School of Computer Science, University of Nottingham

COMP2012/G52LAC Languages and Computation, Spring 2019

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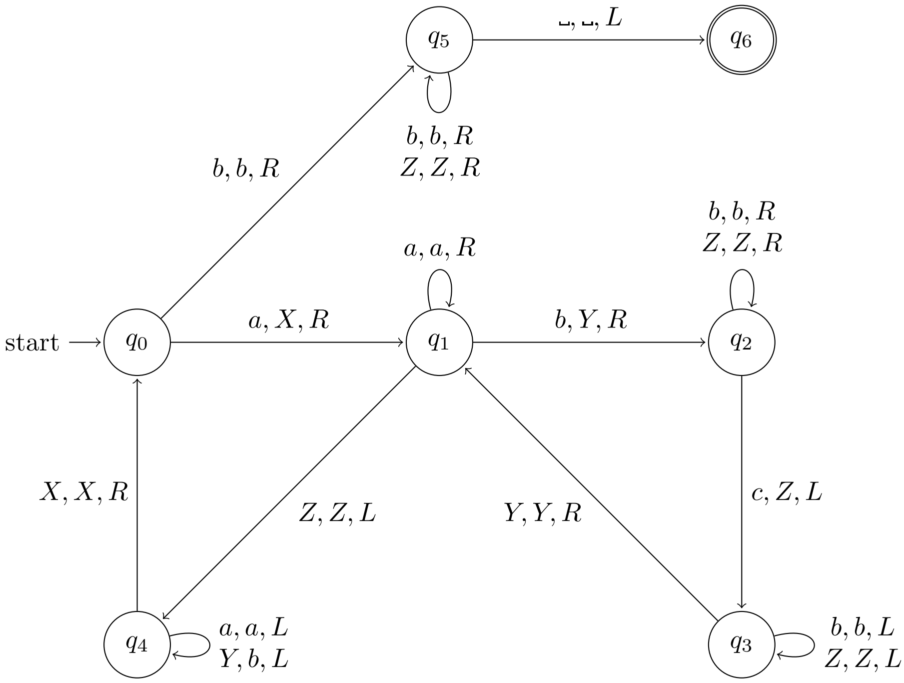
Coursework Problems, Set 3

4 April 2019

Deadline: 12 April 2019, 3 PM

Each question carries equal weight (25 marks).

1. Look at the following drawing of a Turing Machine:



The formal definition of this Turing Machine is $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ with set of states $Q = \{q_0, \ldots, q_6\}$, alphabet $\Sigma = \{a, b, c\}$, tape symbols $\Gamma = \{a, b, c, X, Y, Z, ␣\}$, initial state q_0, blank symbol ␣, and set of final states $F = \{q_6\}$.

Write down the definition of the transition function δ.

2. Using the Turing Machine from the previous point, show all the steps (instantaneous descriptions) in the computation starting with the input word $aabcc$. Is the word accepted or rejected?

Show just the final instantaneous description for the computations starting with each of the following words and state if they are accepted or rejected:

$$aabbcc, \quad aaabbcccccc, \quad abc, \quad ababc.$$

3. What is the language accepted by the machine M?

4. The *Fibonacci numbers* are the sequence of natural numbers a_n recursively defined by the following rules:

$$a_0 = 0, \quad a_1 = 1, \quad a_n = a_{n-2} + a_{n-1} \text{ if } n \geq 2.$$

You will implement them in λ-calculus as a function `fib` such that:

$$\texttt{fib}\,0 = 0$$
$$\texttt{fib}\,1 = 1$$
$$\texttt{fib}\,n = \texttt{fib}\,(n-2) + \texttt{fib}\,(n-1) \text{ if } n \geq 2$$

In order to achieve this, it will be necessary to define two auxiliary functions $\texttt{fib}_{\texttt{step}}$ and $\texttt{fib}_{\texttt{aux}}$ that operate on pairs numbers:

$$\texttt{fib}_{\texttt{step}}\,\langle a, b\rangle = \langle b, a+b\rangle$$

$$\texttt{fib}_{\texttt{aux}}\,0 = \langle 0, 1\rangle$$
$$\texttt{fib}_{\texttt{aux}}\,n = \texttt{fib}_{\texttt{step}}\,(\texttt{fib}_{\texttt{aux}}\,(n-1)) \quad \text{if } n \geq 1$$

Write λ-terms that implement the functions $\texttt{fib}_{\texttt{step}}$, $\texttt{fib}_{\texttt{aux}}$, and $\texttt{fib}$. [You are free to use all the functions defined in the lecture notes, for example, the implementation of pairs and projections and the arithmetic functions.]