad (2004b)
Tabu Search	Chakhlevitch and Cowling (2005)
Simulated Annealing	Bai and Kendall (2005); Bilgin et al (2006); Pisinger and Ropke (2007); Antunes et al (2009)
Simulated Annealing with Reheating	Dowsland et al (2007); Bai et al (2007a)
Late Acceptance	Özcan et al (2009)
Iteration Limited Threshold Accepting (ILTA)	Mısırlı et al. (2009)
Adaptive ILTA	Mısırlı et al. (2009)

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Heuristic Selection – Greedy (GR)



- Apply each low level heuristic to the candidate solution and choose the one that generates the best objective value



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Heuristic Selection – Reinforcement Learning (RL)



- A machine learning technique
- Inspired by related psychological theory
 - Reward and punishment
- Concerned with how an agent ought to take actions in an environment to maximize some notion of long-term reward
- Maintains a score for each heuristic
 - If an improving move then increase, otherwise decrease the score of the heuristic

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Heuristic Selection – Choice Function (CF)

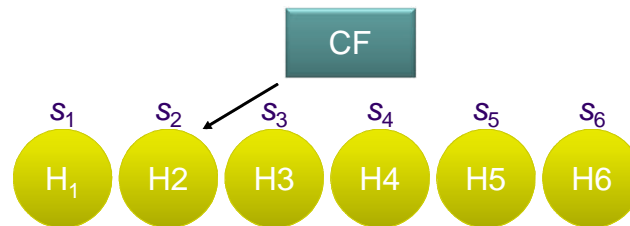


- The choice function maintains a record of the performance of each heuristic. Three criteria are maintained:
 - 1) Its individual performance
 - 2) how well it has performed with other heuristics
 - 3) the elapsed time since the heuristic has been called

$$F_t(h_j) = \alpha_t f_1(h_j) + \beta_t f_2(h_k, h_j) + \gamma_t f_3(h_j)$$

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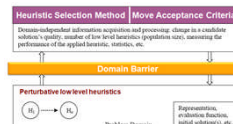
Heuristic Selection – Choice Function (CF)



$s_2 > s_1, s_3, s_4, s_5, s_6$ at step t

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Move Acceptance



Component name	Reference(s)
Deterministic move acceptance	
All Moves	Cowling et al (2000, 2002b)
Only Improvements	Cowling et al (2000, 2002b)
Improving and Equal	Cowling et al (2000, 2002b)
Non-deterministic move acceptance	
Monte Carlo	Ayob and Kendall (2003)
Great Deluge	Kendall and Mohamad (2004a); Bilgin et al (2006)
Record to Record Travel	Kendall and Mohamad (2004b)
Tabu Search	Chakhlevitch and Cowling (2005)
Simulated Annealing	Bai and Kendall (2005); Bilgin et al (2006); Pisinger and Ropke (2007); Antunes et al (2009)
Simulated Annealing with Reheating	Dowsland et al (2007); Bai et al (2007a)
Late Acceptance	Özcan et al (2009)
Iteration Limited Threshold Accepting (ILTA)	Misir et al. (2009)
Adaptive ILTA	Misir et al. (2009)

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Move Acceptance – Simple Criteria



- **AM:** All Moves Accepted
- **OI:** Only Improving Moves accepted
- **IE:** Improving or Equal moves are accepted.

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Move Acceptance – Great Deluge (GD)



- Improving and equal moves are accepted
- Non-improving moves resulting in a fitness value less than a threshold are accepted.
- The threshold is decreased to global minimum with time.
 - N : initial fitness – minimum fitness
 - t : time passed
 - D : Duration

$$f_t < f_{\min} + N \times \left(1 - \frac{t}{D}\right)$$

f_t
 ↓
 current
 fitness

 $f_{\min} + N \times \left(1 - \frac{t}{D}\right)$
 ↓
 threshold

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Move Acceptance – Simulated Annealing



- All improving moves are accepted while the non-improving are accepted based on Metropolis criterion ($e^{-\delta/\tau}$), where τ represents temperature, being decreased at each iteration using a *cooling schedule*, and δ is the change in the solution quality.
- Previous studies show that simulated annealing is one of the best move acceptance criterion

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Some Tools for Heuristic Selection



- SATzilla: algorithm portfolio oriented data-driven framework
- Simple Neighborhood-based Algorithm Portfolio in PYthon (snappy)
- Hyper-heuristics Flexible Interface (HyFlex)
- ParHyFlex extends MPI
- Hyperion provides a white-box framework giving a metaheuristic/hyper-heuristic full access to the problem domain

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A Comprehensive Analysis of Hyper-heuristics

Ender Özcan, Burak Bilgin, Emin Erkan Korkmaz

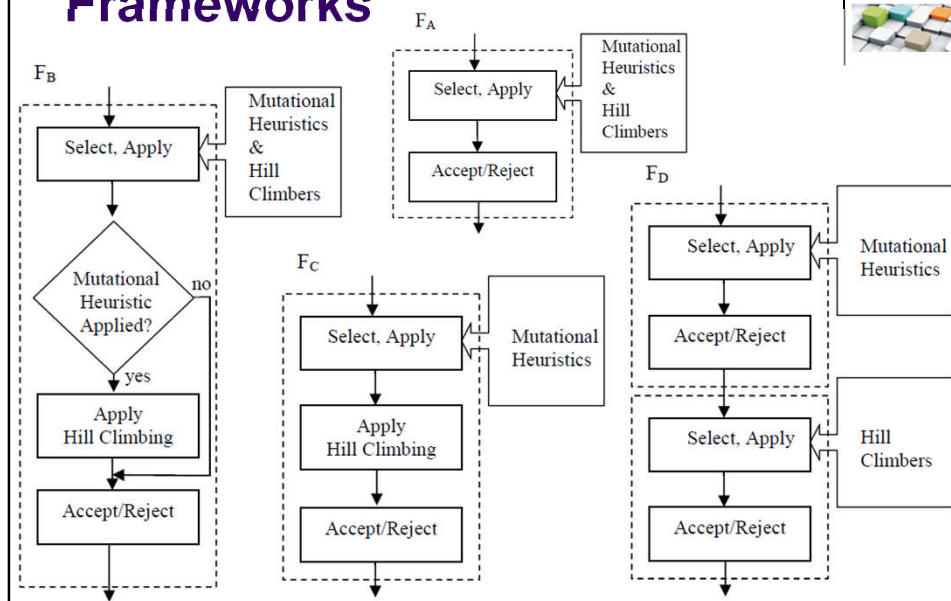
Intelligent Data Analysis, 12:1, pp. 3-23, 2008



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Selection Hyper-heuristic Frameworks



Results



Label	F_A	F_B	F_C	F_D
F1	1.00	1.00	1.00	1.00
F2	0.00	0.00	0.00	0.00
F3	1.00	1.00	1.00	0.00
F4	0.00	0.02	0.02	0.02
F5	0.76	1.00	1.00	0.54
F6	0.08	1.00	1.00	0.00
F7	0.92	0.98	1.00	0.00
F8	0.00	0.30	0.90	0.90
F9	1.00	1.00	1.00	0.96
F10	0.02	0.44	0.54	0.02
F11	0.00	1.00	1.00	0.06
F12	1.00	1.00	1.00	0.00
F13	0.00	1.00	1.00	0.00
F14	0.82	1.00	1.00	0.06
Avr.	0.47	0.77	0.82	0.25

- Binary representation
- 3 mutational, 3 hill climbing heuristics
- F_B and F_C employ DBHC.
- F_D uses CF-AM (mutational) and CF-IE (hill climbing)

