COMP3012/G53CMP: Lecture 3

Syntactic Analysis: Bottom-Up Parsing

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This Lecture

- Parsing strategies: top-down and bottom-up.
- Shift-Reduce parsing theory.
- LR(0) parsing.
- LR(0), LR(k), and LALR(k) grammars

Parsing Strategies

There are two basic strategies for parsing:

- ***Top-down parsing***:
  - Attempts to construct the parse tree ***from the root downward***.
  - Traces out a ***leftmost derivation***.
  - E.g. Recursive-Descent Parsing (see G52LAC).

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- ***Bottom-up parsing***:
  - Attempts to construct the parse tree ***from the leaves working up toward the root***.
  - Traces out a ***rightmost derivation in reverse***.

Top-Down: Leftmost Derivation

Consider the grammar:

$$S \to aABe \qquad A \to bcA \mid c \qquad B \to d$$

Call sequence for predictive parser on $abccde$:

`parseS`	S	$\underset{lm}{\Rightarrow}$	$aABe$
read `a`			
`parseA`		$\underset{lm}{\Rightarrow}$	$abcABe$
read `b`			
read `c`			
`parseA`		$\underset{lm}{\Rightarrow}$	$abccBe$
read `c`			
`parseB`		$\underset{lm}{\Rightarrow}$	$abccde$
read `d`			
read `e`			

Shift-Reduce Parsing

Shift-reduce parsing is a general style of bottom-up syntax analysis:

- Works from the leaves toward the root of the parse tree.
- Has two basic actions:
  - ***Shift*** (read) next terminal symbol.
  - ***Reduce*** a sequence of read terminals and previously reduced nonterminals corresponding to the RHS of a production to LHS nonterminal of that production.

Bottom-Up: Rightmost Der. in Reverse

Consider (again) the grammar:

$$S \rightarrow aABe \qquad A \rightarrow bcA \mid c \qquad B \rightarrow d$$

Reduction steps for the sentence $abccde$ to S

$abccde$	(reduce by $A \rightarrow c$)
$abcAde$	(reduce by $A \rightarrow bcA$)
$aAde$	(reduce by $B \rightarrow d$)
$aABe$	(reduce by $S \rightarrow aABe$)
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Trace out rightmost derivation in reverse:

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Will make this more precise in the following.

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- ***LALR(k)***: ***L***ook***A***head ***LR***, simplified LR parsing

LL, LR, and LALR parsing (2)

By extension, the classes of grammars these methods can handle are also classified as LL(k), LR(k), and LALR(k).

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- Consequently, many parser generator tools based on LALR.
- We will mainly study LR(0) parsing because it is the simplest, yet uses the same fundamental principles as LR(1) and LALR(1).

Shift-Reduce Parsing Theory (1)

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- An ***item*** for a CFG is a production with a dot anywhere in the RHS.

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For example, the items for the grammar

$$S \rightarrow aAc \qquad A \rightarrow Ab \mid \epsilon$$

are

$$\begin{array}{lllllll} S & \rightarrow & \cdot aAc & \qquad & A & \rightarrow & \cdot Ab \\ S & \rightarrow & a \cdot Ac & & A & \rightarrow & A \cdot b \\ S & \rightarrow & aA \cdot c & & A & \rightarrow & Ab \cdot \\ S & \rightarrow & aAc \cdot & & A & \rightarrow & \cdot \end{array}$$

Shift-Reduce Parsing Theory (2)

- Recap: Given a CFG $G = (N, T, P, S)$, a string $\phi \in (N \cup T)^*$ is a ***sentential form*** for G iff $S \overset{*}{\underset{G}{\Rightarrow}} \phi$.

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- A ***right-sentential form*** is a sentential form that can be derived by a rightmost derivation.
- A ***handle*** of a right-sentential form ϕ is a substring α of ϕ such that $S \overset{*}{\underset{rm}{\Rightarrow}} \delta A w \underset{rm}{\Rightarrow} \delta \alpha w$ and $\delta \alpha w = \phi$, where $\alpha, \delta, \phi \in (N \cup T)^*$, and $w \in T^*$.

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For an unambiguous grammar, the rightmost derivation is unique. Thus we can talk about ***"the handle"*** rather than merely "a handle".

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- An item is ***complete*** if the the dot is the rightmost symbol in the item.

Shift-Reduce Parsing Theory (5)

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Shift-Reduce Parsing Theory (6)

Last derivation step $aAde \underset{rm}{\Rightarrow} abcAde$ by production $A \rightarrow bcA$, meaning the handle is bcA.

Valid item for each non-ϵ viable prefix of $abcAde$ considering this particular derivation only?

Viable prefix	Valid item

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Any ***complete*** valid item?

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- If $A \rightarrow \alpha\,\cdot$ is a ***complete*** valid item for a viable prefix $\gamma = \delta\alpha$ of a right-sentential form γw ($w \in T^*$), then it ***appears*** that $A \rightarrow \alpha$ can be used at the last step, and that the previous right-sentential form is $\delta A w$.

