Towards a Formal Semantics for FHM: Part 2

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What are we trying to achieve?

Evaluation of models in FHM.

1. Normalisation of functional level terms
2. Reducing signal level terms
3. Handling of simulation runtime events.
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   \[ \beta\text{-reduction of signal level products.} \]
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Discrete Semantics

We are only interested in the discrete aspects of FHM.

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How do we achieve this?

We use **Normalisation by Evaluation** (NbE), but why?

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2. Symbolic method, enabling partial evaluation
3. We get use Agda as the meta-language!
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Normalisation by Evaluation

What is NbE?

1. Closely related to type-directed partial evaluation
2. Proceeds by interpreting terms into an appropriate model
3. Objects of the model are then reified back into the normal forms that represent them.
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How do we know our normaliser is correct?

The correctness of normalisation can be specified in terms of the equational theory ($\sim_{\beta\eta}$) of the language.

**Soundness**

$$t \sim_{\beta\eta} t' \implies \text{norm } t = \text{norm } t'$$

**Completeness**

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### Completeness

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We consider an equation based language embedded into the simply typed \( \lambda \)-calculus.

### Syntax

\[
\begin{align*}
    t &::= x \\
    &\mid t_1 \ t_2 \\
    &\mid \lambda x.\ t \\
    &\mid \text{sigrel } z \ \text{where } q \\
    &\mid \ldots \text{etc}
\end{align*}
\]

\[
\begin{align*}
    q &::= t \odot s \\
    &\mid s_1 = s_2 \\
    &\mid \text{init } s_1 = s_2 \\
    &\mid \ldots \text{etc}
\end{align*}
\]
The language

With a signal level language as follows:

Syntax

\[
\begin{align*}
  s &::= z \\
  &| t \\
  &| s_1 + s_2 \\
  &| s_1 \ast s_2 \\
  &| \text{zero} \\
  &| \text{suc } s \\
  &| \text{fst } s \\
  &| \text{snd } s \\
  &| \text{pair } s_1 s_2 \\
  &| \ldots \text{etc}
\end{align*}
\]
Types

With a simple language of types:

Syntax

\[
\begin{align*}
\tau ::= & \tau_1 \rightarrow \tau_2 \\
| & \text{SR } \sigma \\
| & \text{Nat} \\
\sigma ::= & \top \\
| & \sigma_1 \times \sigma_2 \\
| & \text{Nat}
\end{align*}
\]
Equational Theory

We need to extend the equational theory ($\sim_{\beta\eta}$):

$\beta$-convertibility at $s : \sigma$.

$$(\text{sigrel } z \text{ where } q) \diamond s \sim_{\beta\eta} q[s/z]$$

$\eta$-convertibility at $t : \text{SR } \sigma$.

$$t \sim_{\beta\eta} \text{sigrel } z \text{ where } (t \diamond z)$$

+ new congruence rules, and $\beta\eta$ for signal level products.
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Term Representation

Some signatures:

**Term representation.**

\[ STerm (\Gamma \Delta : \text{Ctx}) : (\sigma : SType) \rightarrow \text{Set} \]
\[ Term (\Gamma : \text{Ctx}) : (\tau : \text{Type}) \rightarrow \text{Set} \]
\[ EqTerm (\Gamma \Delta : \text{Ctx}) : \text{Set} \]
The Model

A type directed interpretation into the model (roughly!). Nrm is the redex-free representation of terms. First, the signal types.

Model interpretation

\[ SVal : (\Gamma \Delta : Ctx) \rightarrow SType \rightarrow Set \]
\[ SVal \Gamma \Delta \text{ Unit} = SNrm \Gamma \Delta \text{ Unit} \]
\[ SVal \Gamma \Delta \text{ Nat} = SNrm \Gamma \Delta \text{ Nat} \]
\[ SVal \Gamma \Delta (\sigma_1 \times \sigma_2) = SVal \Gamma \Delta \sigma_1 \times SVal \Gamma \Delta \sigma_2 \]
The Model

And the functional types.

Model interpretation

\[
\begin{align*}
Val : (\Gamma : Ctx) \rightarrow Type \rightarrow Set \\
Val \Gamma \text{ Nat} &= Nrm \Gamma \Delta \text{ Nat} \\
Val \Gamma (\tau_1 \rightarrow \tau_2) &= Val \Gamma \tau_1 \rightarrow Val \Gamma \tau_2 \\
Val \Gamma (SR \sigma) &= SVal \Gamma \sigma \rightarrow EqNrm \Gamma \Delta
\end{align*}
\]
It is now possible to give our interpreter.

**Interpreter type signatures**

\[
\llbracket \cdot \rrbracket_s : \mathit{STerm} \Gamma \Delta \sigma \rightarrow \mathit{Env} \; \Gamma \rightarrow \mathit{Env} \; \Delta \rightarrow \mathit{SVal} \; \Delta \; \sigma
\]

\[
\llbracket \cdot \rrbracket : \mathit{Term} \; \Gamma \; \tau \rightarrow \mathit{Env} \; \Gamma \rightarrow \mathit{Val} \; \Gamma \; \tau
\]

\(\mathit{Env}\) is just an environment of values for each variable in the indexing context.
The final step is to take our representative in the model, and convert it back into a normal form.

Reification

\[
\text{reify} : \text{Val} \ \Gamma \ \tau \rightarrow \text{Nrm} \ \Gamma \ \tau \\
\text{reify}_S : \text{SVal} \ \Delta \ \sigma \rightarrow \text{SNrm} \ \Delta \ \sigma
\]

Composition of interpretation and reification is normalisation!
The handling of events is quite rudimentary at the moment. But maybe there is a better way? Coinduction maybe?
Questions?

For further details, see our draft at cs.nott.ac.uk/~jjc