An Investigation into the Effect of Number of Model Parameters on Performance in Type-1 and Type-2 Fuzzy Logic Systems

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Abstract

An investigation was carried out in which the performances of type-1 and type-2 fuzzy logic systems (FLSs) with varying number of tunable parameters were compared in their ability to predict the Mackey-Glass time series with various levels of added noise. Each of the FLSs were tuned to achieve the best possible performance using a standardised gradient descent procedure. These experiments were repeated a number of times in order to establish the mean performance of each FLS. The results show that the best performance was achieved with a type-1 FLS, albeit featuring a high number of tunable parameters. A type-2 FLS with far fewer parameters achieved performance very close to the best.

Keywords: Type-1 fuzzy logic systems, type-2 fuzzy logic systems, time-series forecasting.

1 Introduction

Many decision-making and problem solving tasks are too complex to be understood quantitatively, but by using knowledge that is imprecise rather than precise [1] it is possible to overcome this. Fuzzy logic resembles human reasoning in its use of approximate information and uncertainty to generate decisions. It was specifically designed to represent uncertainty and vagueness and provide formalized tools for dealing with the imprecision in many real problems. Since knowledge can be expressed more naturally by using fuzzy sets, many complex decision problems can be significantly simplified.

Although many applications have been found for type-1 fuzzy logic systems (FLSs), it is its application to rule-based systems that has most significantly shown its importance as a powerful design methodology, but yet it is unable to model and minimize the effect of all uncertainties. To overcome this limitation, type-2 FLSs can be introduced as they can model uncertainties better and minimize their effects. Type-2 FLSs are characterized by IF-THEN rules, but their antecedent or consequent sets are type-1 or type-2. A type-2 fuzzy set can represent and handle uncertain information effectively. More details about type-2 fuzzy sets and FLSs can be found in section 2.

In 1977 Mackey and Glass published a paper in which they associate the onset of disease with bifurcations in the dynamics of first-order differential-delay equation, which model physiological systems. The Mackey-Glass time series has become one of the benchmark problems for time-series prediction in both the neural network and fuzzy logic areas. Mendel and Karnik [5, 24] have carried out experiments into forecasting Mackey-Glass Chaotic Time-series with noisy data by using type-1 and type-2 FLSs. They have suggested that an interval non-singleton type-2 FLS with type-1 fuzzy sets achieves the best performance and can be used in a real-time adaptive environment. Although the authors noted that the type-2 FLS used in their experiment featured more internal tuneable model parameters than the type-1 FLS, they did not go on to investigate whether it was simply the number of model parameters that was responsible for the performance gains achieved.

The purpose of this work was firstly to attempt to reproduce the results of Mendel and Karnik and secondly to perform a careful analysis of whether the performance of the type-2 FLS could be matched or surpassed by type-1 models with a similar or greater number of internal tunable model parameters. Four main classes of FLSs are considered: (i) T1-SFLS - 'conventional' FLSs with singleton inputs and type-1 fuzzy sets throughout; (ii) T1-NFLS - type-1 FLSs with non-singleton (type-1) fuzzy inputs and type-1 fuzzy sets throughout; (iii) T2-SLFS – FLSs featuring interval type-2 sets with singleton inputs; (iv) T2-NSLFS-T1 – FLSs featuring interval type-2 sets with type-1 non-singleton inputs. This work has evolved from recent studies on modelling of non-deterministic reasoning using type-2 FLSs [25]. The software used in this experiment is that provided by Professor Mendel[1].

This paper has been organized into six sections: Introduction presents general information about Fuzzy Logic Systems and Mackey-Glass Chaotic Time-Series. Section 2 gives more details about type-2 fuzzy sets and FLSs. Section 3, Methodology, describes the experimental design and fuzzy model evaluation. Section 4, Results, shows the results from the experiment. Section 5, Discussion, summarizes the results and discusses the importance and limitation of these results. Finally, section 6, Future Work, presents the directions of future work for this research.