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- If this indeed ***always is the case*** for a CFG G, then for any $x \in L(G)$, since x ***is*** a right-sentential from, previous right-sentential forms can be determined until S is reached, giving a right-most derivation of x.

Shift-Reduce Parsing Theory (8)

Of course, if $A \to \alpha \cdot$ is a complete valid item for a viable prefix $\gamma = \delta\alpha$, in general, we only know it ***may be possible*** to use $A \to \alpha$ to derive γw from $\delta A w$. For example:

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- There could be a handle of γw that ***includes symbols of*** w.

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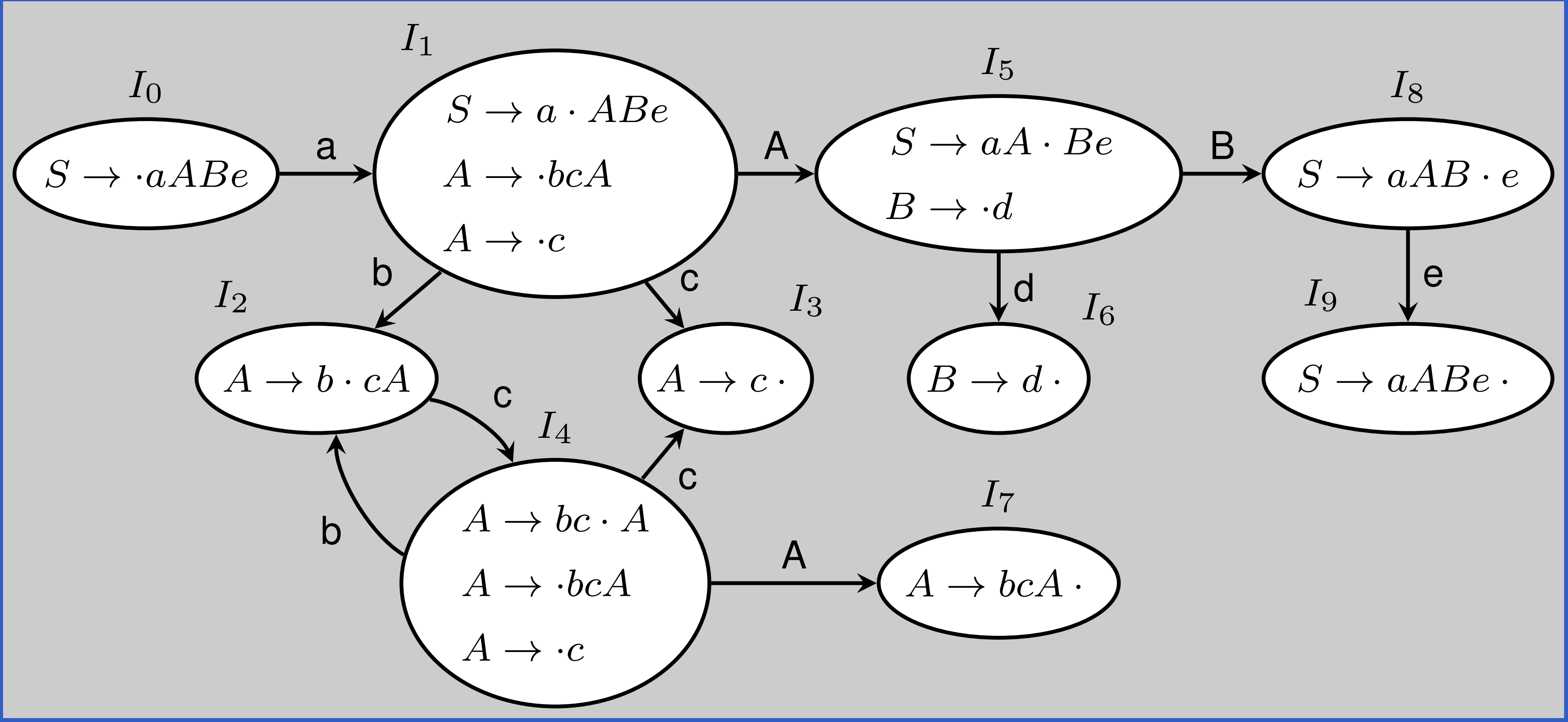
- Thus, an efficient parser can be developed for an LR(0) CFG based on a DFA for recognising viable prefixes and their valid items.
- The states of the DFA are ***sets*** of items valid for a recognised viable prefix.

LR(0) Parsing (2)

A DFA recognising viable prefixes for the CFG

$$S \to aABe \qquad A \to bcA \mid c \qquad B \to d$$



LR(0) Parsing (3)

Drawing conventions for “LR DFAs”:

- For the purpose of recognizing the set of viable prefixes, all drawn states are considered accepting.
- Error transitions and error states are not drawn.

LR(0) Parsing (4)

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- Recall that the viable prefixes for the right-sentential form $abcAde$ are ϵ, a, ab, abc, $abcA$. They are indeed all recognised by the DFA (all states are considered accepting).
- Recall that the item $A \rightarrow bc \cdot A$ is valid for the viable prefix abc. The corresponding DFA state indeed contains that item. (Along with ***more items*** in this case!)

