Mixed-level Embedding and JIT Compilation for an Iteratively Staged DSL

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Outline

- Modelling and simulation from declarative programming point of view
- New language for modelling and simulation embedded in Haskell
- New EDSL implementation approach

Modelling and Simulation

Developing models and studying their properties and behaviour are of immense theoretical and practical importance.

- Science
  - Understanding
  - Prediction
- Engineering
  - Development
  - Optimisation
  - Safety
- Modelling and simulation languages

Causal Modelling

- Ordinary Differential Equation (ODE) in explicit form:

$$\frac{d\vec{x}}{dt} = f(\vec{x}, \vec{u}, t)$$

- Causality (cause-effect relationship) given by the modeller. Cf. Functional Programming.
- Causal modelling is the dominating modelling paradigm (e.g. Simulink).

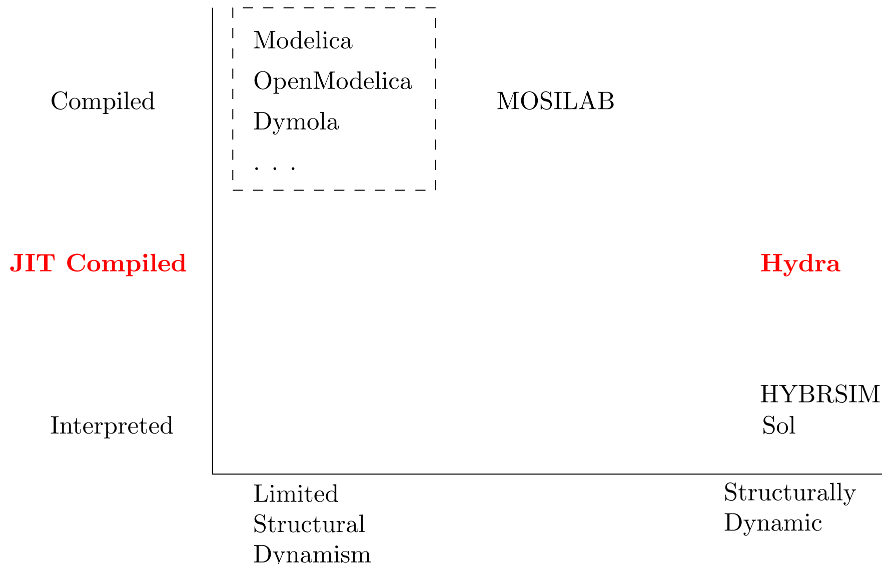
Non-Causal Modelling

- Differential Algebraic Equation (DAE) in implicit form:

$$f(\frac{d\vec{x}}{dt}, \vec{x}, \vec{y}, t) = 0$$

- Causality inferred by simulation tool from usage context. Cf. Logic Programming.
- Non-causal modelling is a fairly recent development (e.g. Modelica)

Non-Causal Modelling Languages



Hydra

- Example of a combined functional and constraint programming language
- Two-level language:
  - Functional level (time-invariant values)
  - Signal level (constrains on time-varying values)
- Functional language primarily serving as a meta language that generates updated constrains given as non-causal DAEs

Hydra

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  - Functional level (time-invariant values)
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- Functional language primarily serving as a meta language that generates updated constrains given as non-causal DAEs
- First class equational constraints allowing for an evolving model structure by repeated generation of updated constraints

Embedded Domain Specific Languages (EDSLs)

Embedding is a powerful and popular way to implement domain-specific languages (DSLs). There are two basic approaches to language embeddings:

- Shallow:
  - Domain-specific notions expressed directly in host language terms
- Deep:
  - Building representation of domain-specific programs as data
  - Providing interpreter or compiler

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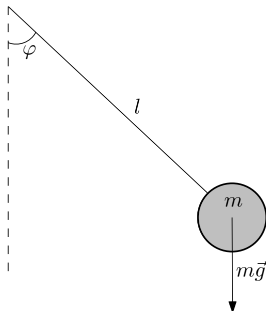
Mixed-level embedding combines the two approaches.