2 Type-2 Fuzzy Sets and FLSs

All fuzzy sets are characterized by membership functions. Type-1 fuzzy sets are characterized by two-dimensional membership functions in which each element of the type-1 fuzzy set has a membership grade that is a crisp number in [0, 1]. Type-2 fuzzy sets are characterized by *fuzzy membership functions* that are three-dimensional. The membership grade for each element of a type-2 fuzzy set is a fuzzy set in [0, 1]. The additional third dimension provides additional degrees of freedom to capture more information about the represented term. Type-2 fuzzy sets are useful in circumstances where it is difficult to determine the exact membership function for a fuzzy set, which is useful for incorporating uncertainties. Type-1 fuzzy sets handle uncertainties by using precise membership functions that the user believes capture the uncertainties. Once the type-1 membership functions have been chosen, all the uncertainty disappears, because type-1 membership functions are totally precise. However, type-2 fuzzy sets handle uncertainties about the meaning of the words that they represent by modelling the uncertainties using type-2 membership functions.

Fuzzy logic systems (FLSs) which are used for representing and inferring with knowledge that is imprecise, uncertain, or unreliable consists of four main interconnected components: *rules, fuzzifier, inference engine, and output processsor*. Once the rules have been established, a FLS can be viewed as a mapping from inputs to outputs. Type-1 FLSs use type-1 fuzzy sets and a FLS which uses at least one type-2 fuzzy set is called a type-2 FLS. Type-2 fuzzy sets let us model the effects of uncertainties in rule-based fuzzy logic systems (FLSs). Figure 1 shows *footprint of uncertainty* to represent the type-2 fuzzy set.

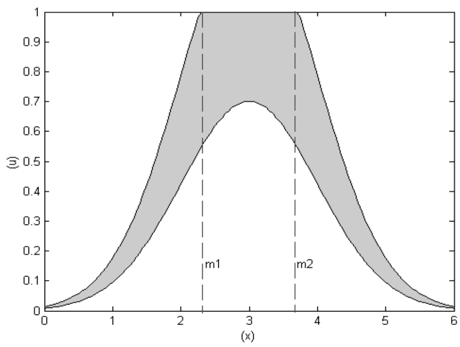

Figure 1 Pictorial representation of a Gaussian type-2 fuzzy set with uncertain of means.

Rule-based type-2 FLSs can be applied to every area where type-1 rule-based FLSs have been applied in which some uncertainty is present. Type-2 fuzzy sets can also be applied to non-rule-based applications of fuzzy sets, again if uncertainty is present.

The concept of a type-2 fuzzy set was introduced by Zadeh [2] as an extension of the concept of an ordinary fuzzy set i.e., a type-1 fuzzy set. Mizumoto and Tanaka studied the set theoretic operations of type-2 fuzzy sets and properties of membership grades of such sets [3]; and examined type-2 fuzzy sets under the operations of algebraic product and algebraic sum [4]. Karnik and Mendel extended the works

[1] http://sipi.usc.edu/~mendel/software/

of Mizumoto and Tanaka and obtained algorithms for performing union, intersection, and complement for type-2 fuzzy sets, and developed the concept of the centroid of a type-2 fuzzy set and provided a practical algorithm for computing it [5].

Dubois and Prade discussed fuzzy valued logic and gave a formula for the composition of type-2 relations as an extension of the type-1 sup-star composition for the minimum t-norm [6]. Karnik et al. presented a general formula for the extended sup-star composition of type-2 relations [7]. Hisdal studied rules and interval sets for higher-than-type-1 FL [8]. Liang and Mendel developed the theory for interval type-2 FLSs for different kinds of fuzzifiers, and showed how the free parameters within such FLSs can be tuned using training data. Additional discussions on the use of interval sets in fuzzy logic can be found in [9-17]. Lee and Lee introduced a ranking method for type-2 fuzzy values and used this result in solving the shortest path problem in a type-2 weighted graph [18]. Some other examples of application of type-2 FLSs are [20-23].

2.1 Type-2 FLS's Inferencing

Suppose that a type-2 fuzzy set in X is $\tilde{A}$, and the membership grade of $x \in X$ in $\tilde{A}$ is $\mu_{\tilde{A}}(x)$, which is a type-1 fuzzy-set whose domain is in [0,1]. The elements of the domain of $\mu_{\tilde{A}}(x)$ are called *primary memberships* of x in $\tilde{A}$ and the memberships in $\mu_{\tilde{A}}(x)$ are called *secondary memberships* of x in $\tilde{A}$. Secondary membership defines the possibilities for the primary membership.

When the secondary membership functions are type-1 interval sets, the type-2 set is called an *interval type-2 set*. Interval type-2 sets are the simplest kind of type-2 sets, and [5] presents fast algorithms to compute the output of a type-2 FLS, which uses interval type-2 sets.