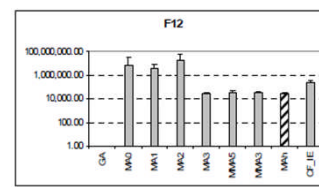
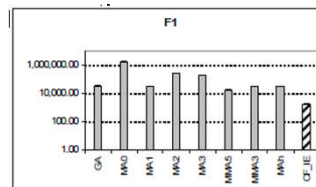
Success rate = (# of runs achieving expected objective value)/(total # of runs)

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Results



- GD, MC and IE performs well with CF and SR
- CF-IE (under F_C) delivers a “similar” performance to multimeme algorithm



- Choice of low level heuristics influences the overall performance of a hyper-heuristic

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[Hyper-heuristics Flexible Interface] HyFlex



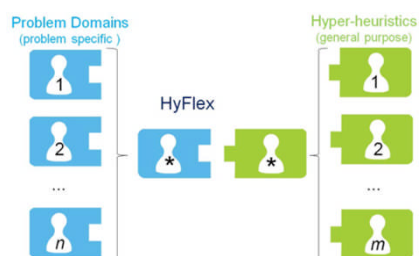
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HyFlex Hyper-heuristics Flexible Interface



- Defines behaviours of components and arranges the interaction between them



Separation between the problem-specific and the general-purpose parts, both of which are reusable and interchangeable through the HyFlex interface

<http://www.hyflex.org/>

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HyFlex v1.0 Java Implementation



- Currently there are 6 problem domain implementations
- heuristic types: mutational, ruin-recreate, local search, crossover
- parameters: intensity, depth of search

MAX-SAT

Bin Packing

Flow Shop

Personnel Scheduling

TSP

VRP

Heuristic IDs	LLH0	LLH1	LLH2	LLH3	LLH4	LLH5	LLH6	LLH7
MAX-SAT	MU ₀	MU ₁	MU ₂	MU ₃	MU ₄	MU ₅	RC ₀	HC ₀
Bin Packing	MU ₀	RC ₀	RC ₁	MU ₁	HC ₀	MU ₂	HC ₁	XO ₀
PS	HC ₀	HC ₁	HC ₂	HC ₃	HC ₄	RC ₀	RC ₁	RC ₂
PFS	MU ₀	MU ₁	MU ₂	MU ₃	MU ₄	RC ₀	RC ₁	HC ₀
TSP	MU ₀	MU ₁	MU ₂	MU ₃	MU ₄	RC ₀	HC ₀	HC ₁
VRP	MU ₀	MU ₁	RC ₀	RC ₁	HC ₀	XO ₀	XO ₁	MU ₂

Heuristic IDs	LLH8	LLH9	LLH10	LLH11	LLH12	LLH13	LLH14
MAX-SAT	HC ₁	XO ₀	XO ₁				
PS	XO ₀	XO ₁	XO ₂	MU ₀			
PFS	HC ₁	HC ₂	HC ₃	XO ₀	XO ₁	XO ₂	XO ₃
TSP	HC ₂	XO ₀	XO ₁	XO ₂	XO ₃		
VRP	HC ₁	HC ₂					

<http://www.hyflex.org/>

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CROSS-DOMAIN HEURISTIC SEARCH CHALLENGE

<http://www.hyflex.org/>

automated scheduling optimisation & planning

CHeSC 2011 benchmark based on HyFlex v1.0

MAX-SAT

- 10 public training instances
- **5 test instances**
(3 training + 2 hidden/all hidden)

Bin Packing

- Set problem instance
- Set time limit (10 min.)
- Perform 31 runs
- Report median

Flow Shop

- Set problem instance
- Set time limit (10 min.)
- Perform 31 runs
- Report median

Personnel Scheduling

- Set problem instance
- Set time limit (10 min.)
- Perform 31 runs
- Report median

TSP

VRP

Hidden

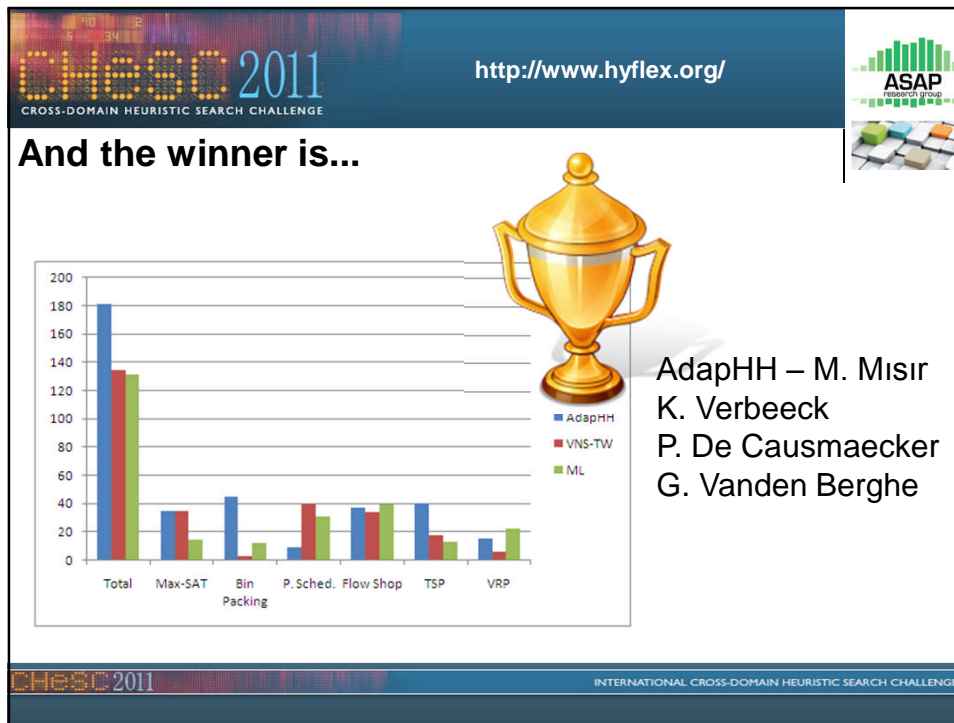
Ranking: Formula 1 scoring system

Organising Partners:

Sponsor:

INTERNATIONAL CROSS-DOMAIN HEURISTIC SEARCH CHALLENGE

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AdapHH – Overview

$$p_i = w_1 \left[(C_{p,best}(i) + 1)^2 (t_{remain}/t_{p,spent}(i)) \right] \times b +$$

$$w_2 \left(f_{p,imp}(i)/t_{p,spent}(i) \right) - w_3 \left(f_{p,vera}(i)/t_{p,spent}(i) \right) +$$

$$w_4 \left(f_{imp}(i)/t_{p,spent}(i) \right) - w_5 \left(f_{vera}(i)/t_{p,spent}(i) \right)$$

$$b = \begin{cases} 1, & \sum_{i=0}^n C_{p,best}(i) > 0 \\ 0, & \text{otherwise} \end{cases} \quad avg = \left\lfloor \left(\sum_i QI_i \right) / n \right\rfloor$$

$$pl = ph_{duration} / t_{subset}$$

$$ph_{duration} = t_{total} / ph_{requested}$$

$$exc(i) = t_{perMove}(i) / t_{perMove}(fastest)$$

$$\sigma > 2.0 ; exc(i) > 2\sigma ; nb > 1$$

$$pr_i = ((C_{best}(i) + 1) / t_{spent})^{(1+3t f^3)}$$

$$k = \begin{cases} ((l-1).k + iter_{elapsed}) / l, & \text{if } cw = 0 \\ ((l-1).k + \sum_{i=0}^{cw} k \cdot 0.5^i \cdot tf) / l, & \text{otherwise} \end{cases}$$

$$tf = (t_{exec} - t_{elapsed}) / t_{exec}$$

$$cw = iter_{elapsed} / k$$

Relay hybridisation

```

Input: listsize = 10;  $\gamma \in (0.02, 50)$ ;  $p, p' \in [0:1]$ 
1  $\gamma = (C_{best,s} + 1) / (C_{best,r} + 1)$ 
2 if  $p \leq (C_{phase}/pl)^\gamma$  then
3   select LLH using a LA and apply to  $S \rightarrow S'$ 
4   if  $size(list_s) > 0$  and  $p' \leq 0.25$  then
5     select a LLH from  $list_s$  and apply to  $S' \rightarrow S''$ 
6   else
7     select a LLH and apply to  $S' \rightarrow S''$ 
8   end
9 end

