# Effect of Type-2 Fuzzy Membership Function Shape on Modelling Variation in Human Decision Making

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***Abstract*— This paper explains how the shape of type-2 fuzzy membership functions can be used to model the variation in human decision making. An interval type-2 fuzzy logic system (FLS) is developed for umbilical acid-base assessment. The influence of the shape of the membership functions on the variation in decision making of the fuzzy logic system is studied using the interval outputs. Three different methods are used to create interval type-2 membership functions. The centre points of the primary membership functions are shifted, the widths are shifted, and a uniform band is introduced around the original type-1 membership functions. It is shown that there is a direct relationship between the variation in decision making and the uncertainty introduced to the membership functions.**

## I. Introduction

Fuzzy logic systems (FLSs) usually employ type-1 fuzzy sets and represent uncertainty by numbers in the range [0,1] which are referred to as degrees of membership. Type-2 fuzzy sets are an extension of type-1 fuzzy sets with an additional dimension that represent the uncertainty about the degrees of membership. Type-2 fuzzy sets are useful in circumstances where it is difficult to determine the exact membership function (mf) for a fuzzy set. Type-1 mfs are precise in the sense that once they have been chosen all the uncertainty disappears. However, type-2 mfs are fuzzy themselves. The simplest type-2 sets are interval type-2 sets whose elements' degree of membership are intervals with secondary membership degree of 1.0.

FLSs consist of four main interconnected components: rules, fuzzifier, inference engine, and output processor. Fig. 1 shows the mechanisms of a type-2 fuzzy logic system.

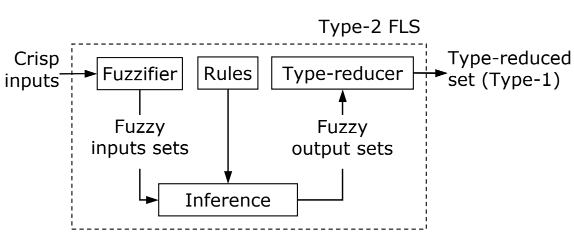


Fig. 1. Components of a type-2 FLS

Once the rules are established, an FLS can be viewed as a mapping from inputs to outputs. A typical rule is like:

IF arterial pH is *low* and venous pH is *low*
THEN Acidemia is *severe*.

Fuzzy sets are associated with the linguistic terms in the rules, shown in italics above, and with the inputs to and the output of the FLS. Type-1 FLSs use type-1 fuzzy sets and an FLS which uses at least one type-2 fuzzy set is called a type-2 FLS. A general type-2 FLS is too complicated, inferencing and output processing are prohibitive [1]. A simplification approach is to use interval type-2 fuzzy sets. There are fast algorithms to compute the output of an interval type-2 FLS (it2FLS) [2].

The concept of type-2 fuzzy sets was introduced by Zadeh [3]. Mizumoto and Tanaka studied the set theoretic operations of type-2 fuzzy sets and properties of membership degrees of such sets [4]; and examined type-2 fuzzy sets under the operations of algebraic product and algebraic sum [5]. Karnik and Mendel obtained algorithms for performing union, intersection, and complement for type-2 fuzzy sets, and developed the concept of the centroid of a type-2 fuzzy set [1]. Dubois and Prade gave a formula for the composition of type-2 relations as an extension of the type-1 sup-star composition for the minimum t-norm [6]. Karnik et al. presented a general formula for the extended sup-star composition of type-2 relations [7]. Hisdal studied rules and interval sets for higher-than-type-1 fuzzy logic [8]. Liang and Mendel developed the theory for different kinds of fuzzifiers for it2FLSs [9].

Type-1 FLSs, like classical expert systems, are deterministic in the sense that for the same inputs the outputs are always the same. However, human experts exhibit a nondeterministic behaviour in decision making. Variation may occur among the decisions of a panel of human experts as well as in the decisions of an individual expert for the same inputs. The terms that are used in an FLS have different meanings for different experts and experts may arrive to different conclusions in their inferencing depending on environmental conditions or over time. Understanding the dynamics of the variation in human decision making could allow the creation of 'truly intelligent' systems that cannot be differentiated from their human counterparts. Moreover, in application areas where having an expert constantly available is not possible, such systems can produce a span of decisions that may be arrived at by a panel of experts. This paper presents the results of the research that studies the relation between the variation in decision making of an FLS and the shape of the type-2 mfs that are used in the FLS.