The membership grades of type-2 sets are type-1 sets; therefore, in order to perform operations like union and intersection on type-2 sets, we need to be able to perform t-conorm and t-norm operations between type-1 sets. This is done using Zadeh's Extension Principle [2] and also proven in [1] using a representation method without having to use the Extension Principle. A binary operation $*$ between two crisp numbers can be extended to two type-1 sets $F = \int_v f(v)/v$ and $G = \int_w g(w)/w$ as

$$F * G = \int_v \int_w \frac{[f(v) \bullet g(w)]}{(v * w)} \qquad (1)$$

where $\bullet$ denotes the chosen t-norm (generally product or minimum t-norm is used). For example, the extension of the t-conorm (generally the maximum t-conorm is used) operation to type-1 sets is:

$$F \sqcup G = \int_v \int_w \frac{[f(v) \bullet g(w)]}{(v \vee w)} \qquad (2)$$

This is called the *join* operation [4]. Similarly, the extension of the t-norm operation to type-1 sets, which is also known as the *meet* operation, is:

$$F \sqcap G = \int_v \int_w \frac{[f(v) \bullet g(w)]}{(v \bullet w)} \qquad (3)$$

The complement of a type-2 set which is called as the *negation* operation is defined as:

$$\neg F = \int_v \frac{f(v)}{1 - v} \qquad (4)$$

where all integrals denote to logical union.

In type-1 FLS, multiple antecedents in rules are connected by the *t-norm*. The membership grades in the input sets are combined with those in the output sets using the sup-star composition. Multiple rules may be combined using the *t-conorm* operation (adapted from [7]). In type-2 FLS, the inference combines rules and gives mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. Extended sup-star composition is the backbone computation for a type-2 FLS, in which *t-norm* is replaced by *meet* and *t-conorm* is replaced by *join*. There is insufficient space to detail type-2 inference here. The interested reader is referred to *Chapter 10 and 11* in Mendel's book [5].

The centroid of a type-1 set *A*, whose domain is discretized into N points is given as

$$C_A = \frac{\sum_{i=1}^{N} x_i \mu_A(x_i)}{\sum_{i=1}^{N} \mu_A(x_i)} \qquad (5)$$

Similarly, the centroid of a type-2 set $\tilde{A}$, whose domain is discretized into N points, can be defined using (5) as follows. If we let $D_i = \mu_{\tilde{A}}(x_i)$, then

$$C_{\tilde{A}} = \int_{\theta_1} \cdots \int_{\theta_n} \frac{[\mu_{D_1}(\theta_1) \bullet \ldots \bullet \mu_{D_n}(\theta_n)]}{\frac{\sum_{i=1}^{N} x_i \theta_i}{\sum_{i=1}^{N} \theta_i}} \qquad (6)$$

where $\theta_i \in D_i$, and all the integrals denote logical union.

3 Methodology

Equation (7) has become known as the Mackey-Glass equation. It is a non-linear delay differential equation [5]:

$$\frac{dx(t)}{dt} = \frac{0.2x(t-\tau)}{1 + x^{10}(t-\tau)} - 0.1x(t) \qquad (7)$$

For τ > 17 is known to exhibit chaos.

This equation is converted into a discrete time-series equation by using Euler's method as shown in (8):

$$f(x,t) = \frac{0.2x(t-\tau)}{1 + x^{10}(t-\tau)} - 0.1x(t) \qquad (8)$$

Then

$$x(t+1) = x(t) + hf(x,t) \qquad (9)$$

Where h = 1 and τ = 30.

The initial values of $x(t)$ where t <= 30 are set randomly. The value of each data is in [0,1].

Five independent data sets with 5 different noise levels (in total 25 data sets 2200 series each) were generated. These data sets were generated by using Mackey-Glass time-series delay differential equation show in (7) above.

After 5 data sets were generated, 5 different level of noise were generated as follows:

Level 1: 0 *noise* (noise free)

Level 2: 0.01 *noise*

Level 3: 0.05 *noise*

Level 4: 0.10 *noise*

Level 5: 0.20 *noise*

where *noise* was a uniformly distributed random number in [-1,1]. Then these 5 different levels of noise were added into the data sets.