```

$$l = l_{base} + (l_{initial} - l_{base} + 1) t f^3$$

AILLA move acceptance

```

Input:  $i = 1, K \geq k \geq 0, l > 0$ 
for  $i=0$  to  $l-1$  do  $best_{list}(i) = f(S_{initial})$ 
1 if  $adapt\_iterations \geq K$  then
2   if  $i < l-1$  then
3      $i++$ 
4   end
5 end
6 if  $f(S') < f(S)$  then
7    $S \leftarrow S'$ 
8    $w\_iterations = 0$ 
9   if  $f(S') < f(S_k)$  then
10     $i = 1$ 
11     $S_k \leftarrow S'$ 
12     $w\_iterations = adapt\_iterations = 0$ 
13     $best_{list}.remove(last)$ 
14     $best_{list}.add(0, f(S_k))$ 
15  end
16 else if  $f(S') = f(S)$  then
17    $S \leftarrow S'$ 
18    $w\_iterations++$ 
19    $adapt\_iterations++$ 
20   if  $w\_iterations \geq k$  and  $f(S') \leq best_{list}(i)$  then
21      $S \leftarrow S'$  and  $w\_iterations = 0$ 
22   end
23 end

```

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AdapHH – Heuristic Selection



- A multi-phase approach adaptively deciding on the subset of low level heuristics to use at a phase and its duration
- Computes quality index for each heuristic based on a weighted average of performance measure based on (i) the number of new best solutions found, the total number of (ii) improvement and (iii) worsening until a given time and (iv, v) during a single phase, (vi) overall remaining time, the time spent by a heuristic (vii) until that moment and (viii) during a phase and excludes the one below the average at a stage
- Excludes relatively slow heuristics
- Uses a probability vector to select a heuristic based on (i), (vi), overall time and time spent

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AdapHH – Heuristic Selection



- A multi-phase approach adaptively deciding on the subset of low level heuristics to use at a phase and its duration
- Computes quality index for each heuristic based on a weighted average of performance measure based on (i) the number of new best solutions found, the total number of (ii) improvement and (iii) worsening until a given time and (iv, v) during a single phase, (vi) overall remaining time, the time spent by a heuristic (vii) until that moment and (viii) during a phase and excludes the one below the average at a stage
- Excludes relatively slow heuristics
- Uses a probability vector to select a heuristic based on (i), (vi), overall time and time spent

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AdapHH – Heuristic Selection



- Relay hybridisation: Keeps track of performance of successive application of heuristic pairs and applies a pair of heuristics with a certain probability. The first heuristic is chosen using a learning automaton which maintains the probability of selecting a given heuristic.
- Heuristic Parameter Adaptation: A reinforcement learning based mechanism is used

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AdapHH – Move acceptance AILLA



- Maintains a list of fitness values for the recently visited new best solutions
- A worsening solution is accepted:
 - If a new best solution cannot be found after a certain number of iterations with consecutive worsening solutions (adapted during search)
 - If its fitness is better than the fitness of the top solution in the list which acts like a threshold level

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CHeSC Results



Rank	Hyper-heuristic	Score	Rank	Hyper-heuristic	Score
1	AdapHH	181.00	11	ACO-HH	39.00
2	VNS-TW	134.00	12	GenHive	36.50
3	ML	131.50	13	DynILS	27.00
4	PHUNTER	93.25	14	SA-ILS	24.25
5	EPH	89.75	15	XCJ	22.50
6	HAHA	75.75	16	AVEG-Nep	21.00
7	NAHH	75.00	17	GISS	16.75
8	ISEA	71.00	18	SelfSearch	7.00
9	KSATS-HH	66.50	19	MCHH-S	4.75
10	HAEA	53.50	20	Ant-Q	0.00

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Limitations of CHeSC/Hyflex



- Deficiencies of standard CHeSC/Hyflex:
 - Pure Blackbox Interface: Hyflex
 - Only allows access to the objective value of current state
 - Many suggestions for extensions to permit more information to be passed
 - Fixed 10mins
 - Independent instances
 - The HH is restarted for each instance and so cannot pass on anything it has learned

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Limitations of CHeSC/Hyflex



- Fixes to deficiencies of standard CHeSC/Hyflex:
 - Blackbox “hyflex” interface
 - Many people have suggested extensions to permit more information to be passed
 - Fixed 10mins
 - Easy to change
 - Independent instances
 - **Batched mode**

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Batched Mode (CHeSC 2014)



- Simple extension to Hyflex/CHeSC
- “Batched mode”:
 - HH is given N instances and a total time T
- Advantages:
 - **Load Balancing:**
 - Allocate more time to harder instances, by stopping earlier on “easy” ones
 - **Inter-instance learning:**
 - Allowed to keep information learned from previous instances

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Potential Future Directions

- Better annotations
- Instance features
- Solution features
- Distance metrics
- Multi-objective support
- Extensions to support generative hyper-heuristics and more...

These are currently being explored and Hyflex being extended to match them.

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Case Study: A Tensor-based Selection Hyper-heuristic for Cross Domain Search

Shahriar Asta, Ender Özcan

Information Sciences, *to appear*



School of
Computer Science



The University of
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Classification of the Approach



Feedback

Nature of the heuristic search space

Online
learning

Hyper-
heuristics

Heuristic selection

Methodologies to select

perturbative
heuristics

131

Two Simple Hyper-heuristics Mixing Heuristics (Stochastic Local Search)



- Simple Random Heuristic Selection – Improving and Equal Move Acceptance (IE)
 - Reject any worsening move
- Simple Random Heuristic Selection – Naïve Move Acceptance (NA)
 - Accept a worsening move with a fixed probability of p (0.5 in this study)

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Proposed Approach – Ideas



- The balance between diversification and intensification is crucial (e.g. ILS)



?

IE

?