Diagnostic medicine, where systematic handling of perceptual uncertainties is crucial to success, is an important application domain for this study. Umbilical acid-base (UAB) assessment of an infant immediately after delivery is an objective measure of labour, and can be used to audit assessment of labour performance. The acidity ($\mathrm{pH}$), partial pressure of oxygen ($\mathrm{pO_2}$) and partial pressure of carbon dioxide ($\mathrm{pCO_2}$) in blood samples taken from the venous and arterial vessels in the clamped umbilical cord can be measured by a blood gas analysis machine. A parameter termed base deficit of extracellular fluid (BDecf) can be derived from the $\mathrm{pH}$ and $\mathrm{pCO_2}$ parameters [10]. This can distinguish the cause of a low $\mathrm{pH}$ between the distinct physiological conditions of respiratory acidosis, due to a short-term accumulation of $\mathrm{CO_2}$, and a metabolic acidosis, due to lactic acid from a longer-term oxygen deficiency. An interpretation is made based on the $\mathrm{pH}$ and BDecf parameters from both arterial and venous blood.

A type-1 FLS was previously developed for the UAB assessment, encapsulating the knowledge of leading obstetricians, neonatologists and physiologists gained over years of acid-base interpretation [11], [12], [13], [14]. This FLS combines knowledge of the errors likely to occur in acid-base measurement, physiological knowledge of plausible results and statistical knowledge of a large database of results. The FLS developed to carry out the research presented in this paper is an extension of the original type-1 FLS.

Preliminary investigations for determining the parameters that define the uncertainties resulting in variation in decision making were presented in [15], [16]. The motivation for this research and proposed method was explained in [15]. In [16], a nondeterministic type-1 FLS (nd1FLS) and an it2FLS used for modelling nondeterminism were presented and the effect of the magnitude of the uncertainty introduced to the centre point of the mfs was studied. This paper presents the results of the further studies where the effect of the shape of the mfs on the variation in the decision making of an FLS is investigated.

An it2FLS is developed by representing the terms used in the type-1 FLS by type-2 fuzzy sets, the developed model is explained further in section II . The variation in decision making of the it2FLS is examined in terms of uncertainty introduced to the original type-1 mfs. The uncertainty is introduced using three methods: by varying the centre point, varying the width, and adding a uniform band around the original type-1 mfs. The results of the study are presented in section III. The paper concludes with discussions of the results and an outline of future work in section IV .

## II. Methodology

The six expert clinicians who took part in the development of the type-1 FLS were asked to rank 50 UAB assessments in terms of perceived likelihood of having suffered brain damage due to lack of oxygen. Fig. 2 shows the rankings of 50 UAB assessments by six experts against the type-1 FLS. A perfect agreement, which would be a straight line from (0,0) to (50,50), is the ideal desired result. However, as can be seen from Fig. 2, there is neither perfect agreement with the FLS nor among the experts. It can also be observed that at the extreme cases the experts tend to agree with each other and the FLS but in the cases that fall in the middle of the range, there is less agreement. The distribution presents the characteristic of an elliptic envelope around the diagonal line from (0,0) to (50,50).