Type-1 SFLS and type-1 NSFLS have been designed with 4 antecedents, 2 and/or 3 membership functions (MFs) for each antecedent, the number of rules are 16 and/or 81 rules (2^4 and/or 3^4) respectively, each rule is characterized by 8 antecedent MF parameters (means and standard deviations), and 1 consequent parameter ($\bar{y}$). The initial location of each Gaussian antecedent MF is based on the mean (m_x) and standard deviation (σ_x) and the mean of MFs are:

2 MFs = [m_x - 2σ_x , m_x + 2σ_x]
3 MFs = [m_x + 2σ_x , m_x , m_x + 2σ_x]

Initially all standard deviation parameters are tuned to σ_x or 2σ_x. Additionally the height defuzzifier and initial centre of each consequent's MF are random numbers in [0,1]. So, the total number of tunable parameter for Type-1 SFLS with 2 and 3 MFs are 144 and 729 respectively. For type-1 NSFLS each of the 4 noisy input measurements are modelled using a Gaussian MF, a different standard deviation is used for each of the 4 input measurement MFs (σ_n). So, the total number of tunable parameters for Type-1 NSFLS with 2 and 3 MFs are 145 and 730 respectively. Finally, 4 different models were created for both type-1 SFLS and type-1 NSFLS for each data set (25 data sets).

Interval type-2 singleton FLS (type-2 SFLS) and type-1 non-singleton type-2 FLS (type-2 NSFLS-T1) have been designed by using the partially dependent approach. First, the best possible singleton and non-singleton type-1 FLSs were designed by tuning their parameters using back-propagation designs, and then some of those parameters were used to initialise the parameters of the interval type-2 SFLS and type-2 NSFLS-T1. They consisted of 4 antecedents for forecasting, 2 MFs for each antecedent and 16 rules. The Gaussian primary MFs of uncertain means for the antecedents were chosen. The means of MFs are:

Mean of MF1 = [m_x - 2σ_x - 0.25σ_n ,
m_x - 2σ_x + 0.25σ_n]

Mean of MF2 = [m_x + 2σ_x - 0.25σ_n ,
m_x + 2σ_x + 0.25σ_n]

where m_x is the mean of the data in the training parts, and σ_n is the standard deviation of the additive noise. Each rule of the type-2 SFLS and type-2 NSFLS-T1 were characterized by 12 antecedent MF parameters: left and right hand bounds on the mean, and the standard deviation for each of 4 Gaussian MFs) and 2 consequent parameters (left and right hand end-points for the centroid of the consequent type-2 fuzzy set). So in total the number of parameters tuned for type-2 SFLS is 224. Standard deviation for each of the 4 input measurement MFs (σ_n) is used in type-2 NFLS-T1. So in total the number of parameters tuned for type-2 SFLS is 225.

Initially the final tuned results were used for the standard deviation of the input, σ_x or $2\sigma_x$, obtained from type-1 NSFLS design, and also $\bar{y}^i$ was obtained from type-1 SFLS and then initial $\bar{y}_r^i$ and $\bar{y}_l^i$ was chosen as:

$\bar{y}_r^i = \bar{y}^i + \sigma_n$

$\bar{y}_l^i = \bar{y}^i - \sigma_n$, where i = 1, 2, …, 16

Finally, two different models for both type-2 SFLS and type-2 NSFLS-T1 were created for each data set (25 data sets).

All designs mentioned above were tuned using steepest descent algorithm in which all of the learning parameters were set equal to the same value, 0.2. Training and testing were carried out for ten epochs. After each epoch the testing data was used to see how each FLS performed, by computing root mean square error (RMSE).

All designs above were also based on 1,000 noisy data points: x(501), x(502), …, x(1500). The First 500 noisy data, x(501), x(502), …, x(1000) were used for training, and the remaining 500 , x(1001), x(1002), …, x(1500), were used for testing the design. 4 antecedents: *x(k-3), x(k-2), x(k-1),* and *x(k)* were used to predict *x(k+1)*.

The performance of all the designs was evaluated using the RMSE as shown below:

$$RMSE = \sqrt{\frac{1}{500}\sum_{k=1000}^{1499}\left[x(k+1) - f(x^{(k)})\right]^2}$$

where $x^{(k)} = [x(k-3), x(k-2), x(k-1), x(k)]^T$.

4 Results

After the tuning had been completed, the performances of the type-1 and type-2 FLSs were compared. The results of performance of 12 different models are shown as follows. Table 1 shows the number of parameters that were used in the experiment for each of the 12 models.

Figures 2-6 show the performance (RMSE) of 12 different models for the 5 different noise levels averaged over five separate runs, while Figure 7 shows the key that applies to Figure 2 – 6. Figures 8 - 12 show the performance of just models M5, M8, and M11 with the *y*-axis expanded for more detail, while Figure 13 shows the key for these Figures. From Figures 2 and 8, it can be seen that M5 and M8 show better results after epoch 3 than M9 (T2-SFLS) and M11 (T2-NSFLS), probably because these data sets are noise free (this finding agrees with Mendel's result).