Iterative Staging and JIT Compilation

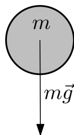
- Compilation: standard tool for EDSLs
  - Program generation, compilation and execution with a fixed number of stages
- Iterative staging: repeated program generation and execution
  - This work: efficient implementation approach for an iteratively staged EDSL using mixed-level embedding and JIT compilation

Breaking Pendulum Example

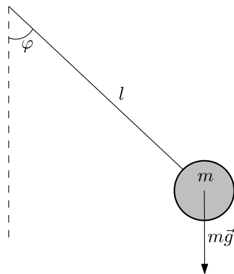
Mode 1 - Pendulum



Mode 2 - Free Fall



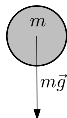
Pendulum Mode



$$
\begin{array}{l}
\textbf{type } \mathit{Coordinate} = (\mathbb{R}, \mathbb{R}) \\
\textbf{type } \mathit{Velocity} = (\mathbb{R}, \mathbb{R}) \\
\textbf{type } \mathit{Body} = (\mathit{Coordinate}, \mathit{Velocity})
\end{array}
$$

$$
\begin{array}{l}
\mathit{pendulum} :: \mathbb{R} \to \mathbb{R} \to \mathit{SR}\ \mathit{Body} \\
\mathit{pendulum}\ l\ \varphi_0 = [\$\mathit{hydra}| \\
\quad \textbf{sigrel}\ ((x, y), (v_x, v_y))\ \textbf{where} \\
\qquad \textbf{init}\ \varphi \qquad\quad = \ \$\, \varphi_0\, \$ \\
\qquad \textbf{init}\ \mathit{der}\ \varphi = 0 \\
\qquad \textbf{init}\ v_x \qquad = 0 \\
\qquad \textbf{init}\ v_y \qquad = 0 \\
\qquad x \qquad\qquad\quad = \qquad \$\, l\, \$ * \mathit{sin}\ \varphi \\
\qquad y \qquad\qquad\quad = \ - \ \$\, l\, \$ * \mathit{cos}\ \varphi \\
\qquad (v_x, v_y) \quad\ = (\mathit{der}\ x, \mathit{der}\ y) \\
\qquad \mathit{der}\ (\mathit{der}\ \varphi) + (\$g\ /\ l\$) * \mathit{sin}\ \varphi = 0 \\
|]
\end{array}
$$

Free Fall Mode



$$
\begin{aligned}
& \mathit{freeFall} :: \mathit{Body} \rightarrow \mathit{SR}\ \mathit{Body} \\
& \mathit{freeFall}\ ((x_0, y_0), (v_{x_0}, v_{y_0})) = [\$\mathit{hydra}| \\
& \quad \mathbf{sigrel}\ ((x, y), (v_x, v_y))\ \mathbf{where} \\
& \qquad \mathbf{init}\ (x, y) \qquad\quad\;\; = (\$x_0\$, \$y_0\$) \\
& \qquad \mathbf{init}\ (v_x, v_y) \qquad\;\; = (\$v_{x_0}\$, \$v_{y_0}\$) \\
& \qquad (\mathit{der}\ x, \mathit{der}\ y) \quad\;\; = (v_x, v_y) \\
& \qquad (\mathit{der}\ v_x, \mathit{der}\ v_y) = (0, -\$g\$) \\
& |]
\end{aligned}
$$

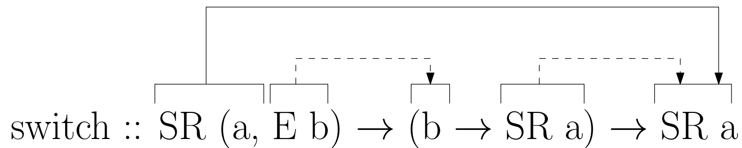
Combining the Two Modes

$$
\begin{array}{l}
\mathit{pendulumBE} :: \mathbb{R} \to \mathbb{R} \to \mathbb{R} \to \mathit{SR}\ (\mathit{Body}, E\ \mathit{Body}) \\
\mathit{pendulumBE}\ t\ l\ \varphi_0 = [\$\mathit{hydra}| \\
\quad \textbf{sigrel}\ (((x, y), (v_x, v_y)), \textbf{event}\ e)\ \textbf{where} \\
\qquad \$\ \mathit{pendulum}\ l\ \varphi_0\ \$\ \diamond ((x, y), (v_x, v_y)) \\
\qquad \textbf{event}\ e = ((x, y), (v_x, v_y))\ \textbf{when}\ \mathit{time} = \$t\ \$ \\
|]
\end{array}
$$

$$
\begin{array}{l}
\mathit{breakingPendulum} :: \mathit{SR}\ \mathit{Body} \\
\mathit{breakingPendulum} = \mathit{switch}\ (\mathit{pendulumBE}\ 10\ 1\ (\mathit{pi}\ /\ 4))\ \mathit{freeFall}
\end{array}
$$

$$
\mathit{switch} :: \mathit{SR}\ (a, E\ b) \to (b \to \mathit{SR}\ a) \to \mathit{SR}\ a
$$

