School of Computer Science, University of Nottingham

COMP2012/G52LAC Languages and Computation, Spring 2019

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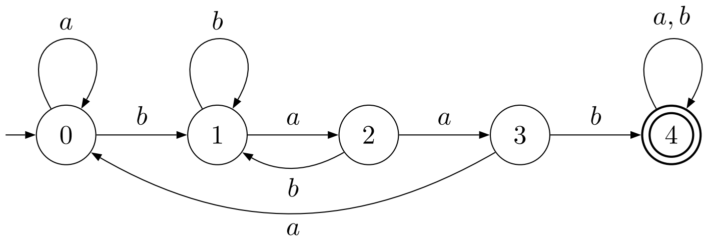
# Solutions to Coursework Problems, Set 1

27 February 2019

1. (a) DFA A




(b)

| $w$ | $w \in L(A)$ |
|---|---|
| $\epsilon$ | no |
| $ababa$ | no |
| $ababba$ | no |
| $aabbaabba$ | yes |

(c)

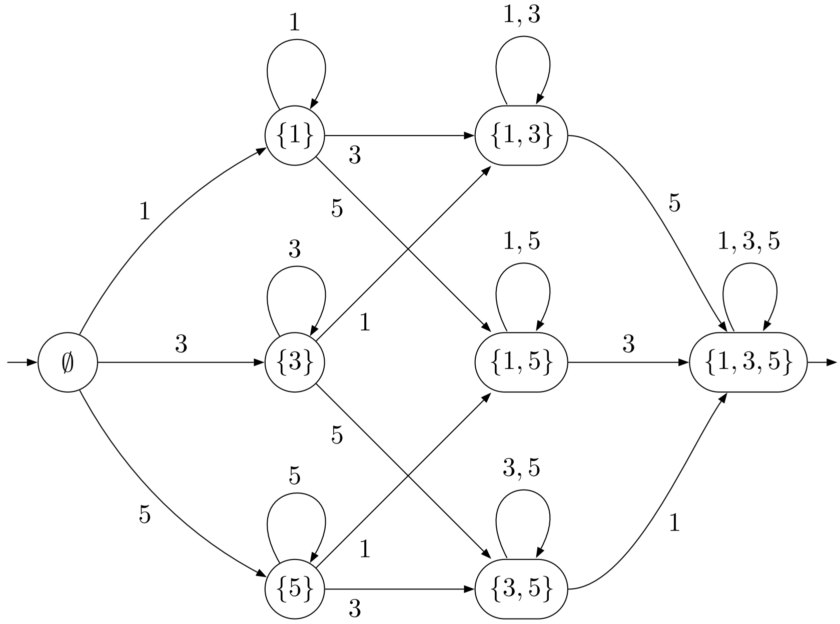
$$\begin{array}{rcll}
\hat{\delta}(0, aab) & = & \hat{\delta}(\delta(0, a), ab) & \text{def. } \hat{\delta} \\
& = & \hat{\delta}(0, ab) & \text{because } \delta(0, a) = 0 \\
& = & \hat{\delta}(\delta(0, a), b) & \text{def. } \hat{\delta} \\
& = & \hat{\delta}(0, b) & \text{because } \delta(0, a) = 0 \\
& = & \hat{\delta}(\delta(0, b), \epsilon) & \text{def. } \hat{\delta} \\
& = & \hat{\delta}(1, \epsilon) & \text{because } \delta(0, b) = 1 \\
& = & 1 & \text{def. } \hat{\delta}
\end{array}$$

$$\begin{array}{rcll}
\hat{\delta}(0, baabb) & = & \hat{\delta}(\delta(0, b), aabb) & \text{def. } \hat{\delta} \\
& = & \hat{\delta}(1, aabb) & \text{because } \delta(0, b) = 1 \\
& = & \hat{\delta}(\delta(1, a), abb) & \text{def. } \hat{\delta} \\
& = & \hat{\delta}(2, abb) & \text{because } \delta(1, a) = 2 \\
& = & \hat{\delta}(\delta(2, a), bb) & \text{def. } \hat{\delta} \\
& = & \hat{\delta}(3, bb) & \text{because } \delta(2, a) = 3 \\
& = & \hat{\delta}(\delta(3, b), b) & \text{def. } \hat{\delta} \\
& = & \hat{\delta}(4, b) & \text{because } \delta(3, b) = 4 \\
& = & \hat{\delta}(\delta(4, b), \epsilon) & \text{def. } \hat{\delta} \\
& = & \hat{\delta}(4, \epsilon) & \text{because } \delta(4, b) = 4 \\
& = & 4 & \text{def. } \hat{\delta}
\end{array}$$

(d) $L(A)$ contains all words over $\{a, b\}$ in which the substring $baab$ occurs at least once.