Table 1: Number of parameters of each design

No.	FLS	# Parameters
M1	T1-SFLS-2mf-2σ	144
M2	T1-SFLS-2mf-σ	144
M3	T1-SFLS-3mf-2σ	729
M4	T1-SFLS-3mf-σ	729
M5	T1-NSFLS-2mf-2σ	145
M6	T1-NSFLS-2mf-σ	145
M7	T1-NSFLS-3mf-2σ	730
M8	T1-NSFLS-3mf-σ	730
M9	T2-SFLS-2mf-2σ	224
M10	T2-SFLS-2mf-σ	224
M11	T2-NSFLS-T1-2mf-2σ	225
M12	T2-NSFLS-T1-2mf-σ	225

Table 2: The mean of RMSE of the best model for 12 different FLS models with 5 different noise levels

RMSE	N1	N2	N3	N4	N5
M1	0.2143	0.2351	0.2236	0.1253	0.1745
M2	0.2864	0.2447	0.2009	0.2071	0.2636
M3	0.1348	0.1326	0.1353	0.1343	0.1364
M4	0.1618	0.1723	0.1860	0.1800	0.1349
M5	**0.0243**	**0.0244**	**0.0274**	**0.0219**	**0.0226**
M6	0.0353	0.0555	0.0530	0.0467	0.0350
M7	0.0469	0.0469	0.0468	0.0466	0.0468
M8	**0.0189**	**0.0213**	**0.0239**	**0.0188**	**0.0211**
M9	0.0508	0.0285	0.0700	0.0529	0.0427
M10	0.1537	0.0450	0.0664	0.1291	0.1239
M11	**0.0264**	**0.0215**	**0.0243**	**0.0202**	**0.0216**
M12	0.0489	0.0263	0.0640	0.0434	0.0834