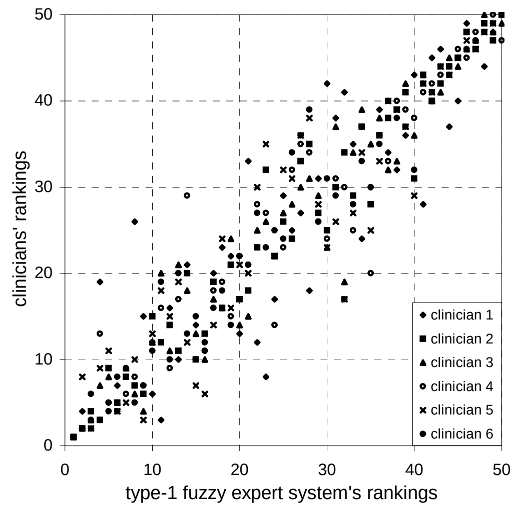

Fig. 2. Variation in rankings of 50 assessments

The aim of this research is to explore the dynamics of the variation in human decision making as explained above. In this paper, the relation between the shape of the mfs and the variation in decision making of the FLS is studied. The rules of the type-1 FLS use fixed type-1 mfs to represent linguistic terms. However, experts have diverse opinions about meanings of linguistic terms and they often provide different consequents for the same antecedents of a rule. Using the precise type-1 mfs does not take into account the vagueness present in the terms used. In this study the vagueness inherent in the linguistic terms is introduced by using interval type-2 mfs. The resulting it2FLS is explained in more detail in section II-A.

The type-1 FLS produces a health measure for every input case. These health measures are then used to rank the cases in terms of perceived likelihood of having suffered brain damage due to lack of oxygen. The health measure that is produced by the it2FLS is an interval. In order to demonstrate the effect of uncertainty in the linguistic terms of the rules, ranking is done for every combination of the cases using the upper and lower bounds of the health measure interval. This results in $2^{50}$ rankings for 50 UAB assessments.

The results of the experiments for analyzing the effect of uncertainty in the interval mfs on variation in decision making is presented in the section III.

### A. *Interval Type-2 Fuzzy Logic System (it2FLS)*

Using a type-2 FLS can effectively provide a natural mechanism to present the vagueness inherent in linguistic terms used in FLSs.

The type-1 FLS was extended by converting the rule set directly by using interval type-2 fuzzy sets. The type-1 FLS uses sigmoidal membership functions. Fig. 3 shows three type-1 sigmoidal functions. From left to the right in Fig. 3, the mfs are created using the following functions respectively:

$$mf_l(x) = \frac{1}{1+e^{\frac{(x-c_n)\times 5}{w_d}}} \quad (1)$$

$$mf_c(x) = \frac{1}{\left(1+e^{\frac{(x-c_n-w_d/3)\times 15}{w_d}}\right)\left(1+e^{\frac{(c_n-w_d/3-x)\times 15}{w_d}}\right)} \quad (2)$$

$$mf_r(x) = \frac{1}{1+e^{\frac{(c_n-x)\times 5}{w_d}}} \quad (3)$$

where $c_n$ is the centre point and $w_d$ is the width of the sigmoidal functions.

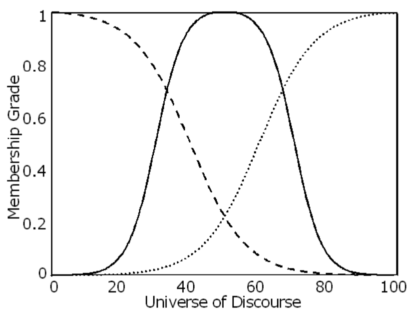


Fig. 3. Three type-1 sigmoidal mfs

In the it2FLS, the type-2 mfs are created by deviating the parameters of the original type-1 mfs by a percentage of the universe of discourse of the variables that they are associated to. Three different methods are used to create these type-2 mfs: by varying the centre point, varying the width and adding a uniform band around the original type-1 mf. This results in interval type-2 fuzzy sets with sigmoidal primary mfs. The centre and width are varied 1-5% of the universe of discourse and the uniform band around the primary mf is created by adding $\pm\, 0.01 - 0.05$ to the degree of membership of the original type-1 membership function. Fig. 4 shows three interval type-2 sigmoidal mfs, where $c_i$ and $c_i'$ ($i = 1, 2, 3$) are the centre point pairs for each type-2 mf. Fig. 5 shows three interval type-2 sigmoidal mfs which are obtained by varying the width of the primary mf. Fig. 6 shows the three interval type-2 sigmoidal mfs created by adding a uniform band around the original type-1 mf.

The mechanisms of the developed it2FLS is presented in Fig. 1. The inference and defuzzification methods of the type-1 FLS were updated to work with the type-2 fuzzy terms. The fuzzifier of the type-1 FLS, which turns the crisp input values into type-1 fuzzy input sets in order to compensate for the errors in readings of the blood gas analysis machine, is not changed. By Mendel's classification, the resulting FLS is a type-1 nonsingleton it2FLS because the inputs are type-1 fuzzy sets and all the antecedent and consequent sets of the rule base are interval type-2 fuzzy sets [1].