LR(0) Parsing (5)

- Recall that item $A \to bcA\cdot$ is a ***complete*** valid item for the viable prefix $abcA$. The corresponding DFA state indeed contains that item (and ***only*** that item).

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  - Move to new state by following the edge labelled by the read terminal.

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  - The handle is just the ***RHS*** of the valid item.

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- In a state with a ***single complete item***: ***Reduce***
  - The top of the parse stack contains the ***handle*** of the current right-sentential form (since we have recognised a viable prefix for which a single ***complete*** item is valid).
  - The handle is just the ***RHS*** of the valid item.
  - Reduce to the previous right-sentential form by ***replacing the handle*** on the parse stack with the ***LHS*** of the valid item.

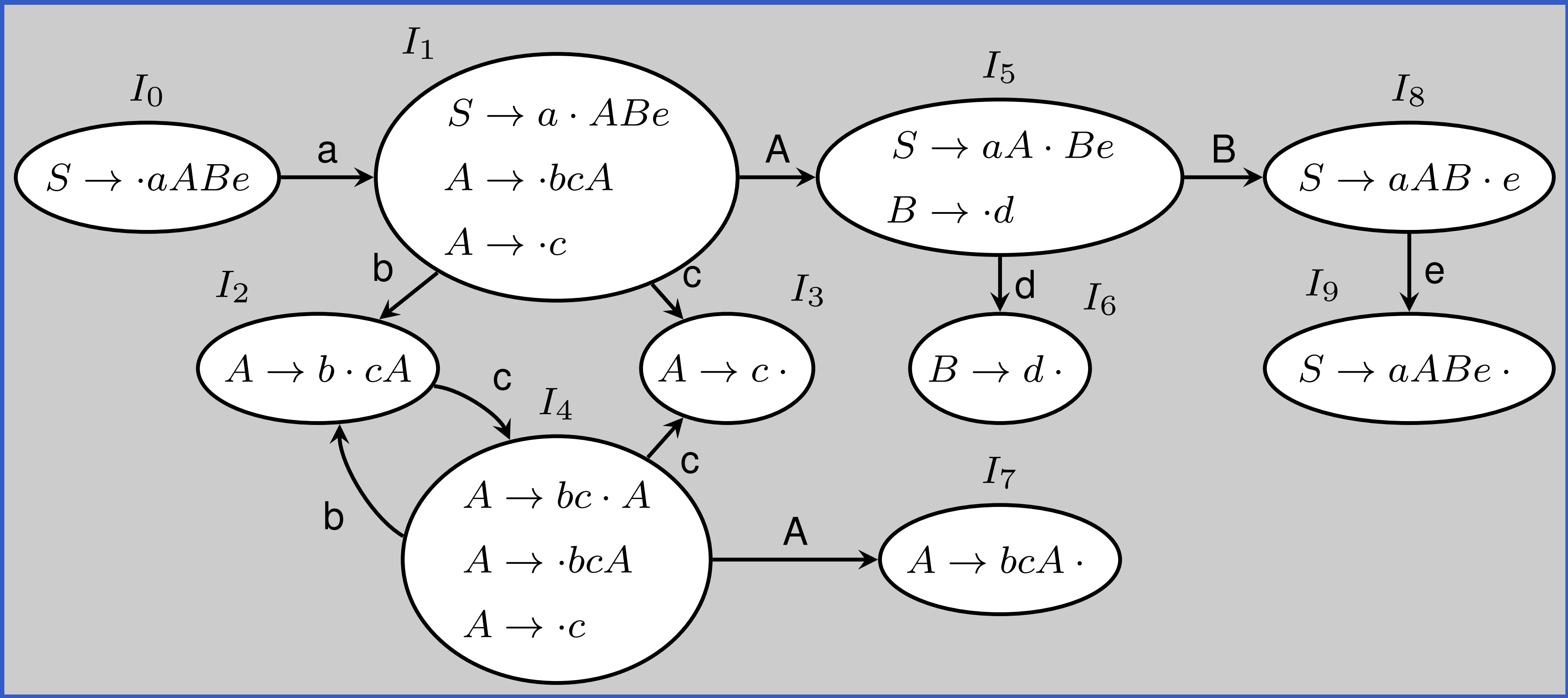
LR(0) Parsing (7)

- In a state with a ***single complete item***: ***Reduce***
  - The top of the parse stack contains the ***handle*** of the current right-sentential form (since we have recognised a viable prefix for which a single ***complete*** item is valid).
  - The handle is just the ***RHS*** of the valid item.
  - Reduce to the previous right-sentential form by ***replacing the handle*** on the parse stack with the ***LHS*** of the valid item.
  - ***Move*** to the state indicated by the new viable prefix on the parse stack.

LR(0) Parsing (8)

- If a state contains both complete and incomplete items, or if a state contains more than one complete item, then the grammar ***is not LR(0)***.