[Marking: (a): 6; (b): 4 (one mark each); (c): 8 (3 marks and 5 marks); (d): 2. 20 marks in total.]

2. We need to keep track of which digits from the set $\{1, 3, 5\}$ that have been entered so far. Thus we need one state for each possible subset of this set; i.e., $2^3 = 8$ states. We name each state by the corresponding subset. The (one) start state is thus $\emptyset$ and the (one) accepting state is $\{1, 3, 5\}$. The automaton transitions from one state to another on any additional digit from the code, while it loops back to the current state on any digit that already has been entered.




Note that the outgoing, unlabelled arrow from the state $\{1, 3, 5\}$ is an alternative convention for denoting accepting states.

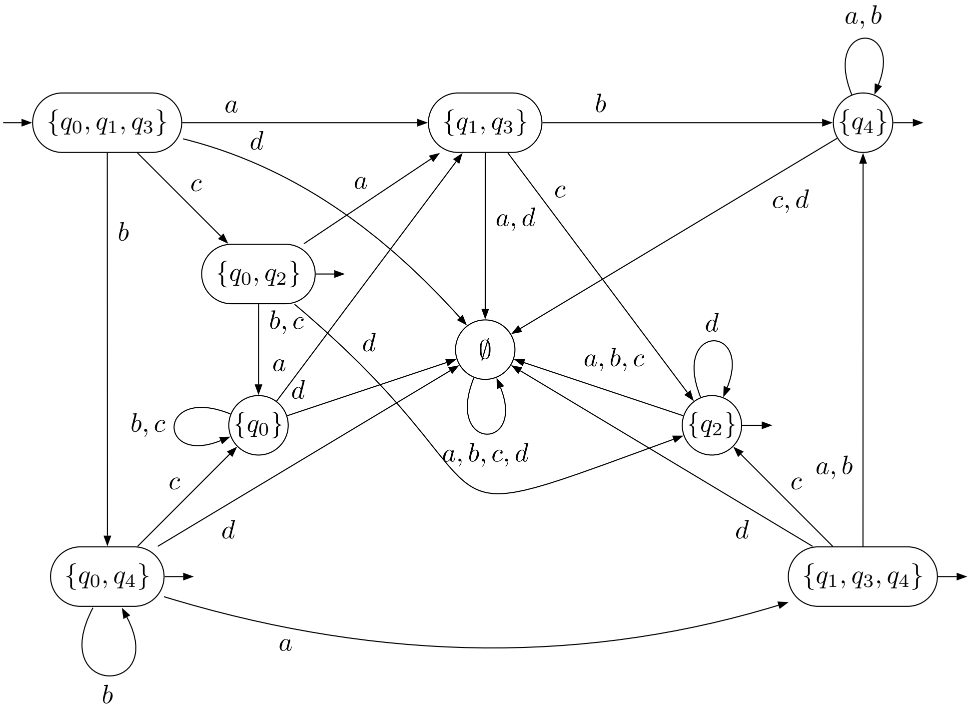
[Marking: 10 marks for a good explanation of the design; 20 marks for a complete and correct transition diagram. 30 marks in total.]

