Exercises, Set 4
17 February 2015
Deadline: 20 March 2015, 4 PM

1. Consider the following Context-Free Grammar (CFG) Exp:

\[
\begin{align*}
T & \rightarrow T + T | F \\
F & \rightarrow F * F | P \\
P & \rightarrow N(A) | (T) | I \\
N & \rightarrow f | g | h \\
A & \rightarrow T | \epsilon \\
I & \rightarrow DI | D \\
D & \rightarrow 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
\end{align*}
\]

\(T, F, P, N, A, I, D\) are nonterminals; \(+, *, f, g, h, (, ), 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\) are terminals; \(T\) is the start symbol.

(a) Derive the following words in the grammar Exp where possible. If it is possible to derive the word, give the entire \(\text{left-most derivation; i.e. always expand the left-most non-terminal of the sentential form}^1\). If it is not possible to derive the word, give a brief explanation as to why not.

i. \((789)\)

ii. \(7 + g(3 * 5) * (f())\)

iii. \(1 + 2 * 3)\)

iv. \(1 + 7(9)\)

(b) Draw a derivation tree for the word \(7 + (8 * h(1)) + 9\) in the grammar Exp.

(c) Draw another derivation tree for the word \(7 + (8 * h(1)) + 9\) from 1b. What does the fact that there are two different derivation trees for one word tell about the grammar Exp?

2. (a) Construct a simple, unambiguous grammar according to the following:

- The integer literals are the only primitive expressions. An integer literal is either 0, or a non-empty word of decimal digits (0, 1, \ldots, 9) not starting with 0 and with a single optional minus sign (\(-\)) in front. E.g. 0, 1, 42, \(-234\) are all valid integer literals, but \(-0\), \(--1\) are not.

- There are three binary operators:
  - \(=\): non associative, lowest precedence
  - \(\oplus\): left associative, intermediate precedence
  - \(\otimes\): right associative, highest precedence

- Additionally, it should be possible to use parentheses for grouping in the standard way.

(b) Draw the derivation tree for \(42 = 0 \otimes -10 \oplus (1 \oplus 7)\) and verify that its structure reflects the specification above.

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\(^1\)Sentential form: word derivable from the start symbol.
3. Consider the following Pushdown Automaton (PDA) $P_1$:

$$P_1 = (Q = \{q_0, q_1\}, \Sigma = \{a, b, c\}, \Gamma = \{0, 1, \#\}, \delta, q_0, Z_0 = \#, F = \{q_1\})$$

where the transition function $\delta$ is given by

- $\delta(q_0, a, \#) = \{(q_0, 0\#), (q_0, 11\#)\}$
- $\delta(q_0, a, 0) = \{(q_0, 00), (q_0, 110)\}$
- $\delta(q_0, a, 1) = \{(q_0, 01), (q_0, 111)\}$
- $\delta(q_0, b, 0) = \{(q_0, \epsilon)\}$
- $\delta(q_0, c, 1) = \{(q_0, \epsilon)\}$
- $\delta(q_0, \epsilon, \#) = \{(q_1, \#)\}$
- $\delta(q, w, z) = \emptyset$ everywhere else

Acceptance is by final state.

Which of the following words are accepted by the PDA $P_1$?

- (a) $ab$
- (b) $aababcc$
- (c) $ac$

For those words that are accepted, provide a sequence of Instantaneous Descriptions (IDs) leading to an accepting configuration as evidence. For those words that are not accepted, explain why there is no sequence of IDs leading to an accepting configuration.