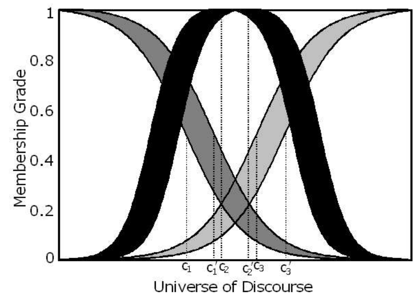


Fig. 4. Three interval type-2 sigmoidal mfs, centre point shifted

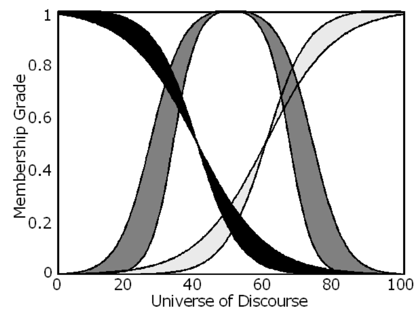


Fig. 5. Three interval type-2 sigmoidal mfs, width shifted

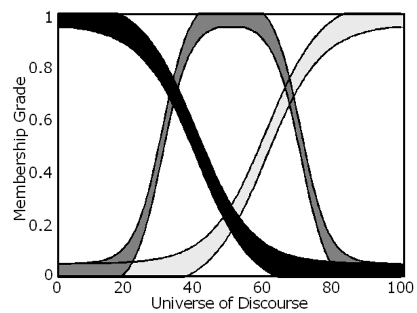


Fig. 6. Three interval type-2 sigmoidal mfs, uniform band added

The input, antecedent, consequent and implication operations use minimum t-norm and maximum t-conorm. The result of antecedent operations is an interval which is fed into the consequent. Firing of rules result in type-2 fuzzy sets which are combined into a single type-2 fuzzy set by minimum t-norm. Mendel [1] has established theoretical results to effectively determine the lower and upper bounds of the centroid of a type-2 set and has provided algorithms [2] for carrying out the necessary calculations. That is, the centroid is an interval, the mean value of the upper and lower bounds of which can be taken as a single crisp centroid, if required.

A type-2 FLS must reduce to a type-1 FLS when the uncertainty about the shape of mfs is zero. This was verified by running the it2FLS with 0% deviation in the parameters which were being varied to create the type-2 mfs.

The it2FLS produces for every input case an interval of health measure which is used to rank the cases in terms of the perceived likelihood of having suffered brain damage due to lack of oxygen. Ranking of 50 UAB assessments is done for every combination of the upper and lower bounds of the interval outputs which results in $2^{50}$ rankings. In section III, the variation in rankings of the it2FLS models is presented to demonstrate the effect of uncertainty in the mfs.

## III. Results

In this section, the effect of introducing uncertainty to the linguistic terms used in an FLS is presented. The uncertainty is introduced by using different shapes of the type-2 mfs that are associated to the linguistic terms.

It can be observed from these trials that there is a direct relationship between the uncertainty in the mfs used in the it2FLS and the variation in the rankings. As the uncertainty about the linguistic terms used in the it2FLS is increased, the variation in decision making is observed to increase. Another important observed feature is in the nature of the variation, the extreme UAB cases are ranked most of the time the same but in the cases that fall in the middle of the range, there is less agreement which results in a cloud of data bounded in an elliptic envelope along the diagonal. This is in parallel to the behaviour exhibited by the panel of experts presented in Fig. 2.

Variation of all three parameters have resulted in similar behaviour. However, type-2 mfs created by shifting the centre point of the original type-1 mfs have caused greater variation in the rankings in comparison to the other two models which were created by varying the parameters of the original type-1 mfs with similar quantities.

### *A. Varying the Center Points*

Fig. 7 - 9 show the variation in rankings as the deviation in the centre point of the mfs used in the it2FLS is increased from 1% to 5% of the universe of discourse of the variable they are associated with. In comparison with the results presented in the following two sections, the variation in ranking increases rapidly with the increasing shift in the centre points.