The Switch Combinator



Internal (Untyped) Representation

```
data SigRel =
      SigRel          Pattern [Equation]
    | SigRelSwitch SigRel   (Expr → SigRel)
```

```
data Equation =
      EquationInit Expr Expr
    | EquationEq   Expr Expr
    | EquationEvent String Expr Expr
    | EquationSigRelApp SigRel Expr
```

Typed Combinators

- Phantom types
- Typing EDSL in Haskell
- Haskell type checking (GHC)

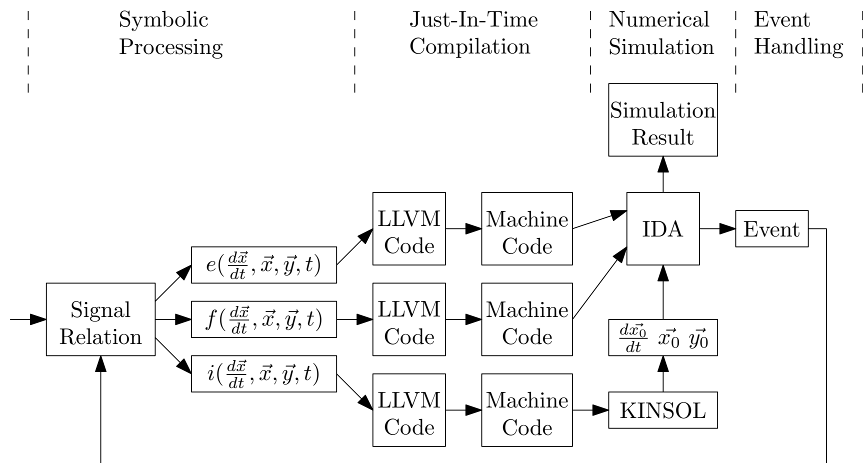
$$
\begin{aligned}
&\mathbf{data}\ SR\ a && = SR && SigRel\\
&\mathbf{data}\ PatternT\ a && = PatternT && Pattern\\
&\mathbf{data}\ ExprT\ a && = ExprT && Expr\\
&\mathbf{data}\ E\ a
\end{aligned}
$$

$$
\begin{aligned}
&sigrel :: PatternT\ a \rightarrow [Equation] \rightarrow SR\ a\\
&sigrel\ (PatternT\ p)\ eqs = SR\ (SigRel\ p\ eqs)
\end{aligned}
$$

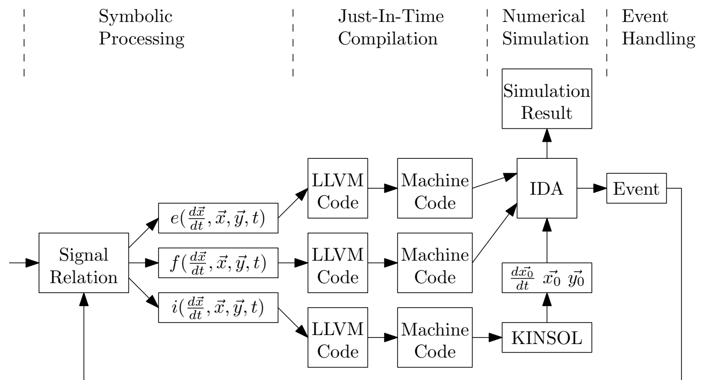
$$
switch :: SR\ (a, E\ b) \rightarrow (b \rightarrow SR\ a) \rightarrow SR\ a
$$

$$
\begin{aligned}
&equationEq :: ExprT\ a \rightarrow ExprT\ a \rightarrow Equation\\
&equationSigRelApp :: SR\ a \rightarrow ExprT\ a \rightarrow Equation
\end{aligned}
$$

Execution model of Hydra



Future Work



- Code reuse
- Mode switching overhead reduction
- (Soft) real-time simulation

Related Work

- Flask: EDSL for programming sensor networks [Mainland et al.]
- Accelerate: EDSL for data-parallel array computations on GPUs [Chakravarty et al.]
- Functional Reactive Programming (FRP), particularly Yampa [Nilsson et al.]
- DSLs embedded in multi-stage host language (e.g. MetaOCaml) [Taha et al.]
- Modelling and simulation languages:
    - MKL [Broman et al.]
    - MOSILAB [Nytsch-Geusen et al.]
    - Sol [Zimmer et al.]