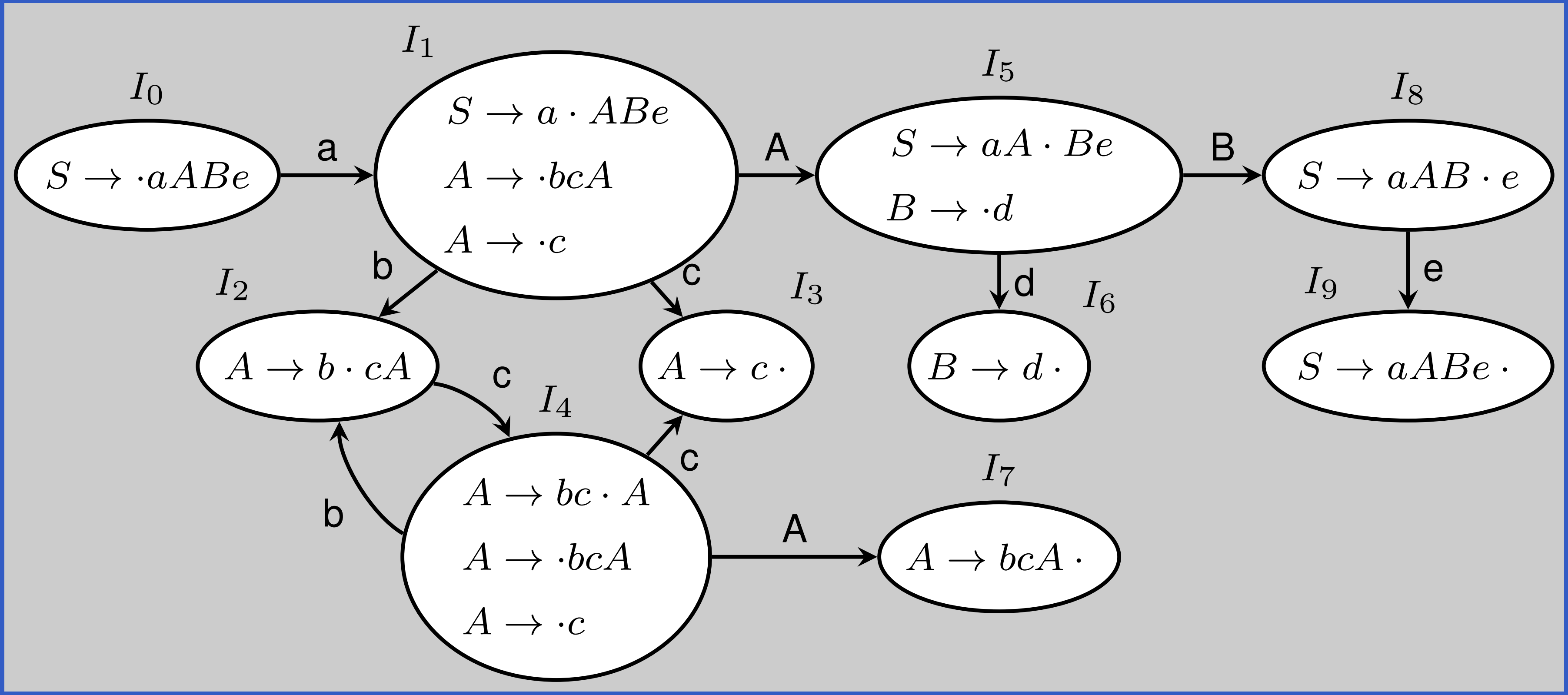
LR(0) Parsing (9)



Note: γw is the current right-sentential form.