### *B. Varying the Widths*

Fig. 10 - 12 show the variation in rankings as the deviation in the width of the mfs used in the it2FLS is increased from 1% to 5% of the universe of discourse of the variable they are associated with. In comparison to the effect of shifting the centre points, the variation in ranking increases less rapidly with the increasing shift in the widths of the type-2 mfs. However, in comparison to similar amount of changes in the width of the uniform bands introduced around type-1 mfs, the behaviour is very similar.

### *C. Creating a Uniform Band Around the Type-1 mfs*

The uniform bands are created by adding to and subtracting from the membership degree of the original type-1 mfs a fixed value. Fig. 13 - 15 show the variation in rankings as the width of the band introduced around the original type-1 mfs is increased from 1% to 5% of the span of membership grade. In comparison to the effect of shifting the centre points, the variation in ranking increases less rapidly with the increasing width of the bands. However, in comparison to type-2 mfs created with similar amount of changes in the widths of the original type-1 mfs, the behaviour is very similar.

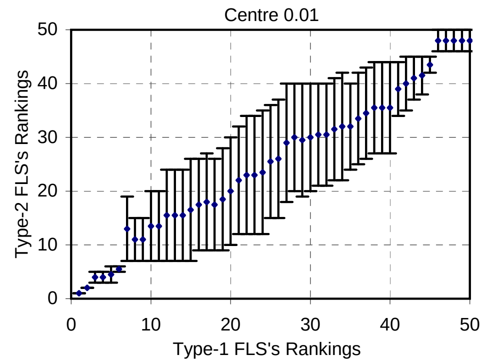


Fig. 7. Ranking variation with 1% shift in centre points

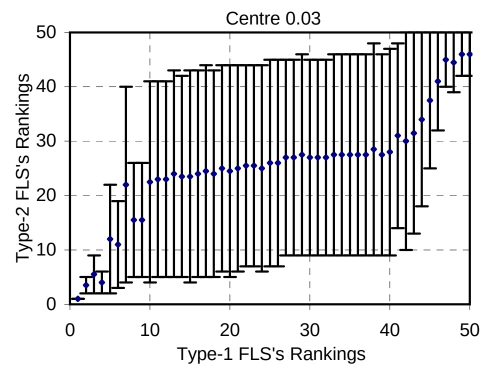


Fig. 8. Ranking variation with 3% shift in centre points

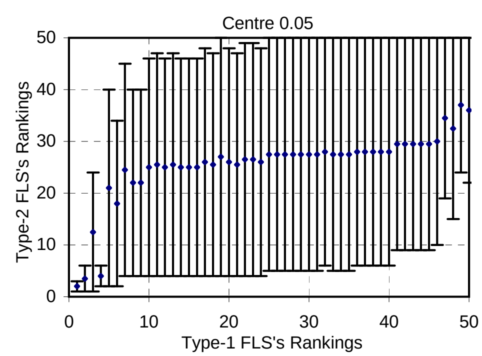


Fig. 9. Ranking variation with 5% shift in centre points

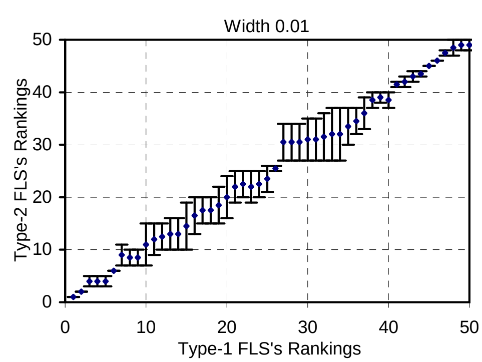


Fig. 10. Ranking variation with 1% shift in widths

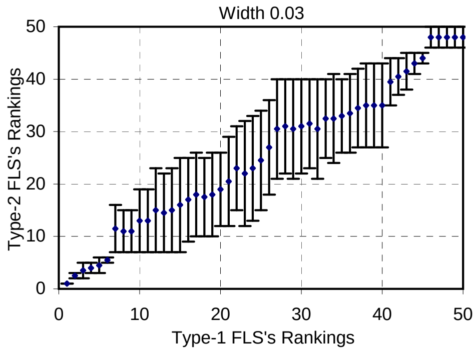


Fig. 11. Ranking variation with 3% shift in widths

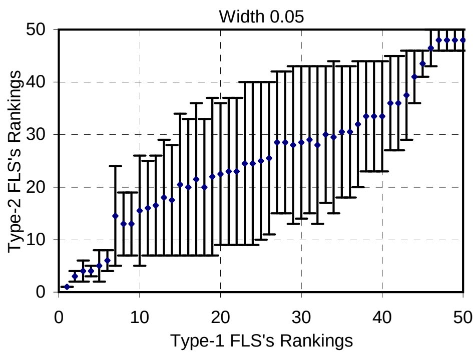


Fig. 12. Ranking variation with 5% shift in widths

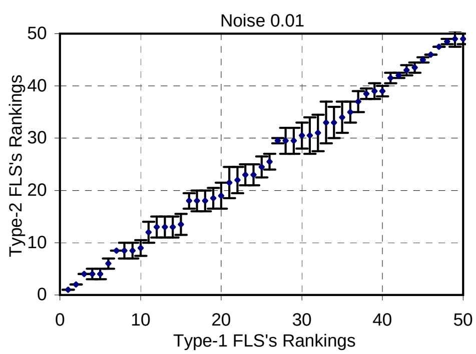


Fig. 13. Ranking variation after 1% uniform band

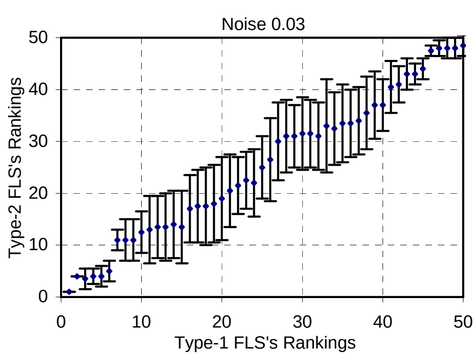


Fig. 14. Ranking variation after 3% uniform band

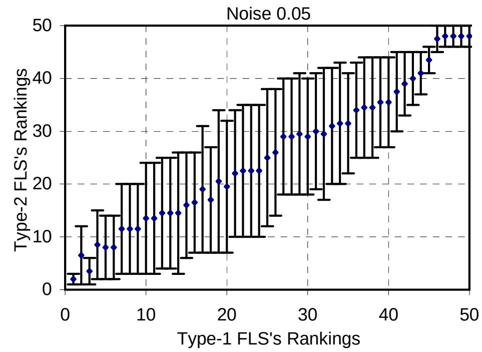


