1. Let the alphabet $\Sigma = \{1, 2, 3\}$, and let the language $L = \{w | w \in \Sigma^*, 1 \leq |w| \leq 2\}$. (If $w$ is a word, $|w|$ denotes the length of that word. If $X$ is a finite set, like an alphabet or finite language, $|X|$ denotes the number of elements in that set, its cardinality.) Answer the following questions:

(a) Describe $L$ in plain English.
(b) Enumerate all the words in $L$.
(c) In general, for an arbitrary alphabet $\Sigma$ and $0 \leq m \leq n$, how many words are there in the language $L_1 = \{w | w \in \Sigma^*_1, m \leq |w| \leq n\}$? That is, write down an expression for $|L_1|$.
(d) How many words would there be in $L_1$ if $\Sigma_1 = \Sigma$, $m = 0$, and $n = 4$?

2. Let the alphabet $\Sigma = \{a, b, c\}$ and let $L_1 = \{\epsilon, b, cc\}$ and $L_2 = \{a, b, c\}$ be two languages over $\Sigma$. Enumerate the words in the following languages:

(a) $L_3 = L_1 \cap L_2$
(b) $L_4 = L_2 L_1$
(c) $L_5 = L_4 \emptyset L_4$

3. Let the alphabet $\Sigma = \{a, b\}$ and consider the following DFA $A$:

$A = (Q_A = \{0, 1, 2, 3\}, \Sigma, \delta_A, q_0^A = 0, F_A = \{0, 3\})$

$\delta_A = \{((0, a), 1), ((0, b), 2), ((1, a), 0), ((1, b), 3), ((2, a), 3), ((2, b), 0), ((3, a), 2), ((3, b), 1)\}$

For the DFA $A$:

(a) Draw its transition diagram.

(b) Determine which of the following words belong to $L(A)$:

i. $\epsilon$
ii. $ababa$
iii. $ababba$
iv. $aabbaabba$

(c) Explicitly calculate $\hat{\delta}_A(0, bab)$.

(d) Describe the language that the automaton recognises in your own words.

4. Construct a DFA $B$ over $\Sigma = \{a, b, c\}$ accepting all words where the number of $a$'s is a multiple of 3. E.g. $abaca \in L(B)$ (3 $a$'s), but $caabaa \notin L(B)$ (4 $a$'s, 4 is not a multiple of 3). Explain your construction. In particular, explain why you chose to have the number of states you did, and explain the purpose (or “meaning”) of each state.