State	Stack (γ)	Input (w)	Move
I_0	ϵ	$abccde$	

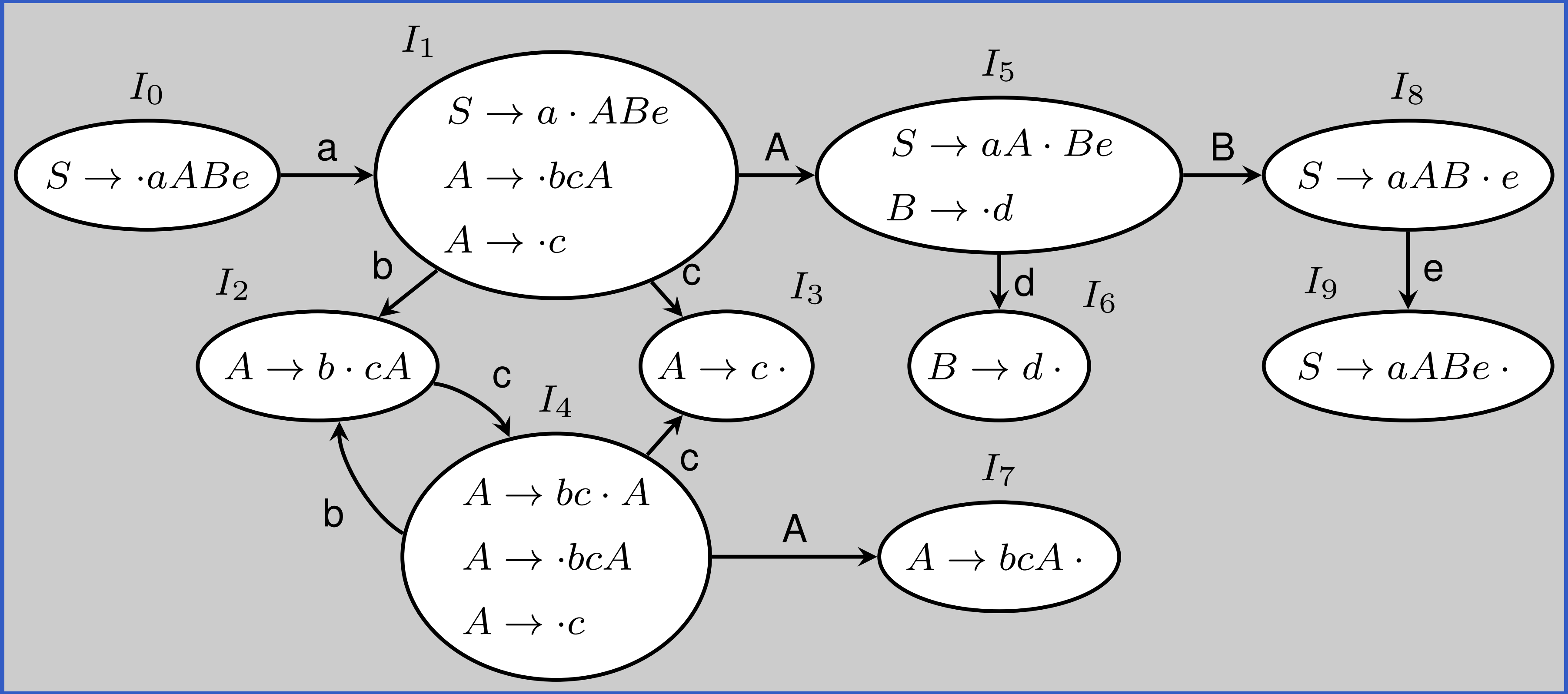
LR(0) Parsing (9)



Note: γw is the current right-sentential form.

State	Stack (γ)	Input (w)	Move
I_0	ϵ	$abccde$	Shift

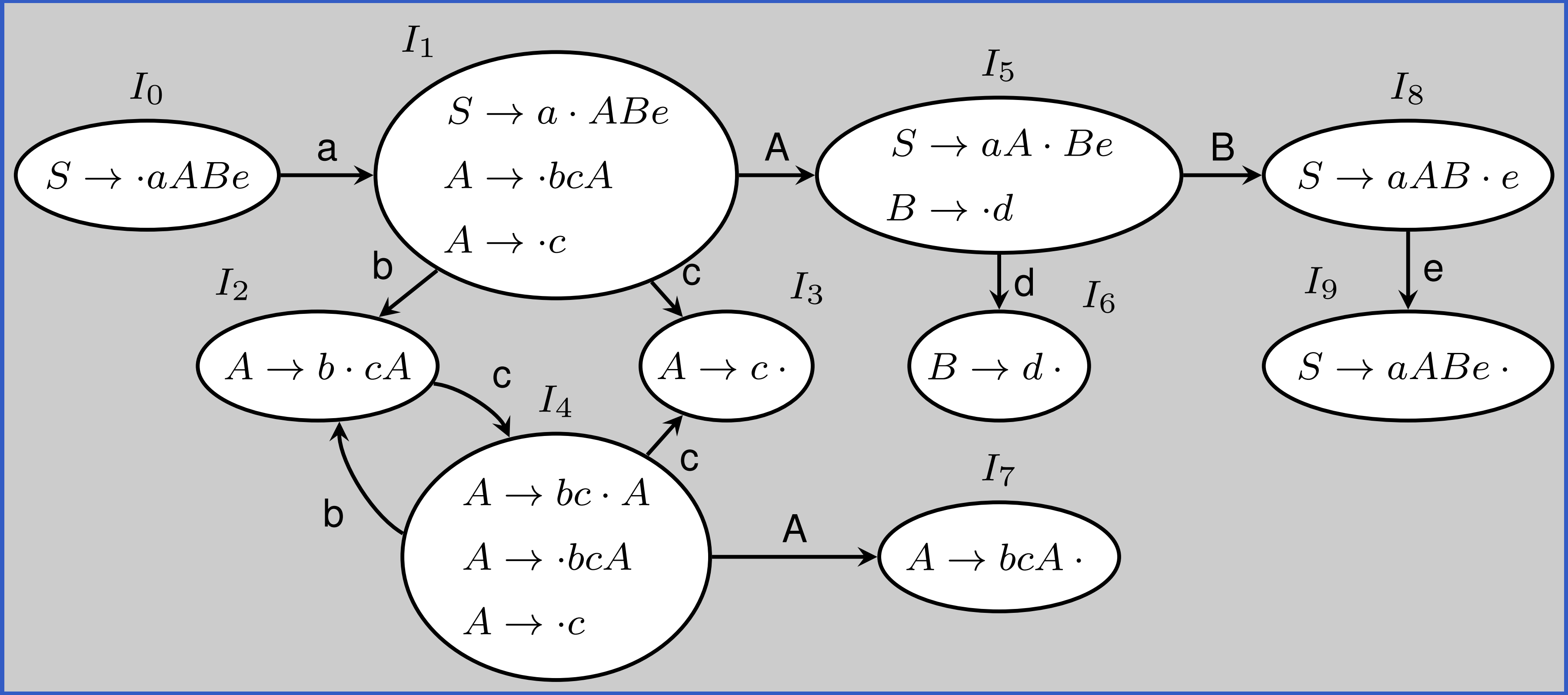
LR(0) Parsing (9)



Note: γw is the current right-sentential form.