Fig. 15. Ranking variation after 5% uniform band

## IV. Conclusions

Most success of fuzzy logic is in fuzzy logic control, but this success has not yet been carried over to modelling human reasoning - Zadeh's Computing with words Paradigm [17].

In this paper, the work done on modelling the variation in human expert opinion is presented. Specifically, the relation of the uncertainty in the mfs used in the FLSs and the variation in decision making is explored. It is shown that it is possible to capture the variation in human decision making using an it2FLS. It is observed that the level of variation is directly related to the amount of uncertainty about the linguistic terms used in decision making. This kind of nondeterministic behaviour can be modelled by using type-2 mfs which can be derived from type-1 mfs associated to the terms used in the rule base of an FLS by varying their parameters.

The variation in decision making by an FLS can be controlled using the level of uncertainty in its mfs. This can be used in creating intelligent systems that can mimic their human counterparts better. An example of the major benefits that this may provide may be in application areas where having an expert constantly available is not possible. Such systems can produce a span of decisions that may be arrived at by a panel of experts.

The research on understanding and modelling the dynamics of variation in human decision making is ongoing. In future, the possibility of modelling nondeterminism using a general type-2 FLS will be explored. A general type-2 FLS is too complicated, inferencing and output processing are prohibitive [1]. One simplification approach is to use interval type-2 fuzzy sets, which was done in developing the it2FLS models used for the research presented in this paper. The main aim of the future work is to develop other approximation methods which simplifies the inferencing and output processing.

## Acknowledgment

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