Recap: Formal Definition of NFA (1)

Formally, a Nondeterministic Finite Automaton or NFA is defined by a 5-tuple

\[(Q, \Sigma, \delta, S, F)\]

where

- \(Q\) : Finite set of States
- \(\Sigma\) : Alphabet (finite set of symbols)
- \(\delta \in Q \times \Sigma \rightarrow \mathcal{P}(Q)\) : Transition Function
- \(S \subseteq Q\) : Initial States
- \(F \subseteq Q\) : Accepting (or Final) States

Recap: Formal Definition of NFA (2)

Note:

- The transition function maps a state and an input symbol to zero or more successor states. Thus an NFA has “choice”; hence “nondeterministic”.

- However, nothing ambiguous about the language defined by an NFA! Not the case that some word \(w \in L(A)\) sometimes, and \(w \notin L(A)\) other times for some NFA \(A\).

- How? By considering all possible states simultaneously.

Recap: Extended Transition Function

For an NFA, the Extended Transition Function is defined on a set of states and a word (string of symbols).

For a NFA \(A = (Q, \Sigma, \delta, S, F)\), the extended transition function is defined by:

\[
\hat{\delta} \in \mathcal{P}(Q) \times \Sigma^* \rightarrow \mathcal{P}(Q)
\]

\[
\hat{\delta}(P, \varepsilon) = P
\]

\[
\hat{\delta}(P, xw) = \hat{\delta}(\bigcup\{\delta(q, x) | q \in P\}, w)
\]

where \(P \in \mathcal{P}(Q)\) (or \(P \subseteq Q\)), \(x \in \Sigma\), \(w \in \Sigma^*\).
**Recap: Language of an NFA**

The *language* $L(A)$ defined by an NFA $A$ is the set or words **accepted** by the NFA. For an NFA $A = (Q, \Sigma, \delta, S, F)$ the language is defined by

$$L(A) = \{ w \in \Sigma^* \mid \hat{\delta}(S, w) \cap F \neq \emptyset \}$$

**The Subset Construction (1)**

Observations:
- An NFA can be in one of a *set* of states.
- When reading an input symbol, the machine enters one of a *new set* of states.
- Which are the *sets* of possible states?
- Each set is a subset of $Q$, so the set of possible states is (at most) $\mathcal{P}(Q)$.
- But $Q$ is finite. Thus $\mathcal{P}(Q)$ is *finite* too!
- There may be *lots* of states as $|\mathcal{P}(Q)| = 2^{|Q|}$. But the number of states is finite!

**The Subset Construction (2)**

• We can thus **convert** an NFA into a DFA by considering each possible set of NFA states as a single DFA state!

**The Subset Construction (3)**

Given an NFA $A$:

$$A = (Q, \Sigma, \delta, S, F)$$

we construct the **equivalent** DFA $D(A)$ as:

$$D(A) = (\mathcal{P}(Q), \Sigma, \delta_{D(A)}, S, F_{D(A)})$$

where

$$\delta_{D(A)}(P, x) = \bigcup \{ \delta(q, x) \mid q \in P \}$$

$$F_{D(A)} = \{ P \in \mathcal{P}(Q) \mid P \cap F \neq \emptyset \}$$

(Cf. def. $\hat{\delta}$ and language for NFA!)