This Lecture

- Types and Type Systems
- Language safety
- Achieving safety through types
  - Introduction: relation between static and dynamic semantics
  - Operational dynamic semantics of a small example language.

Much of this lecture follows parts of the first few chapters of B. C. Pierce 2002 *Types and Programming Languages* closely.
Types and Type Systems (1)

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A plausible definition (Pierce):

*A type system is a tractable syntactic method for proving the absence of certain program behaviors by classifying phrases according to the kinds of values they compute.*
Types and Type Systems (2)

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- **Static checking** implied as the goal is to **prove** absence of certain errors.
- Done by **classifying** syntactic phrases (or **terms**) according to the **kinds** of value they compute: a type system computes a **static approximation** of the run-time behaviour.
Types and Type Systems (3)

Example: if known that two program fragments $exp_1$ and $exp_2$ compute integers (classification), then it is safe to add those numbers together (absence of errors):

$$exp_1 + exp_2$$
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Also known that the result is an integer. While not known exactly which integers are involved, at least known they are integers and nothing else (static approximation).
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Example. In a dynamically checked language, $exp_1 + exp_2$ would be evaluated as follows:

- Evaluate $exp_1$ and $exp_2$
- Add results together in a manner depending on their types (integer addition, floating point addition, \ldots), or signal error if not possible.
A type system is necessarily *conservative*: some well-behaved programs will be rejected.
### Types and Type Systems (5)

- A type system is necessarily **conservative**: some well-behaved programs will be rejected.

For example, typically

```plaintext
if complex test then S else type error
```

will be rejected as ill-typed, even if `complex test` actually always evaluates to true because that cannot be automatically proved in general.
A type system checks for *certain* kinds of bad program behaviour, or *run-time errors*. Exactly which depends on the type system and the language design.
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- A type system checks for certain kinds of bad program behaviour, or run-time errors. Exactly which depends on the type system and the language design.

For example: current main-stream type systems typically

**do check** that arithmetic operations only are done on numbers

**do not check** that the second operand of division is not zero, that array indices are within bounds.
The safety or soundness of a type system must be judged with respect to its own set of run-time errors.
Language safety is a contentious notion. A possible definition (Pierce):

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For example: a Java object should behave as an object; e.g. it would be bad if it was destroyed by creation of some *other* object.
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Other examples: lexical scope rules, visibility attributes *(public, protected, ...).*
Language Safety (2)

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- Scheme is a dynamically checked safe language.
- Even statically typed languages usually use some dynamic checks; e.g.:
  - checking of array bounds
  - down-casting (e.g. Java)
  - checking for division by zero
  - pattern-matching failure
Some examples of statically and dynamically checked safe and unsafe high-level languages:

<table>
<thead>
<tr>
<th></th>
<th>Statically chkd</th>
<th>Dynamically chkd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safe</td>
<td>ML, Haskell, Java</td>
<td>Lisp, Scheme, Perl, Python, Postscript</td>
</tr>
<tr>
<td>Unsafe</td>
<td>C, C++</td>
<td>Certain Basic dialects</td>
</tr>
</tbody>
</table>
Advantages of Typing (1)

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- Enforcing disciplined programming
  Type systems are the backbone of
  - Modules
  - Classes
Advantages of Typing (2)

- Documentation
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- **Efficiency**
  - First use of types in computing was to distinguish between integer and floating point numbers.
  - Elimination of many of the dynamic checks that otherwise would have been needed to guarantee safety.
Disadvantages of Typing

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Increasingly sophisticated type systems, which keep track of more invariants, can help.