Conclusions

- Novel approach to the implementation of non-causal modelling languages supporting structural dynamism
  - Modelling systems that main-stream non-causal language can not handle
  - First compiled implementation of a non-causal modelling language that supports highly structurally dynamic systems
- EDSL approach
  - Mixed-level embedding
  - Compilation of iteratively staged EDSL
  - JIT compilation through LLVM framework
- Paper and implementation: `http://cs.nott.ac.uk/~ggg/`

Thank You

Questions?

Bonus Slides

Performance

	Pendulum $t \in [0, 10)$		Free Fall $t \in [10, 20]$	
	CPU Time		CPU Time	
	s	%	s	%
Symbolic Processing	0.0001	0.2	0.0000	0.0
JIT Compilation	0.0110	18.0	0.0077	9.1
Numerical Simulation	0.0500	81.8	0.0767	90.9
Event Handling	0.0000	0.0	-	-
Total	0.0611	100.0	0.0844	100.0

Table: Time profile of the breaking pendulum simulation

Performance

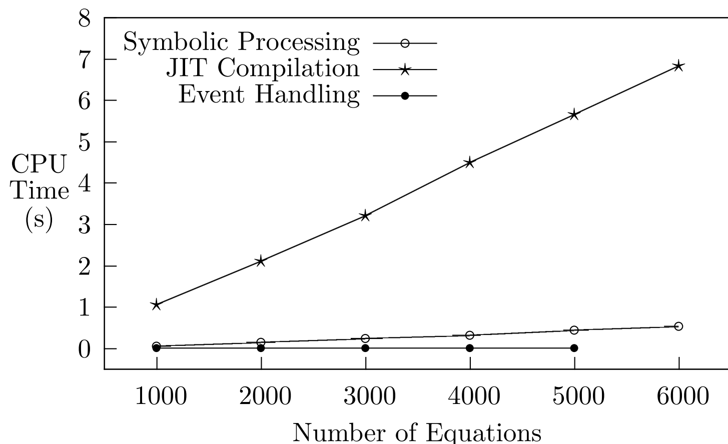


Figure: Plot demonstrating how CPU time spent on mode switches grows as number of equations increase in structurally dynamic RLC circuit simulation

Shallow

$$\textbf{type}\ \mathit{Region} = (\mathbb{R}, \mathbb{R}) \rightarrow \mathit{Bool}$$

$$\mathit{circle} :: \mathbb{R} \rightarrow \mathit{Region}$$
$$\mathit{circle}\ r = \lambda(x, y) \rightarrow \mathit{sqrt}\ (x \uparrow 2 + y \uparrow 2) \leqslant r$$

$$\mathit{rectangle} :: \mathbb{R} \rightarrow \mathbb{R} \rightarrow \mathit{Region}$$

$$\mathit{union} :: \mathit{Region} \rightarrow \mathit{Region} \rightarrow \mathit{Region}$$

$$\ldots$$

Deep

$$\begin{array}{ll}\textbf{data}\ \mathit{Region} = & \\ & \mathit{Circle}\ \mathbb{R} \\ \mid & \mathit{Rectangle}\ \mathbb{R}\ \mathbb{R} \\ \mid & \mathit{Union}\ \mathit{Region}\ \mathit{Region} \\ \ldots & \end{array}$$

Given a parametrised signal relation:

$$sr1 :: \mathbb{R} \rightarrow SR\ ((\mathbb{R}, \mathbb{R}), E\ \mathbb{R})$$

we can recursively define a signal relation sr that describes an overall behaviour by "stringing together" the behaviours described by $sr1$:

$$sr :: \mathbb{R} \rightarrow SR\ (\mathbb{R}, \mathbb{R})$$
$$sr\ x = switch\ (sr1\ x)\ sr$$