NA

$$h_{IE} \cup h_{NA} = h \quad (h_{IE} \cap h_{NA} = \emptyset)$$

h : set of low level heuristics
(MU+RC+LS)

- Mix move acceptance methods
- Use machine learning to partition the low level heuristics associated with each method

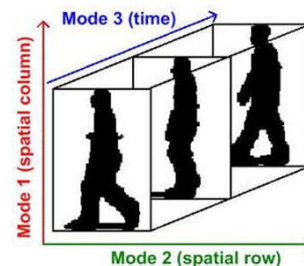
133

Tensors



- Many real-world data are multidimensional
 - Very high-dimensional (big) with a large amount of redundancy
- Multi-dimensional arrays representing such data describe a tensor

Many applications in signal processing, psychometrics, and more



SOURCE: http://en.wikipedia.org/wiki/File:Video_represented_as_a_third-order_tensor.jpg

134

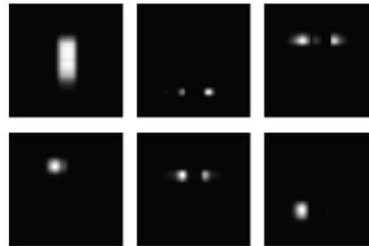
Tensor Factorisation



- There are different decomposition methods, we use Canonical Polyadic (CP) factorisation

$$\hat{\mathcal{T}} = \sum_{k=1}^K \lambda_k \mathbf{a}_k \circ \mathbf{b}_k \circ \mathbf{c}_k$$

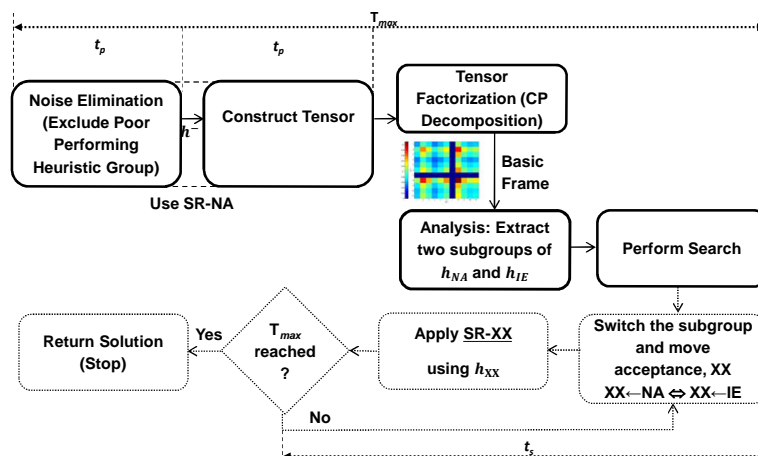
- This gives a projection of 3D data onto 1D vectors
- Helps to discover latent structures in data, quantifying the relationship between pairs of different components



SOURCE: B. Krausz, C. Bauckhage, Action recognition in videos using nonnegative tensor factorization., in: ICPR, IEEE, 2010, pp. 1763–1766.

135

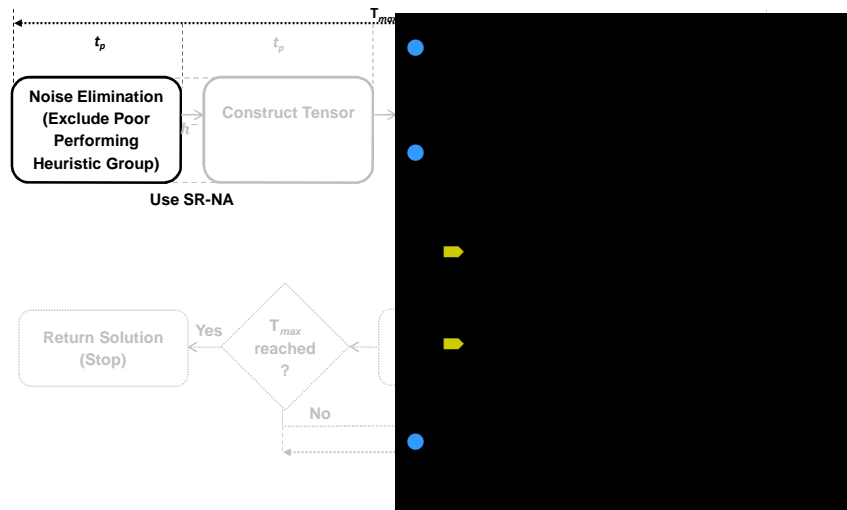
Proposed Approach – TeBHA-HH



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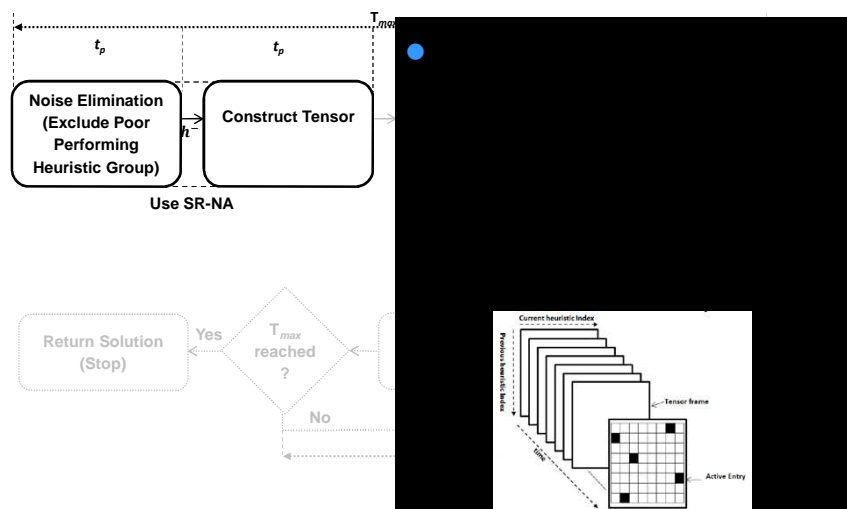
TeBHA-HH:

1. Noise Elimination Phase



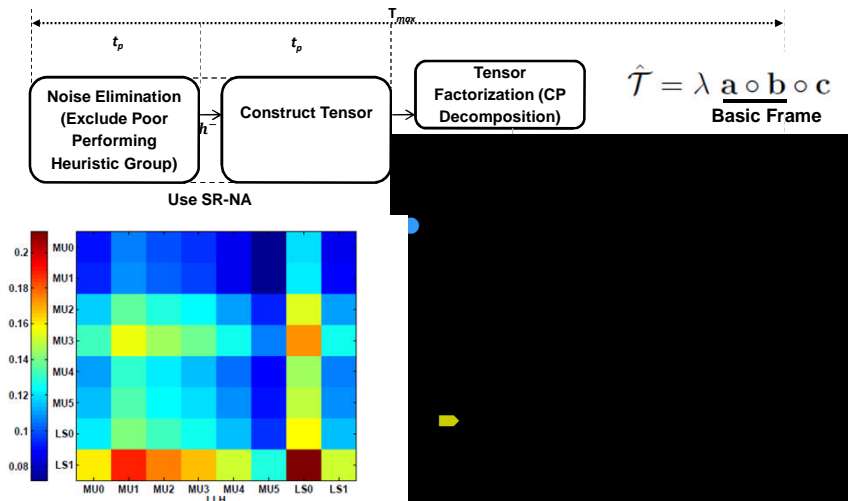
TeBHA-HH:

2. Tensor Construction Phase



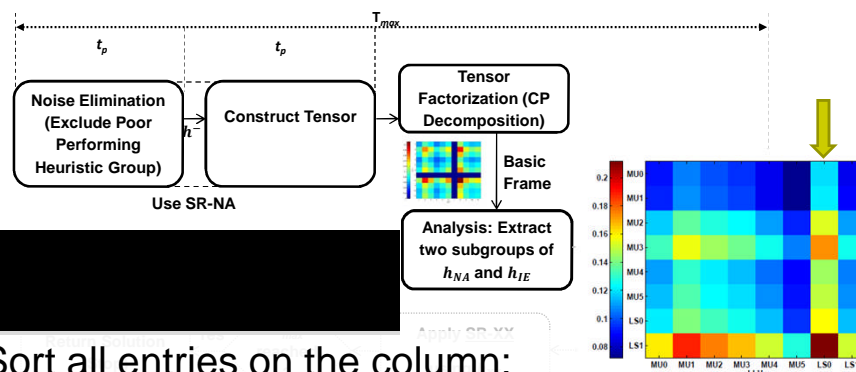
TeBHA-HH:

3. Tensor Factorization



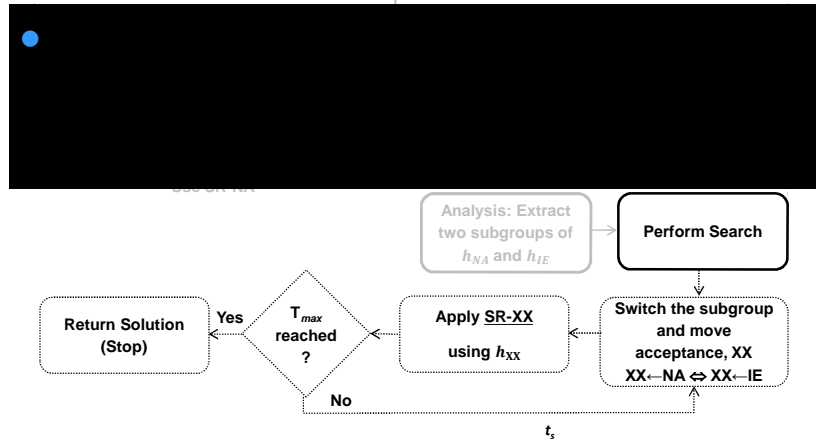
TeBHA-HH:

4. Tensor Analysis



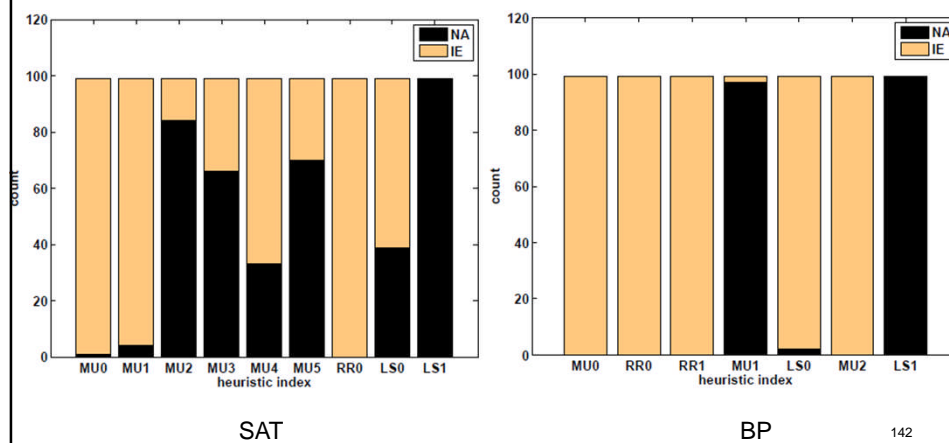
- Sort all entries on the column: (LS0, LS1, MU3, MU2, MU5, MU4, MU1, MU0)
- Top half goes to h_{NA} , the rest to h_{IE}

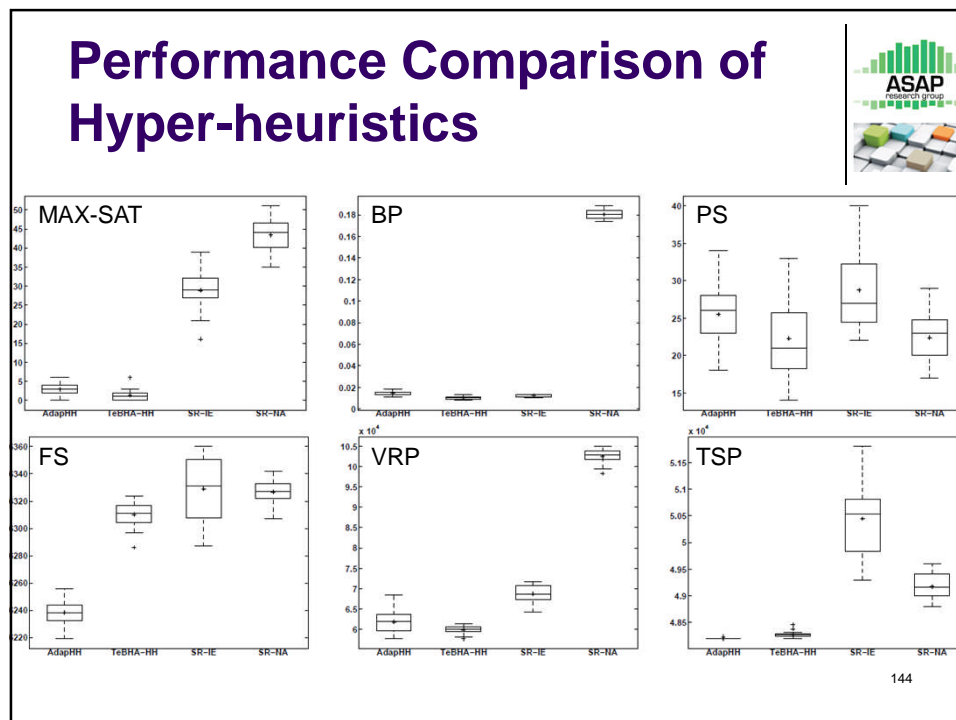
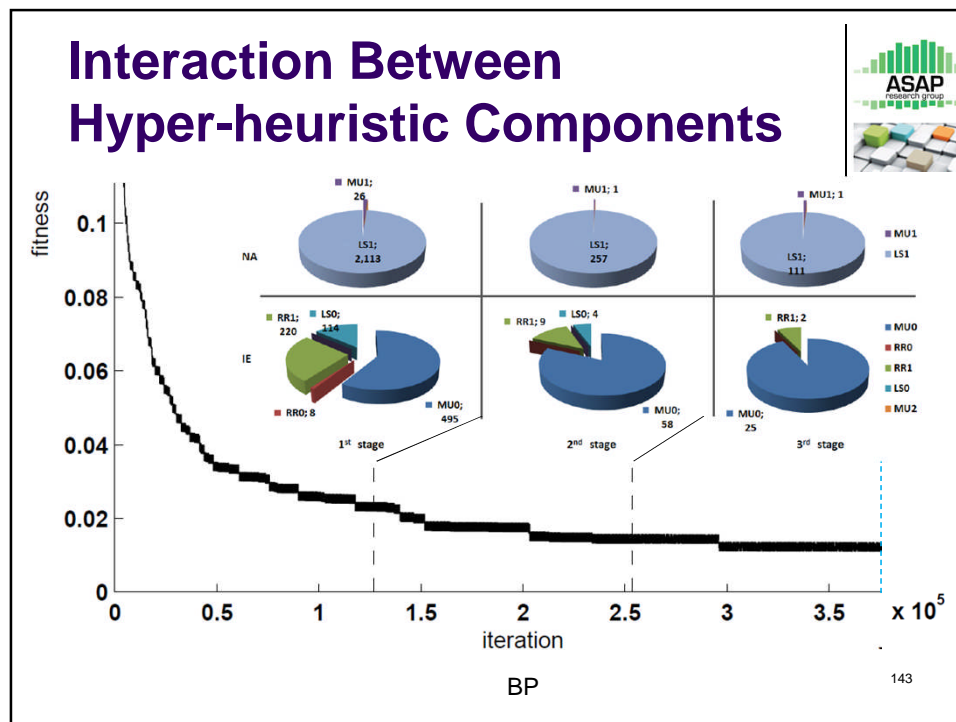
TeBHA-HH: 5. Final Phase: Perform Search