But that can make the type systems harder to understand and less automatic.
In summary:

- A type system *statically* proves properties about the *dynamic* behaviour of a program.
- To make precise exactly what these properties are, and formally *prove* that a type system achieves its goals, both the
  - *static semantics*
  - *dynamic semantics*
must first be formalized.
Example Language: Abstract Syntax

Example language. (Will be extended later.)

\[ t \rightarrow \text{terms:} \]

- **true**
- **false**
- **if** \( t \) **then** \( t \) **else** \( t \)
- **0**
- **succ** \( t \)
- **pred** \( t \)
- **iszero** \( t \)

constant true
constant false
conditional
constant zero
successor
predecessor
zero test
The **values** of a language are a subset of the terms that are **possible results of evaluation**.
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The *values* of a language are a subset of the terms that are *possible results of evaluation*. I.e. the values are the *meaning* of terms according to the *dynamic semantics* of the language.

- The evaluation rules are going to be such that no evaluation is possible for values.
- A term to which no evaluation rule applies is a *normal form*.
- All values are normal forms.
Values (2)

\[ v \rightarrow \text{values:} \]
\[ \text{true} \quad \text{true value} \]
\[ \text{false} \quad \text{false value} \]
\[ n_v \quad \text{numeric value} \]

\[ n_v \rightarrow \text{numeric values:} \]
\[ 0 \quad \text{zero value} \]
\[ \text{succ } n_v \quad \text{successor value} \]
One Step Evaluation Relation (1)

$t \rightarrow t'$ is an *evaluation relation* on terms. Read:

$t$ evaluates to $t'$ in one step.
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\text{if true then } t_2 \text{ else } t_3 \rightarrow t_2 \quad \text{(E-IFTRUE)}
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\text{if false then } t_2 \text{ else } t_3 & \rightarrow t_3 \quad \text{(E-IFFALSE)}
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\[
\frac{t_1 \rightarrow t'_1}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3} \quad \text{(E-IF)}
\]
• Recall that a *mathematical relation* can be understood as a (possibly infinite) set of pairs of “related things”. For example:

\[
\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3)\} \subseteq (\leq)
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- The idea of our “one step evaluation relation” is that the “related things” are *terms* and that one term is related to another iff the first evaluates to the second in one step.

- For example:

  \((\text{if true then succ 0 else 0, succ 0}) \in (\rightarrow)\)
Evaluation Relation??? (2)

- But the evaluation relation is infinite . . .
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- But the evaluation relation is infinite . . . so we can’t enumerate all pairs.
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- But the evaluation relation is infinite . . . so we can’t enumerate all pairs.
- Instead, (schematic) *inference rules* are used to specify relations:

\[
\begin{array}{cccccc}
\text{Premise}_1 & \text{Premise}_2 & \ldots & \text{Premise}_n \\
\hline
\text{Conclusion}
\end{array}
\]

(en.wikipedia.org/wiki/Rule_of_inference)
Evaluation Relation??? (2)

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(\text{en.wikipedia.org/wiki/Rule_of_inference})

- If there are no premises, the line is often omitted:

\[
\begin{array}{c}
\text{Conclusion} \quad \text{or} \quad \text{Conclusion}
\end{array}
\]
Evaluation Relation??? (3)

- **Schematic** means that universally quantified variables are allowed in the rules. For example:

  \[
  \text{if true then } t_2 \text{ else } t_3 \rightarrow t_2
  \]
Evaluation Relation???(3)

- **Schematic** means that universally quantified variables are allowed in the rules. For example:

  \[
  \text{if true then } t_2 \text{ else } t_3 \rightarrow t_2
  \]

- Such a **rule schema** actually stands for an **infinite set** of rules:

  \[
  \ldots
  \]

  \[
  \text{if true then 0 else 0} \rightarrow 0 \\
  \text{if true then succ 0 else 0} \rightarrow \text{succ 0} \\
  \text{if true then true else false} \rightarrow \text{true}
  \]

  \[
  \ldots
  \]
The *domain* of a variable is often specified by *naming conventions*. E.g. the name of a variable may indicate some specific *syntactic category* such as $t$, $v$, or $nv$:

\[
\begin{align*}
  t_1 & \rightarrow t'_1 \\
  \text{if } t_1 \text{ then } t_2 \text{ else } t_3 & \rightarrow \text{if } t'_1 \text{ then } t_2 \text{ else } t_3 \\
  \text{pred}(\text{succ } nv_1) & \rightarrow nv_1
\end{align*}
\]
One Step Evaluation Relation (2)

\[
\frac{\text{succ } t_1 \rightarrow t'_1}{\text{succ } t_1 \rightarrow \text{succ } t'_1}
\]

