G52MAL
Machines and Their Languages
Lecture 8
Proving Languages Not to Be Regular

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Are there Non-regular Languages?

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Why? Intuitively: Need to count ***arbitrarily far*** to check if any given word is accepted. We cannot count arbitrarily far if we only have a finite memory!

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- In theory, no! Anything we physically build is necessarily finite.
- In practice, of course! It doesn't take that many bits to count as far as we could possibly want.

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About 10^{19} years, or 780 million times the currently estimated age of the universe (13.8 billion years).

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As an aside, the question if we can write a ***program*** to decide L is more subtle:

- A ***programming language specification*** can conceivably be very abstract and not mention any specific limits on sizes.
- A correct program can then be expressed in that it in theory could count arbitrarily far.
- However, when this program is run we would sooner or later hit some limitation either due to the ***implementation*** of the language or due to the hardware we are running it on.

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Key observation: Because a Finite Automaton has limited memory, any sufficiently long word in the language must contain repetitive patterns.