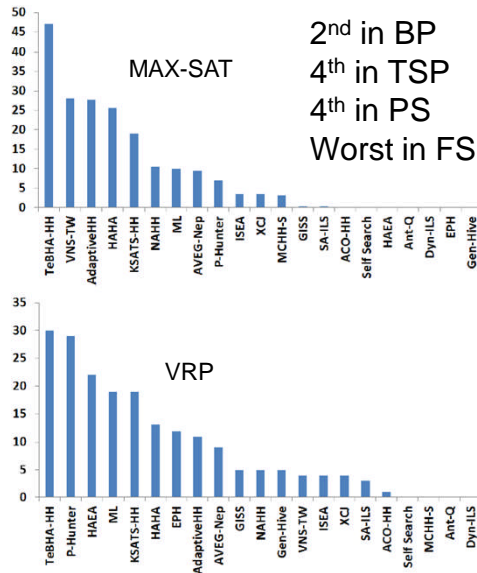
h_{IE} vs. h_{NA}

• Histograms





Results—CHeSC2011



2nd in BP
4th in TSP
4th in PS
Worst in FS

Rank	Name	Score
1	AdaptiveHH	162.83
2	TeBHA-HH	148.85
3	VNS-TW	118.83
4	ML	117.50
5	P-Hunter	84.75
6	EPH	83.25
7	NAHH	68.50
8	HAHA	65.58
9	ISEA	62.50
10	KSATS-HH	52

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Case Study: A Data Mining Embedded Hyper-heuristic

Sahriar Asta, Ender Ozcan



The University of
Nottingham

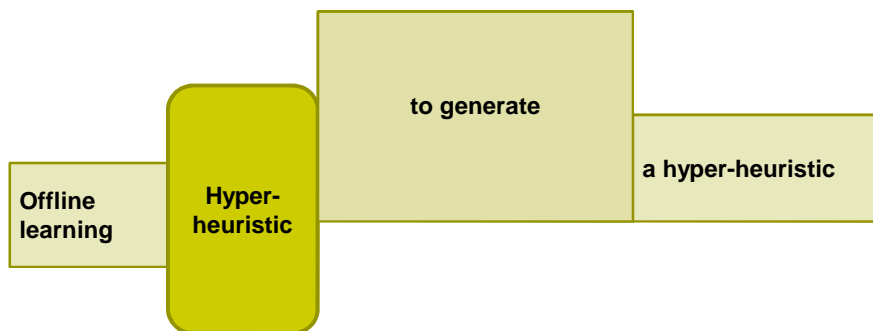
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Classification of the Approach



Feedback

Nature of the heuristic search space

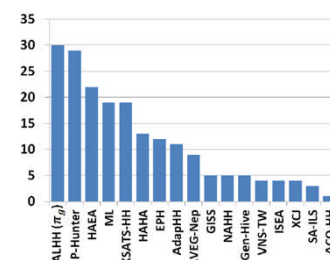


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An Apprenticeship Learning Hyper-Heuristic for Vehicle Routing in HyFlex (SSCI 2014, to appear)



- **Basic idea:** Learn from an expert (AdapHH – winner of CHeSC 2011) how to make decisions on heuristic selection, parameter setting and move acceptance for building a selection hyper-heuristic
- C4.5 to construct decision trees



Solomon Instances						Hombberger Instances					
	0	1	2	3	4	5	6	7	8	9	
AdapHH (π_c)	μ	5093.4	20656.9	13388.0	5321.9	14293.0	146791.0	62809.4	161638.5	153164.3	147360.5
	min	4230.2	20651.6	13296.9	5275.5	14270.7	144040.9	58521.6	160074.4	146584.7	145139.3
	median	5125.7	20655.4	13349.9	5320.8	14291.1	146906.7	61985.8	161596.3	153083.7	147550.3
	σ	161.7	4.2	183.8	24.5	13.9	1369.0	4609.4	982.0	1841.3	1047.6
Apprentice (π_g)	μ	4954.6	20792.8	13266.7	5365.2	14113.8	147017.6	60101.9	161491.5	153132.2	147414.9
	min	4178.8	20653.3	12300.2	5305.2	13277.0	144037.7	58352.6	160084.5	149227.1	145478.3
	median	5156.4	20661.2	13365.5	5366.7	14294.0	146988.0	60163.0	161529.8	153000.2	147480.9
	σ	394.2	340.9	310.9	29.4	481.3	1780.5	790.0	842.7	1663.1	956.8
P-Hunter [30]	min	-	20650.8	12263.0	-	-	143663.9	61139.3	-	-	146472.9
	median	-	20650.8	12290.0	-	-	146944.4	64717.8	-	-	148659.0
AdOr-ILS [31]	μ	5281.7	21291.9	13605.0	6564.4	14280.8	155305.5	77302.7	163177.7	158941.9	149447.7
	σ	334.614	482.56	451.64	554.77	319.54	6154.24	3384.83	2100.09	2460.71	1500.9

Case Study: A Multi-stage Selection Hyper-heuristic

Ahmed Kheiri, Ender Ozcan

EJOR, in review



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Classification of the Approach



Feedback

Nature of the heuristic search space

Online
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Hyper-
heuristics

Heuristic selection

Methodologies to select

perturbative
heuristics

150

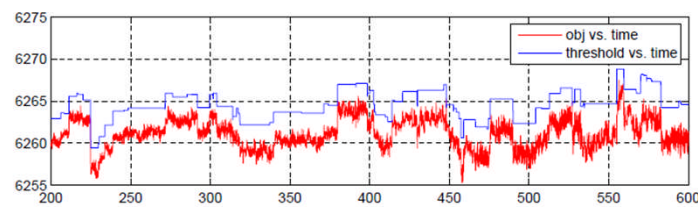
Stage 1 Hyper-heuristic



- Select a low level heuristic i with probability

$$score_i / \sum_k (score_k)$$

- Apply the chosen heuristic
- Accept/reject based on an adaptive threshold acceptance method

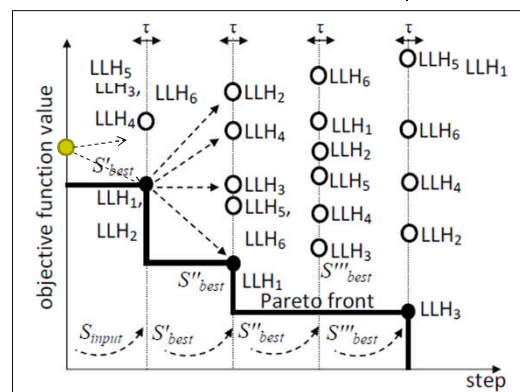


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Stage 2 Hyper-heuristic



- Uses relay hybridisation
Given LLH_1 and LLH_2 :
 $LLH_3 = LLH_1 + LLH_1, \dots$
 $LLH_6 = LLH_2 + LLH_1$
- Reduces the set of low level heuristics
- Adjusts heuristic scores according to a Greedy and dominance based approach



$LLH_1=2, LLH_2=1, LLH_3=1$ $2+4 \text{ LLHs} \rightarrow 3 \text{ LLHs}$
50% 25% 25%