3. Starting from $S = \{q_0, q_1, q_3\}$, the *set* of start states of NFA B and thus *the* (one) start state of $D(B)$, we compute $\hat{\delta}(S, x)$ for each $x \in \Sigma$. Whenever we encounter a state $P \subseteq Q$ of $D(B)$ that has not been considered before, we add $P$ to the table and proceed to tabulate $\hat{\delta}_A(P, x)$ for each $x \in \Sigma$. We repeat the process until no new states are encountered. Finally, we identify the initial state ($\rightarrow$ to the left of the state) and all accepting states ($*$ to the left of the state). Note that a DFA state is accepting if it contains at least one accepting NFA state (as this means it is *possible* to reach at least one accepting state on a given word, which means that word is considered to be in the language of the NFA).

| | $\delta_{D(B)}$ | $a$ | $b$ | $c$ | $d$ |
|---|---|---|---|---|---|
| $\rightarrow$ | $\{q_0, q_1, q_3\}$ | $\{q_1, q_3\} \cup \emptyset \cup \emptyset$ $= \{q_1, q_3\}$ | $\{q_0\} \cup \emptyset \cup \{q_4\}$ $= \{q_0, q_4\}$ | $\{q_0\} \cup \{q_2\} \cup \emptyset$ $= \{q_0, q_2\}$ | $\emptyset \cup \emptyset \cup \emptyset = \emptyset$ |
| | $\{q_1, q_3\}$ | $\emptyset \cup \emptyset = \emptyset$ | $\emptyset \cup \{q_4\} = \{q_4\}$ | $\{q_2\} \cup \emptyset = \{q_2\}$ | $\emptyset \cup \emptyset = \emptyset$ |
| $*$ | $\{q_0, q_4\}$ | $\{q_1, q_3\} \cup \{q_4\}$ $= \{q_1, q_3, q_4\}$ | $\{q_0\} \cup \{q_4\}$ $= \{q_0, q_4\}$ | $\{q_0\} \cup \emptyset = \{q_0\}$ | $\emptyset \cup \emptyset = \emptyset$ |
| $*$ | $\{q_0, q_2\}$ | $\{q_1, q_3\} \cup \emptyset$ $= \{q_1, q_3\}$ | $\{q_0\} \cup \emptyset = \{q_0\}$ | $\{q_0\} \cup \emptyset = \{q_0\}$ | $\emptyset \cup \{q_2\} = \{q_2\}$ |
| | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $*$ | $\{q_4\}$ | $\{q_4\}$ | $\{q_4\}$ | $\emptyset$ | $\emptyset$ |
| $*$ | $\{q_2\}$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | $\{q_2\}$ |
| $*$ | $\{q_1, q_3, q_4\}$ | $\emptyset \cup \emptyset \cup \{q_4\}$ $= \{q_4\}$ | $\emptyset \cup \{q_4\} \cup \{q_4\}$ $= \{q_4\}$ | $\{q_2\} \cup \emptyset \cup \emptyset$ $= \{q_2\}$ | $\emptyset \cup \emptyset \cup \emptyset = \emptyset$ |
| | $\{q_0\}$ | $\{q_1, q_3\}$ | $\{q_0\}$ | $\{q_0\}$ | $\emptyset$ |

(Note that we only needed to consider 9 states. That is a lot fewer than the $2^5 = 32$ possible states in this case. $32 - 9 = 23$ states are thus not reachable from the initial state, and we do not need to worry about those.)

We can now draw the transition diagram for $D(A)$:




Accepting states have been marked by outgoing arrows in this case. That is an alternative to the double circle.

[Marking: (a): 20 for complete and correct table that shows the calculations in some detail; (b): 10 for complete and correct diagram. 30 marks in total.]

4. (a)

$$
\begin{array}{lll}
 & L((\mathbf{ab}+\mathbf{c}+\epsilon)\mathbf{dd}) & \\
= & & \{L(EF) = L(E)L(F)\} \\
 & L(\mathbf{ab}+\mathbf{c}+\epsilon)L(\mathbf{d})L(\mathbf{d}) & \\
= & & \{L(E+F) = L(E) \cup L(F)\} \\
 & (L(\mathbf{ab}) \cup L(\mathbf{c}) \cup L(\epsilon))L(\mathbf{d})L(\mathbf{d}) & \\
= & & \{L(EF) = L(E)L(F)\} \\
 & (L(\mathbf{a})L(\mathbf{b}) \cup L(\mathbf{c}) \cup L(\epsilon))L(\mathbf{d})L(\mathbf{d}) & \\
= & & \{L(\mathbf{x}) = \{x\}\} \\
 & (\{a\}\{b\} \cup \{c\} \cup \{\epsilon\})\{d\}\{d\} & \\
= & & \{\text{Concatenation of Languages}\} \\
 & (\{ab\} \cup \{c\} \cup \{\epsilon\})\{dd\} & \\
= & & \{\text{Set union}\} \\
 & \{ab, c, \epsilon\}\{dd\} & \\
= & & \{\text{Concatenation of Languages}\} \\
 & \{abdd, cdd, dd\} &
\end{array}
$$