State	Stack (γ)	Input (w)	Move
I_0	ϵ	$abccde$	Shift
I_1	a	$bccde$	

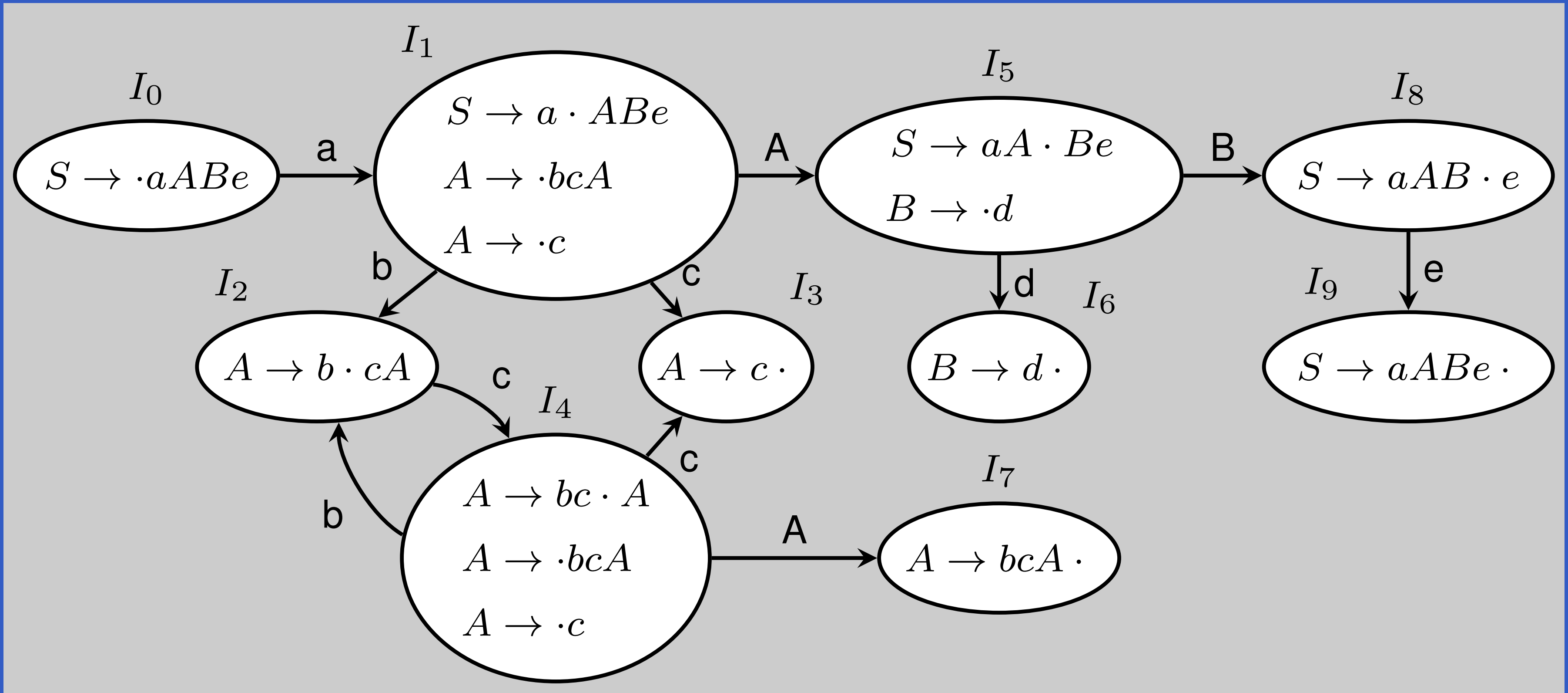
LR(0) Parsing (9)



Note: γw is the current right-sentential form.