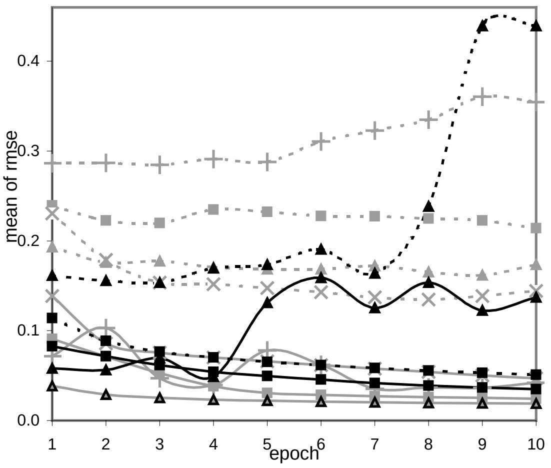

Figure 2: Graph of mean of RMSE of 12 models for noise level 1

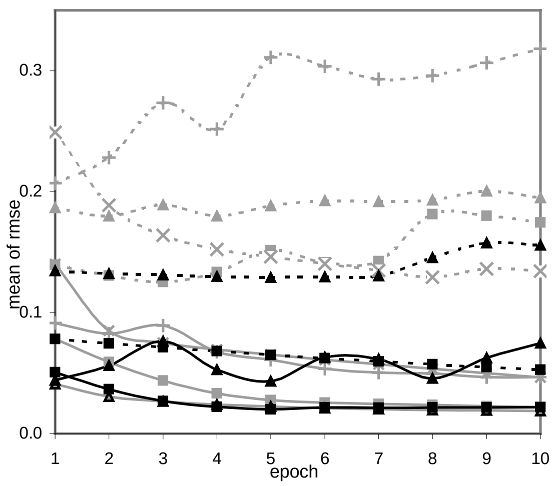

Figure 5: Graph of mean of RMSE of 12 models for noise level 4

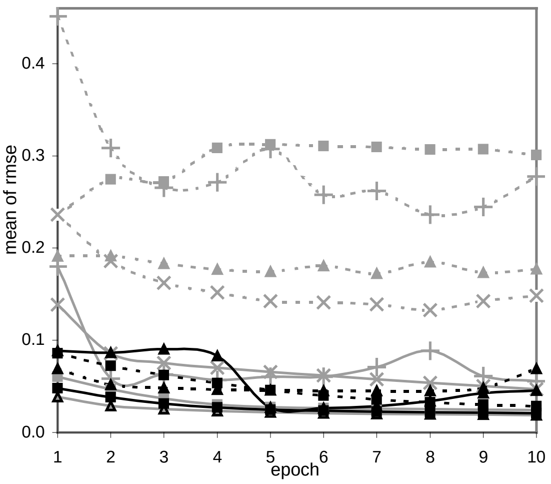

Figure 3: Graph of mean of RMSE of 12 models for noise level 2

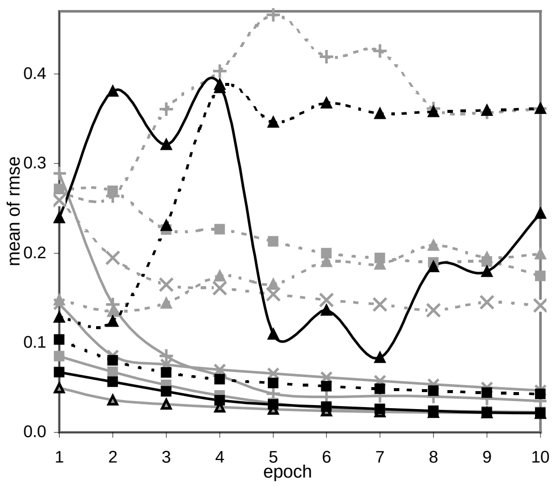

Figure 6: Graph of mean of RMSE of 12 models for noise level 5

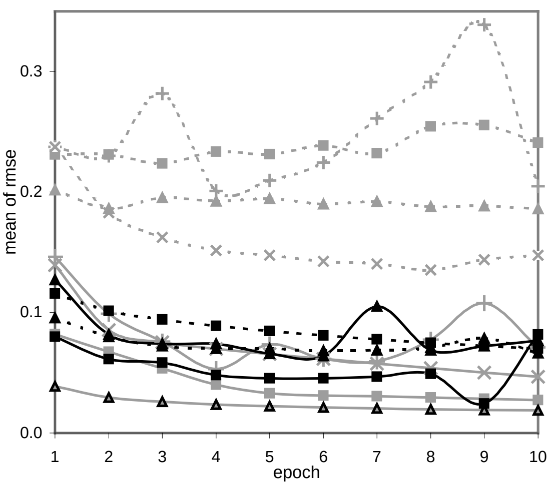

Figure 4: Graph of mean of RMSE of 12 models for noise level 3

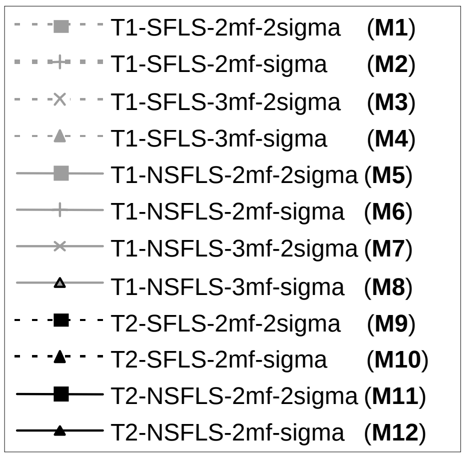

Figure 7: Key for figures 2-6

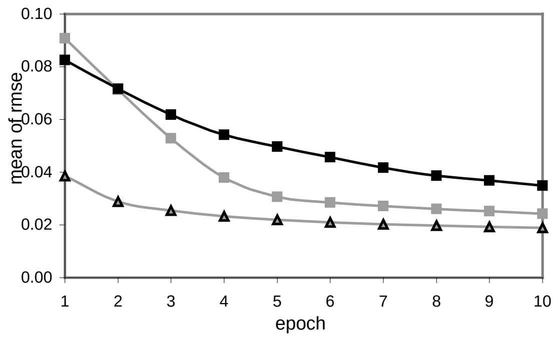

Figure 8: Graph of mean of RMSE of M5, M8, and M11 for noise level 1

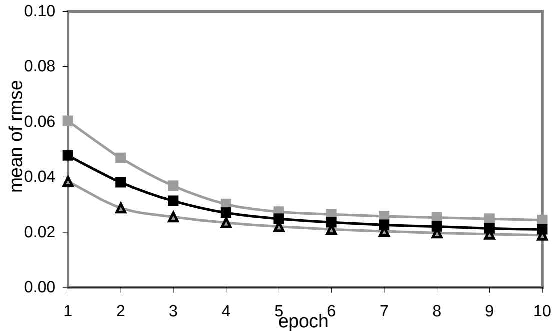

Figure 9: Graph of mean of RMSE of M5, M8, and M11 for noise level 2

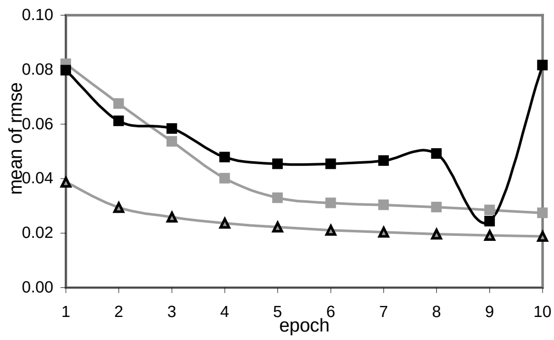

Figure 10: Graph of mean of RMSE of M5, M8, and M11 for noise level 3

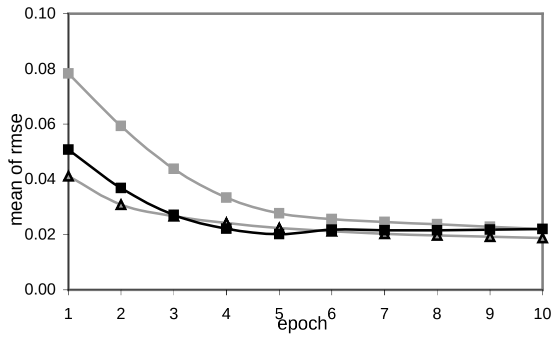

Figure 11: Graph of mean of RMSE of M5, M8, and M11 for noise level 4

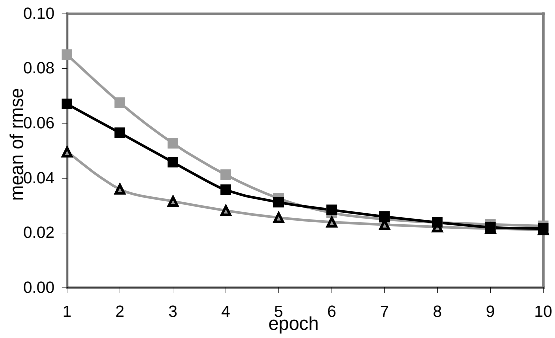

Figure 12: Graph of mean of RMSE of M5, M8, and M11 for noise level 5

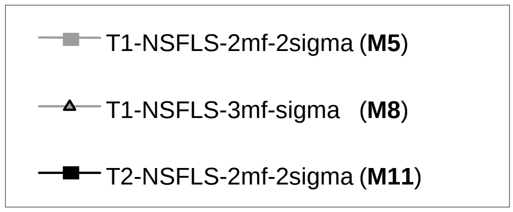

Figure 13: Keys for figures 8-12

5 Discussion

In all cases the performance of type-1 FLSs with singleton inputs (M1 – M4), the most common found in practice, is worse than for the type-1 non-singleton FLSs and the type-2 FLSs. This is regardless of the number of parameters in the systems. Particularly, it should be noted that M3 and M4, each featuring 729 tunable parameters, whilst better than M1 and M2, achieve far worse performance than type-1 non-singleton or type-2 FLSs with far fewer parameters (M5, M6, M9 and M10). This suggests that a high number of model parameters is not in itself sufficient to produce good performance.

With zero noise, M5 (with only 145 parameters) achieves better performance than M11 or M12. This agrees with Mendel's previous findings that in the absence of noise a type-1 FLS with non-singleton inputs is an adequate model for capturing the uncertainty.

The best overall performance is achieved with M8. This is a type-1 FLS with non-singleton inputs and with 3 membership functions for each variable, leading to a high number of tunable model parameters (730). From this, we may tentatively suggest that while type-2 FLSs may not strictly be necessary in order to achieve 'optimal' performance, their benefit may lie more in achieving good performance in a more tractable model. Note also that M5 (T1-NSFLS with 'only' 145 tuneable parameters) achieves very good performance, albeit slightly worse than the best models.

Finally, we emphasise that these finding are for one particular data set (MG-Time series) only and hence, no general conclusions can be made from them alone. In order to reach general conclusions it would be necessary to carry out similar experiments on a wide variety of data sets. There is **no** evidence at present to suggest that the similar results would necessarily be obtained for other kinds of data.