(E-SUCC)
One Step Evaluation Relation (2)

\[
\begin{array}{c}
t_1 \rightarrow t'_1 \\
\text{succ } t_1 \rightarrow \text{succ } t'_1
\end{array}
\]  
\text{(E-SUCC)}

\[
\begin{array}{c}
\text{pred } 0 \rightarrow 0
\end{array}
\]  
\text{(E-PREDZERO)}
One Step Evaluation Relation (2)

\[
\frac{t_1 \rightarrow t'_1}{\text{succ } t_1 \rightarrow \text{succ } t'_1} \quad (\text{E-SUCC})
\]

\[
\text{pred } 0 \rightarrow 0 \quad (\text{E-PREDZERO})
\]

\[
\text{pred } (\text{succ } n v_1) \rightarrow n v_1 \quad (\text{E-PREDSUCC})
\]
One Step Evaluation Relation (2)

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\begin{align*}
  t_1 \rightarrow t'_1 & \quad \text{(E-SUCC)} \\
  \text{succ } t_1 \rightarrow \text{succ } t'_1 & \\
  \text{pred } 0 \rightarrow 0 & \quad \text{(E-PREDZERO)} \\
  \text{pred } (\text{succ } n\nu_1) \rightarrow n\nu_1 & \quad \text{(E-PREDSUCC)} \\
  t_1 \rightarrow t'_1 & \quad \text{(E-PRED)} \\
  \text{pred } t_1 \rightarrow \text{pred } t'_1 &
\end{align*}
\]
One Step Evaluation Relation (3)

\[ \text{iszero } 0 \rightarrow \text{true} \quad \text{(E-ISZEROZERO)} \]
One Step Evaluation Relation (3)

\[
\text{iszero } 0 \rightarrow \text{true} \quad (E-ISZEROZERO)
\]

\[
\text{iszero (succ } n v_1) \rightarrow \text{false} \quad (E-ISZEROSUCC)
\]
One Step Evaluation Relation (3)

\[
iszero \ 0 \ \rightarrow \ true \quad \quad (E-ISZEROZERO)
\]

\[
iszero (\text{succ} \ n v_1) \ \rightarrow \ false \quad \quad (E-ISZEROSUCC)
\]

\[
t_1 \ \rightarrow \ t'_1 \\
\hline
iszero \ t_1 \ \rightarrow \ iszero \ t'_1 \
\]

\[
t_1 \ \rightarrow \ t'_1 \\
\hline
iszero \ t_1 \ \rightarrow \ iszero \ t'_1 \quad \quad (E-ISZERO)
\]
Let’s evaluate some terms according to →:

- \( \text{pred} (\text{pred} 0) \)
- \( \text{if} (\text{iszero} (\text{pred} (\text{succ} 0))) \text{ then } (\text{pred} 0) \text{ else } (\text{succ} 0) \)
- \( \text{if } 0 \text{ then } 0 \text{ else } 0 \)

(On the whiteboard.)
Values and Stuck Terms

Note that:

- **Values** cannot be evaluated further. E.g.:
  - true
  - 0
  - `succ (succ 0)`
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- **Values** cannot be evaluated further. E.g.:
  - `true`
  - `0`
  - `succ (succ 0)`

- Certain “obviously nonsensical” states are **stuck**: the term cannot be evaluated further, but it is not a value. For example:

  `if 0 then pred 0 else 0`
Stuckness and Run-Time Errors

- We let the notion of getting stuck *model* run-time *errors*.
Stuckness and Run-Time Errors

- We let the notion of getting stuck model run-time errors.

- The goal of a type system is thus to guarantee that a program never gets stuck!
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- The goal of a type system is thus to guarantee that a program never gets stuck!

These ideas will be made more precise next time.