The Quantum IO Monad

QIO

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Quantum Computation

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- Qubits have 2 base states (|0⟩ and |1⟩), but can exist in a *superposition* of both states simultaneously, until measured.
- Multiple qubits can become entangled, meaning that an n-qubit system has $2^n$ base states, and can be in a superposition of all these $2^n$ states.
Quantum Computation

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• An arbitrary state ($|\phi\rangle$) can be described by

$$|\phi\rangle = \alpha |00\rangle + \beta |01\rangle + \gamma |10\rangle + \delta |11\rangle$$

where $\alpha, \beta, \gamma, \delta \in \mathbb{C}$ are the complex amplitudes of each base state.
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- Thus the condition that $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$ always holds.
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- Arbitrary qubit states cannot be copied, however they may teleported to another qubit using entangled pairs known as Bell states.
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- Haskell is a Pure functional programming language, and thus introduces Monads to encapsulate effects.
- For example, the IO Monad.
class Monad m where

(\gg\gg) \in m\ a \rightarrow (a \rightarrow m\ b) \rightarrow m\ b

return \in a \rightarrow m\ a

such that the following equations hold

\text{return a} \gg\gg f = f\ a

\text{c} \gg\gg \text{return} = c

(\text{c} \gg\gg f) \gg\gg g = c \gg\gg \lambda a \rightarrow f\ a \gg\gg g
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The IO Monad

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- For example, echoing a Character to the Screen

\[
\begin{align*}
getChar & \in IO \ Char \\
putChar & \in Char \rightarrow IO () \\
echo & \in IO () \\
echo & = getChar \gg (\lambda c \rightarrow putChar c) > > echo
\end{align*}
\]
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\begin{align*}
\text{getChar} & \in \text{IO Char} \\
\text{putChar} & \in \text{Char} \rightarrow \text{IO ()} \\
\text{echo} & \in \text{IO ()} \\
\text{echo} & = \text{getChar} \Rightarrow (\lambda c \rightarrow \text{putChar } c) > > \text{echo}
\end{align*}
\]

- Haskell provides syntactic sugar for Monadic Programming. (in the form of do notation)

\[
\begin{align*}
\text{echo} & = \text{do } c \leftarrow \text{getChar} \\
& \hspace{1em} \text{putChar } c \\
& \hspace{1em} \text{echo}
\end{align*}
\]
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So the QIO Monad can be used to encapsulate the behaviour that would be given by a quantum register.
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  - The first uses a random number generator to measure the qubits, so the outcome is equivalent to running the quantum computation.
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- A quantum computation can be constructed in the QIO Monad (using do notation), and then evaluated using either of the available evaluators.
$rbit \in QIO \; \text{Bool}$

$rbit = \begin{array}{l}
do \quad x \leftarrow \text{mkQbit} \\
\quad \text{apply} \; U \; (\text{rotate} \; x \; rh) \\
\quad b \leftarrow \text{meas} \; x \\
\text{return} \; b
\end{array}$
bell \in QIO (Bool, Bool)
bell = do x \leftarrow mkQbit
        applyU (rotate x rh)
y \leftarrow mkQbit
        applyU (x \mid rotate y rx)
b \leftarrow meas x
c \leftarrow meas y
return (b, c)
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The future of the QIO monad

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- It should also be possible to use larger quantum data structures than individual qubits, creating them in the same way that classical data structures are defined from classical bits.
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It should also be possible to use larger quantum data structures than individual qubits, creating them in the same way that classical data structures are defined from classical bits.

It should also be possible to construct QIO programs from QML programs.
The End

Thank you all for listening!