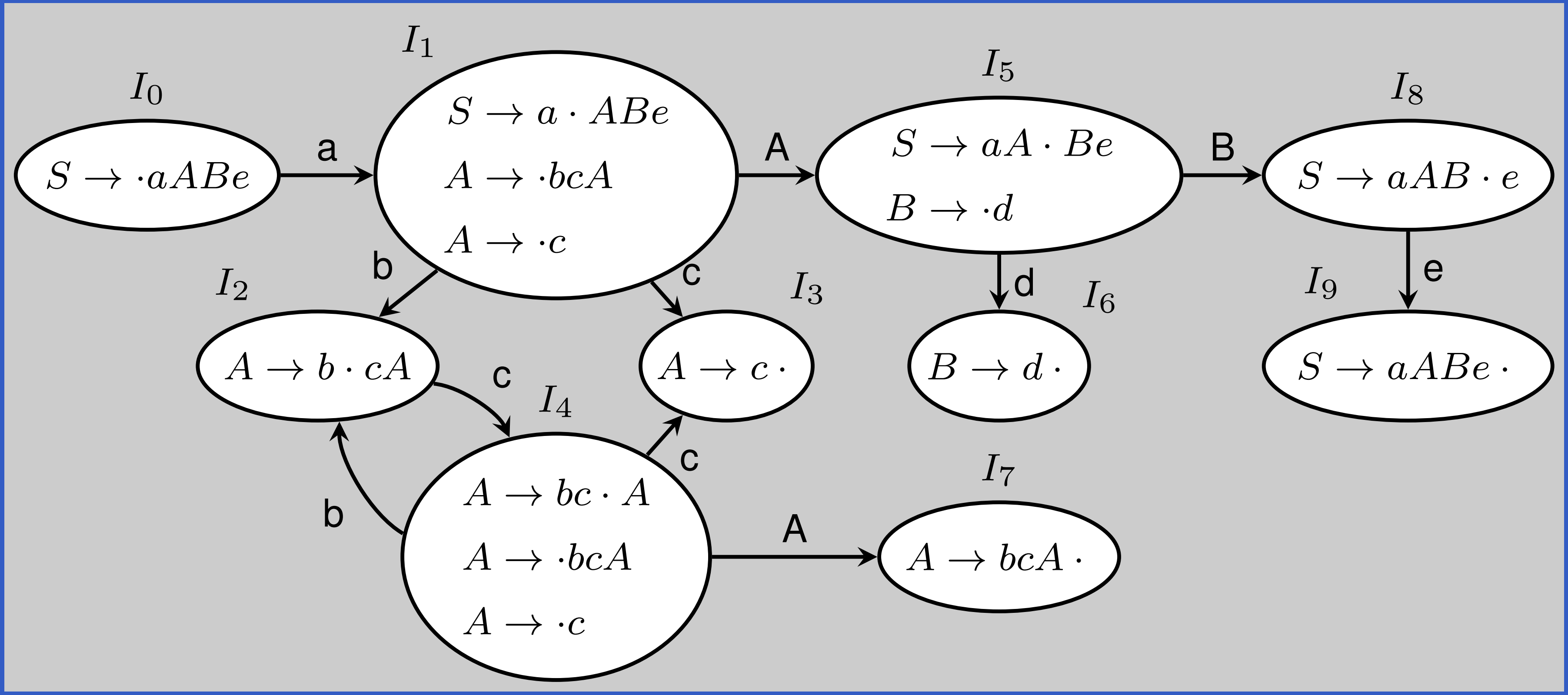
State	Stack (γ)	Input (w)	Move
I_0	ϵ	$abccde$	Shift
I_1	a	$bccde$	Shift

LR(0) Parsing (10)



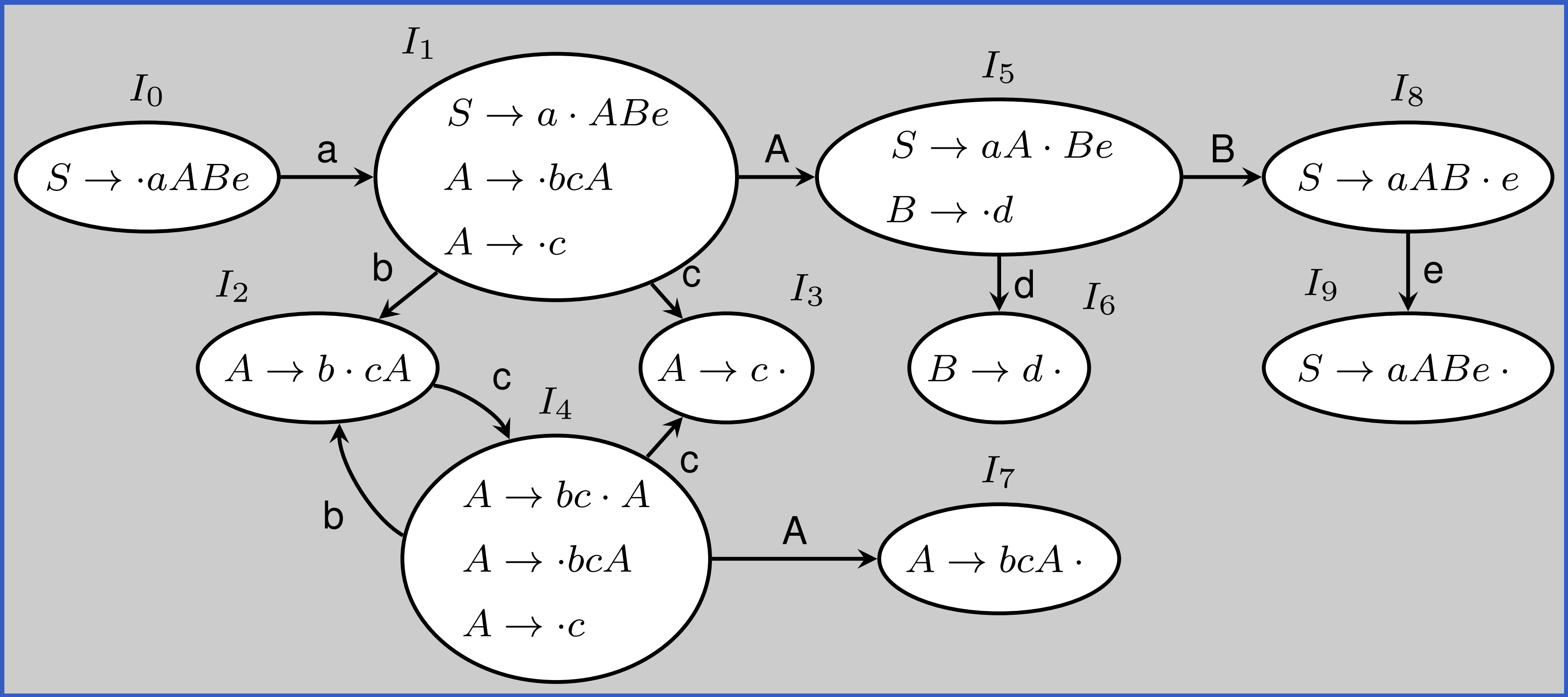
State	Stack (γ)	Input (w)	Move
I_2	ab	$ccde$	

