

Coursework Problems, Set 1

20 February 2019

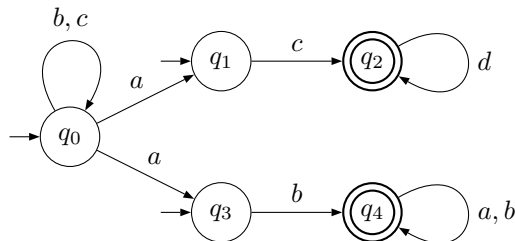
Deadline: 27 February 2019, 3 PM

1. Let the alphabet $\Sigma = \{a, b\}$ and consider the following DFA A :

$$\begin{aligned}
 A &= (Q = \{0, 1, 2, 3, 4\}, \Sigma, \delta, q_0 = 0, F = \{4\}) \\
 \delta &= \{((0, a), 0), ((0, b), 1), ((1, a), 2), ((1, b), 1), ((2, a), 3), ((2, b), 1), \\
 &\quad ((3, a), 0), ((3, b), 4), ((4, a), 4), ((4, b), 4)\}
 \end{aligned}$$

Do the following for DFA A :

- (a) Draw its transition diagram.
 - (b) Determine which of the following words belong to $L(A)$:
 - i. ϵ
 - ii. $ababa$
 - iii. $ababba$
 - iv. $aabbaabba$
 - (c) Explicitly calculate $\hat{\delta}(0, aab)$ and $\hat{\delta}(0, baabb)$.
 - (d) Describe the language this automaton recognises in your own words.
2. We wish to construct a digital door lock that unlocks once all the digits of the code, but no others, have been entered. Thus the order in which the digits of the code are entered, and whether a digit of the code is entered more than once, does not matter. Construct a nondeterministic finite automaton (NFA) over the alphabet $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ that accepts all the words corresponding to the code 135. For example, the words 531 and 511333 should both be accepted, while 3, 15, 1354 should be rejected. Explain your construction; in particular explain the meaning of each state. Give your final answer in the form of a transition diagram for your NFA. Do not forget to indicate the start and accepting states.
3. Consider the following NFA B over $\Sigma = \{a, b, c, d\}$:



- (a) Construct a DFA $D(B)$ equivalent to B using the “subset construction”. Clearly show each step of your calculations in a transition table, and only consider states reachable from the initial DFA state.
- (b) Draw the transition diagram for the resulting DFA $D(B)$.

4. Using the formal definition of the meaning of regular expressions, compute the sets denoted by the following regular expressions, simplifying as far as possible. Provide a step-by-step account of your derivations.
- (a) $(\mathbf{ab} + \mathbf{c} + \epsilon)\mathbf{dd}$
 - (b) $(\mathbf{ab})^*$
 - (c) $(\mathbf{a}^*\emptyset + \epsilon\mathbf{b})\mathbf{c}$
5. Give regular expressions defining the following languages over the alphabet $\Sigma = \{a, b, c\}$. Keep your answers simple; unnecessarily complicated answers will not get full marks.
- (a) All words that contain at least one b .
 - (b) All words such that all a 's appear before all c 's.
 - (c) All words that contain at least one b and at least one c .
 - (d) All words that do not contain the sequence aa .
 - (e) All words that do not contain the sequence bbb .