MSHH



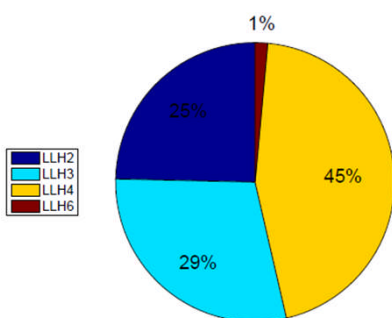
		MSHH					SIHH					S2HH		
Domain	Instance	avg.	std.	median	min.	vs.	avg.	std.	min.	vs.	avg.	std.	min.	
SAT	Inst1	0.9	0.7	1.0	0.0	>	6.4	4.5	1.0	>	15.0	4.6	3.0	
	Inst2	3.1	3.9	2.0	1.0	>	21.3	13.3	3.0	>	44.9	9.8	18.0	
	Inst3	0.7	0.5	1.0	0.0	>	7.1	7.7	0.0	>	26.3	14.0	1.0	
	Inst4	1.7	1.0	1.0	1.0	>	5.7	4.3	1.0	>	20.0	4.6	12.0	
	Inst5	7.6	0.9	7.0	7.0	>	10.4	1.5	7.0	>	15.4	1.7	13.0	
BP	Inst1	0.0163	0.0014	0.0163	0.0136	<	0.0159	0.0010	0.0137	>	0.0198	0.0015	0.0160	
	Inst2	0.0037	0.0015	0.0030	0.0025	<	0.0061	0.0015	0.0034	>	0.0104	0.0021	0.0077	
	Inst3	0.0050	0.0015	0.0049	0.0025	<	0.0054	0.0012	0.0027	>	0.0128	0.0011	0.0104	
	Inst4	0.1084	0.0000	0.1084	0.1083	<	0.1084	0.0000	0.1083	>	0.1084	0.0000	0.1084	
	Inst5	0.0050	0.0019	0.0044	0.0032	<	0.0055	0.0021	0.0032	>	0.0210	0.0015	0.0187	
PS	Inst1	25.5	4.5	25.0	16.0	>	28.8	4.7	18.0	>	31.6	4.9	22.0	
	Inst2	9668.9	217.8	9638.0	9184.0	<	9645.3	159.6	9334.0	<	9645.8	106.7	9391.0	
	Inst3	3283.7	93.3	3270.0	3132.0	<	3304.8	99.6	3134.0	<	3309.9	110.2	3172.0	
	Inst4	1786.3	172.1	1760.0	1545.0	<	1801.0	142.3	1570.0	<	1836.0	291.1	1400.0	
	Inst5	353.2	21.2	350.0	315.0	<	724.4	657.3	320.0	>	810.7	621.5	360.0	
PFS	Inst1	6239.8	14.9	6239.0	6212.0	>	6287.6	21.9	6249.0	>	6353.3	29.8	6301.0	
	Inst2	26895.2	55.3	26889.0	26775.0	<	26873.2	30.7	26822.0	>	26976.9	54.7	26849.0	
	Inst3	6333.8	19.0	6325.0	6303.0	<	6360.5	16.4	6323.0	<	6405.5	23.7	6369.0	
	Inst4	11363.8	32.7	11359.0	11320.0	<	11429.9	43.8	11357.0	<	11529.3	35.9	11436.0	
	Inst5	26711.9	47.0	26709.0	26630.0	<	26693.1	40.7	26608.0	<	26779.1	49.8	26702.0	
TSP	Inst1	48208.1	31.8	48194.9	48194.9	>	50032.0	571.1	49263.1	>	50326.5	606.6	49221.6	
	Inst2	2.09e+7	9.05e+4	2.09e+7	2.07e+7	>	2.14e+7	1.12e+5	2.12e+7	>	2.13e+7	1.05e+5	2.11e+7	
	Inst3	6809.1	7.1	6808.8	6796.6	>	7012.5	30.4	6964.6	>	7040.2	31.3	6988.6	
	Inst4	66840.2	276.5	66843.6	66236.8	>	68908.4	382.4	68159.9	>	70241.9	704.6	68791.0	
	Inst5	53011.4	469.7	52910.2	52341.3	>	54411.1	595.1	53686.0	>	55814.8	946.4	53992.4	
VRP	Inst1	70998.4	3840.3	70506.5	63948.2	<	70223.0	2960.2	64273.2	<	84103.9	7225.8	68958.3	
	Inst2	13421.8	251.6	13359.6	13303.9	<	13658.0	471.4	13319.6	<	13695.8	473.9	13320.0	
	Inst3	148498.2	1625.8	148436.2	145466.5	<	148232.6	1935.3	145426.5	<	149553.2	2377.8	145362.7	
	Inst4	21016.4	488.2	20671.4	20650.8	<	20991.3	478.0	20653.5	<	21131.9	510.3	20657.5	
	Inst5	148813.7	1272.5	149193.7	146334.6	<	148999.1	1217.1	146844.9	<	150282.6	1616.3	146666.9	

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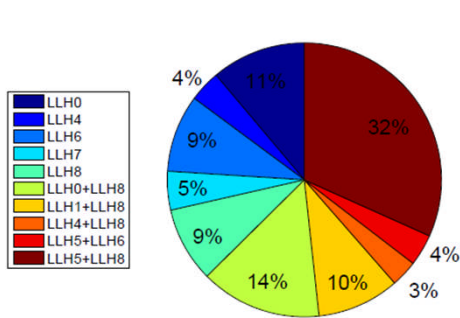
Relay Hybridisation



PS



TSP



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Result

Label	SAT	BP	PS	PFS	TSP	VRP	Overall
MSHH	48.00	38.00	6.00	25.00	42.60	4.00	163.60
AdapHH	27.58	44.00	8.00	33.00	34.60	14.00	161.18
VNS-TW	27.08	2.00	39.50	30.00	13.60	6.00	118.18
ML	10.00	8.00	31.00	36.50	10.00	22.00	117.50
PHUNTER	7.00	2.00	11.50	6.00	21.60	33.00	81.10
EPH	0.00	6.00	10.50	18.00	30.60	12.00	77.10
HAHA	25.58	0.00	24.50	2.83	0.00	14.00	66.92
NAHH	10.50	16.00	2.00	19.50	9.00	6.00	63.00
ISEA	3.50	25.00	14.50	3.50	7.00	4.00	57.50
KSATS-HH	19.00	7.00	8.50	0.00	0.00	22.00	56.50
HAEA	0.00	1.00	1.00	7.33	8.00	27.00	44.33
GenHive	0.00	10.00	6.50	7.00	2.00	6.00	31.50
ACO-HH	0.00	17.00	0.00	6.33	6.00	1.00	30.33
SA-ILS	0.25	0.00	18.50	0.00	0.00	4.00	22.75
AVEG-Nep	9.50	0.00	0.00	0.00	0.00	9.00	18.50
XCJ	3.50	10.00	0.00	0.00	0.00	5.00	18.50
DynILS	0.00	9.00	0.00	0.00	8.00	0.00	17.00
GISS	0.25	0.00	10.00	0.00	0.00	6.00	16.25
SelfSearch	0.00	0.00	3.00	0.00	2.00	0.00	5.00
MCHH-S	3.25	0.00	0.00	0.00	0.00	0.00	3.25
Ant-Q	0.00	0.00	0.00	0.00	0.00	0.00	0.00



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A Memetic Algorithm for Solving a Project Scheduling Problem

S. Asta, D. Karapetyan, A. Kheiri, E. Özcan, and A.J. Parkes, Combining Monte-Carlo and Hyper-heuristic methods for the Multi-mode Resource-constrained Multi-project Scheduling Problem, Journal of Scheduling, in review.