6 Future Work

It is anticipated that further work will be carried out using different data in order to investigate the generality of these findings.

References

[1] JM Mendel, RI John, Type-2 fuzzy sets made simple, *IEEE Transactions on Fuzzy Systems*, 10/2, 117-127, 2002.

[2] LA Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, *Information Sciences*, 8, 199-249, 1975.

[3] M Mizumoto, K Tanaka, Some properties of fuzzy sets of type-2, *Information and Control*, 31, 312–340, 1976

[4] M Mizumoto, K Tanaka, Fuzzy sets of type-2 under algebraic product and algebraic sum, *Fuzzy Sets Systems*, 5, 277–290, 1981.

[5] JM Mendel, *Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions*, NJ, Prentice-Hall, 2001.

[6] D Dubois, H Prade, *Fuzzy Sets and Systems: Theory and Applications*, New York, Academic, 1980.

[7] NN Karnik, JM Mendel, Q Liang, Type-2 fuzzy logic systems, *IEEE Transactions Fuzzy Systems*, **7**, 643–658, Dec. 1999.

[8] E Hisdal, The IF THEN ELSE statement and interval-values fuzzy sets of higher type, *International Journal of Man-Machine Studies*, 15, 385–455, 1981.

[9] Q Liang, JM Mendel, Interval type-2 fuzzy logic systems: Theory and design, *IEEE Transactions Fuzzy Systems*, 8, 535–550, 2000.

[10] DG Schwartz, The case for an interval-based representation of linguistic truth, *Fuzzy Sets Systems*, 17, 153–165, 1985.

[11] I Turksen, Interval valued fuzzy sets based on normal forms, *Fuzzy Sets Systems*, 20, 191–210, 1986.

[12] MB Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, *Fuzzy Sets Systems*, 21, 1–17, 1987.

[13] RN Lea, V Kreinovich, R Trejo, Optimal interval enclosures for fractionally-linear functions and their application to intelligent control, *Reliable Computing*, 2, 265–285, 1996.

[14] HT Nguyen, V Kreinovich, Q Zuo, Interval-valued degrees of belief: Applications of interval computations to expert systems and intelligent control, *Int. J. Uncertainty, Fuzziness, Knowledge-Based Systems*, 5, 317–358, 1997.

[15] S Mabuchi, An interpretation of membership functions and the properties of general probabilistic operators as fuzzy set operators, II: Extension to three-valued and interval-valued fuzzy sets, *Fuzzy Sets Systems*, 92, 31–50, 1997.

[16] V Kreinovich, A Lakeyev, J Rohn, P Kahl, *Computational Complexity and Feasibility of Data Processing and Interval Computations*, MA, Boston: Kluwer, 1998, ch. 10.

[17] H Bustince, P Burillo, Mathematical analysis of interval-valued fuzzy relations: Application to approximate reasoning, *Fuzzy Sets Systems*, 113, 205–219, 2000.

[18] S Lee, KH Lee, A ranking method for type-2 fuzzy values, *Journal of Korea Fuzzy and Intelligent Systems Society*, 12/4, 341-346, 2002.

[19] S Lee, KH Lee, Shortest path problem in a type-2 weighted graph, *Journal of Korea Fuzzy and Intelligent Systems Society*, 11/6, 528-531, 2001.

[20] NN Karnik, JM Mendel, MPEG VBR video traffic modelling and classification using fuzzy techniques, *IEEE Transactions on Fuzzy Systems*, 9, 183–193, 2001.

[21] RR Yager, Fuzzy subsets of type II in decisions, *Journal of Cybernetics*, 10, 137–159, 1980.

[22] DA Chiang, LR Chow, NC Hsien, Fuzzy information in extended fuzzy relational databases, *Fuzzy Sets Systems*, 92, 1–20, 1997.

[23] M Wagenknecht, K Hartmann, Application of fuzzy sets of type-2 to the solution of fuzzy equation systems, *Fuzzy Sets Systems*, 25, 183–190, 1988.

[24] NN Karnik, JM Mendel, Applications of type-2 fuzzy logic systems to forecasting of time-series, *Information Sciences*, 120, 89-111, 1999.

[25] T Ozen, JM Garibaldi, Investigating adaptation in type-2 fuzzy logic systems applied to umbilical acid-base assessment, *European Symposium on Intelligent Technologies, Hybrid Systems and their implementation on Smart Adaptive Systems*, Oulu, Finland, 2003.