(b)

$$
\begin{array}{lll}
 & L((\mathbf{ab})^*) & \\
= & & \{L(E^*) = (L(E))^*\} \\
 & (L(\mathbf{ab}))^* & \\
= & & \{L(EF) = L(E)L(F)\} \\
 & (L(\mathbf{a})L(\mathbf{b}))^* & \\
= & & \{L(\mathbf{x}) = \{x\}\} \\
 & (\{a\}\{b\})^* & \\
= & & \{\text{Concatenation of Languages}\} \\
 & \{ab\}^* & \\
= & & \left\{L^* = \bigcup_{n=0}^{\infty} L^n\right\} \\
 & \bigcup_{n=0}^{\infty} \{ab\}^n &
\end{array}
$$

(c)

$$
\begin{array}{lll}
 & L((\mathbf{a}^*\emptyset + \epsilon\mathbf{b})\mathbf{c}) & \\
= & & \{L(EF) = L(E)L(F)\} \\
 & L(\mathbf{a}^*\emptyset + \epsilon\mathbf{b})L(\mathbf{c}) & \\
= & & \{L(E+F) = L(E) \cup L(F)\} \\
 & (L(\mathbf{a}^*\emptyset) \cup L(\epsilon\mathbf{b}))L(\mathbf{c}) & \\
= & & \{L(EF) = L(E)L(F)\} \\
 & (L(\mathbf{a}^*)L(\emptyset) \cup L(\epsilon)L(\mathbf{b}))L(\mathbf{c}) & \\
= & & \{L(\emptyset) = \emptyset, L(\epsilon) = \{\epsilon\}\} \\
 & (L(\mathbf{a}^*)\emptyset \cup \{\epsilon\}L(\mathbf{b}))L(\mathbf{c}) & \\
= & & \{L(E^*) = (L(E))^*\} \\
 & ((L(\mathbf{a}))^*\emptyset \cup \{\epsilon\}L(\mathbf{b}))L(\mathbf{c}) & \\
= & & \{L(\mathbf{x}) = \{x\}\} \\
 & (\{a\}^*\emptyset \cup \{\epsilon\}\{b\})\{c\} & \\
= & & \{L\emptyset = \emptyset, \{\epsilon\}L = L\} \\
 & (\emptyset \cup \{b\})\{c\} & \\
= & & \{\emptyset \cup L = L\} \\
 & \{b\}\{c\} & \\
= & & \{\text{Concatenation of Languages}\} \\
 & \{bc\} &
\end{array}
$$

[Marking: (a): 3; (b): 3; (c): 4; 10 marks in total.]

5. Note: these are not the only possibilities, nor necessarily the "simplest" in any formal sense. But they are all fairly simple, and your answers should not be much more complicated.

   (a) $(\mathbf{a}+\mathbf{c})^*\mathbf{b}(\mathbf{a}+\mathbf{b}+\mathbf{c})^*$

   (b) $(\mathbf{a}+\mathbf{b})^*(\mathbf{b}+\mathbf{c})^*$

   (c) $\mathbf{a}^*(\mathbf{b}(\mathbf{a}+\mathbf{b})^*\mathbf{c}+\mathbf{c}(\mathbf{a}+\mathbf{c})^*\mathbf{b})(\mathbf{a}+\mathbf{b}+\mathbf{c})^*$

   (d) $((\epsilon+\mathbf{a})(\mathbf{b}+\mathbf{c}))^*(\epsilon+\mathbf{a})$

   (e) $((\epsilon+\mathbf{b}+\mathbf{bb})(\mathbf{a}+\mathbf{c}))^*(\epsilon+\mathbf{b}+\mathbf{bb})$

   [Marking: 2 marks each. 10 marks in total.]