TeamID#3




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MISTA 2013 Challenge

- Aim: Develop an algorithm that produces the best possible solution to any given problem in 300 sec.
 - Problem instances are not known in advance.
- 21 teams registered, 16 teams qualified after the first round, 9 teams qualified after the final round.
- We designed a memetic algorithm – construct and improve


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Problem Description

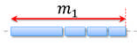
Resource-Constrained Project Scheduling

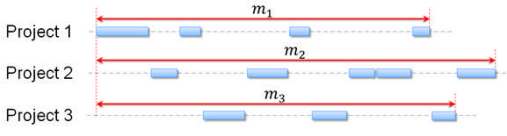
- Schedule given jobs
- Limited resources
- Precedence relations
- Minimise makespan

Multi-mode Resource-constrained Multi-project Scheduling

- Multiple modes for each job
- Multiple projects
- Local and global resources
- Minimise the sum of makespans

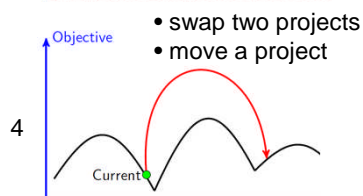
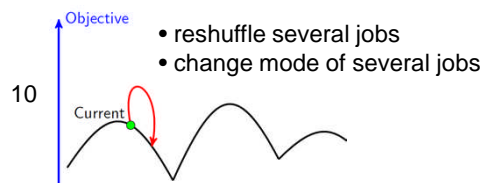
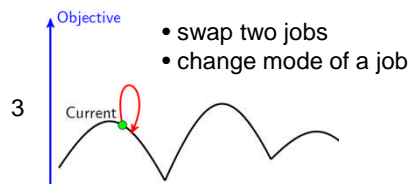
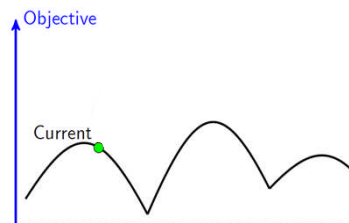
Project 1





A Multi-stage Hyper-heuristic

Low Level Heuristics/Operators



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A Memetic Algorithm

Monte Carlo Tree Search

Initialisation

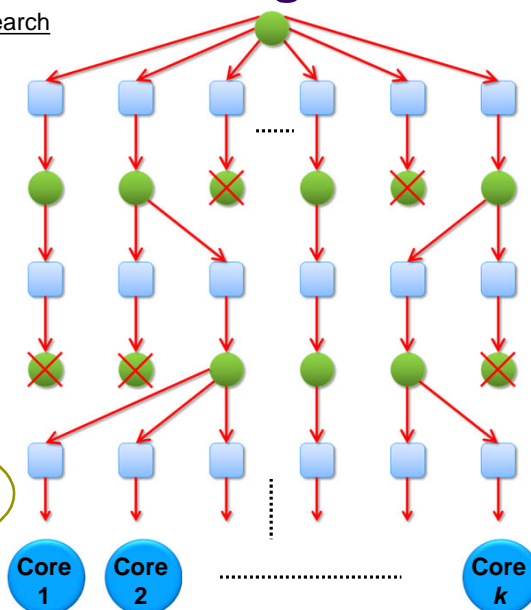
Hyper-heuristic
Improvement

Mutation

Hyper-heuristic
Improvement

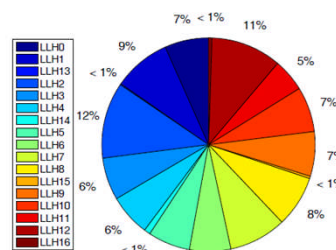
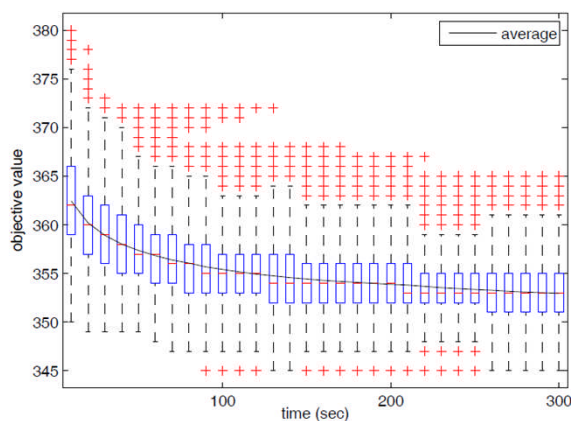
Mutation

Hyper-heuristic
Improvement



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Results

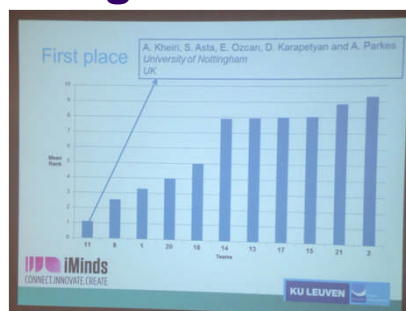


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MISTA 2013 Challenge – Result



Competition results			
Instance	TPD	TMS	Found by
B-1	349	127	3
B-2	434	160	1
B-3	545	210	3
B-4	1274	289	2
B-5	820	254	3
B-6	912	227	3
B-7	792	228	3
B-8	3176	533	3
B-9	4192	746	3
B-10	3249	456	3
X-1	392	142	1
X-2	349	163	3
X-3	324	192	3
X-4	955	213	3
X-5	1768	374	3
X-6	719	232	3
X-7	861	237	3
X-8	1233	283	3
X-9	3268	643	3
X-10	1600	381	3



- We produced the best solutions for 17 out of the 20 instances
- On the 12th second our algorithm becomes the winner

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Summary (and Potential Future Research Directions)



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Summary



- Hyper-heuristic research originated from a job shop scheduling application and has been rapidly growing since then.
- Generation hyper-heuristics are commonly used in the area
 - Train and test fashion
 - Does the selected subset of training instances is sufficiently representative of the test set?
 - Training is time-consuming (delta/incremental evaluation, surrogate functions)
 - The evolved heuristics might not be easy to interpret, yet they can outperform human designed heuristics

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Summary (cont.)



- There is empirical evidence that machine learning/analytics/ data science help to improve the hyper-heuristic search process
 - ▀ Problem features vs solution/state features
 - ▀ Offline versus online learning – Life long learning
- There is still a lack of benchmarks
 - ▀ Problem domains are needed
- Multi-criteria, multi-objective and dynamic problems

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Summary (cont.)



- Domain barrier issues
 - ▀ What constitutes as domain independent information
- More/less number of heuristics
 - ▀ Minimal heuristic set
- Multistage hyper-heuristics
 - ▀ Which hyper-heuristics to combine?
 - ▀ How to switch from one to another?
 - ▀ How to decide on the low level heuristic set?
 - ▀ Is there an end to the recursion/levels?
(hyper^Nheuristic)

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Summary (cont.)



- Finding common representations or description formats that unify different but related problems
 - Example: grouping hyper-heuristics
 - Design a solver for the problems with binary/permutation/integer packed representation
- How do we compare hyper-heuristics?
 - Fairness issues: Termination criteria
 - If we test a hyper-heuristic on
 - new problem instances
 - new problem domains

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Summary (cont.)



- Automated design of search methodologies is extremely challenging
 - Addressed in almost complete absence of a mathematical and theoretical understanding
- Heuristic Understanding
 - How can we analyse the search space of heuristics?
 - How can we visualise the search space of heuristics?
 - Is it possible to learn from small examples and apply to large instances?

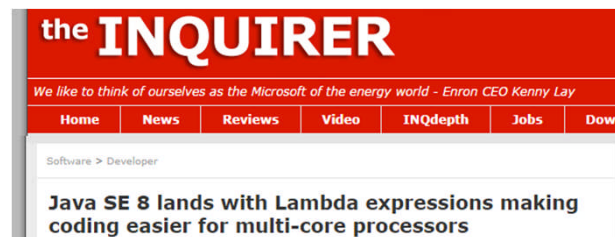
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Summary (cont.)



Intel reportedly eyes future 18-core 'Broadwell' chip

What kind of high-performance silicon is Intel planning in the not-too-distant future? A site that covers chips says that future includes an 18-core processor.



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Q&A



Thank you.

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