LR(0) Parsing (10)



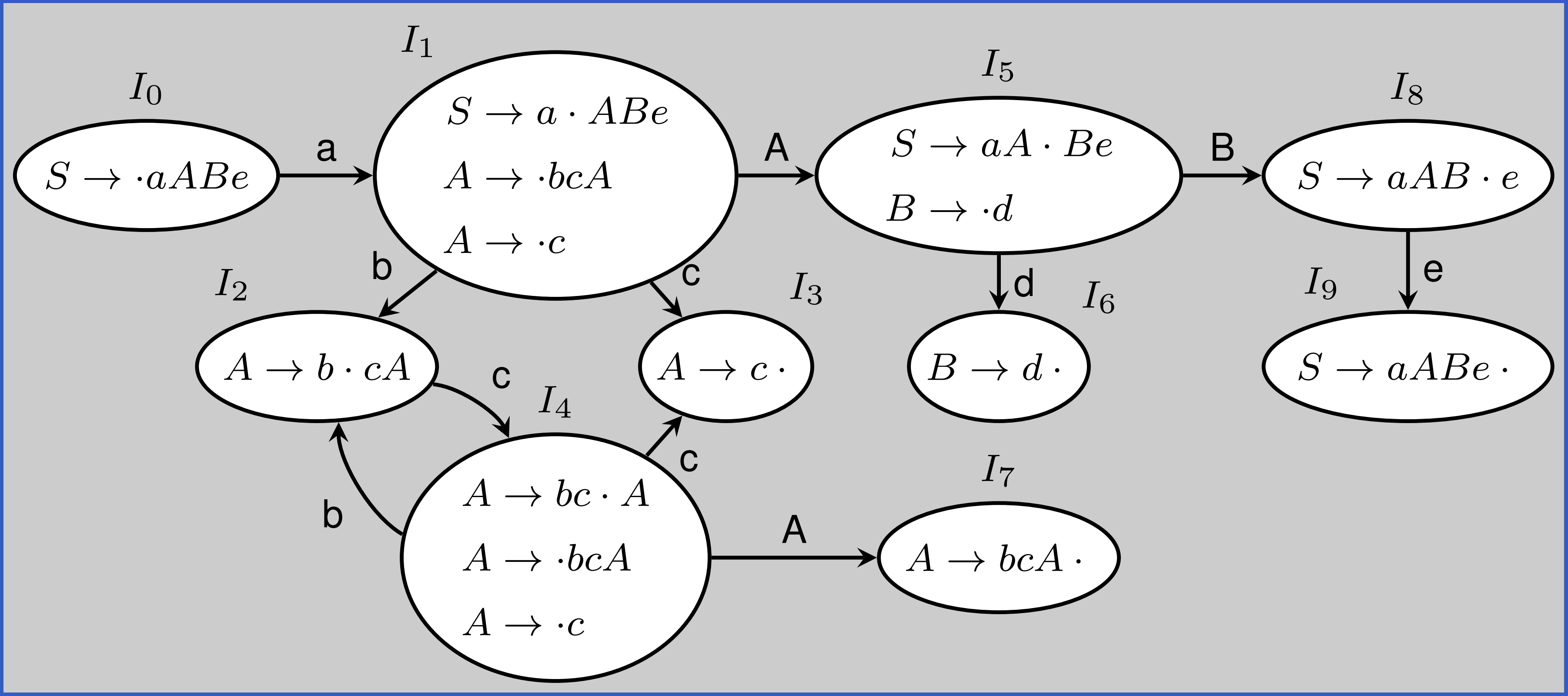
State	Stack (γ)	Input (w)	Move
I_2	ab	$ccde$	Shift

LR(0) Parsing (10)



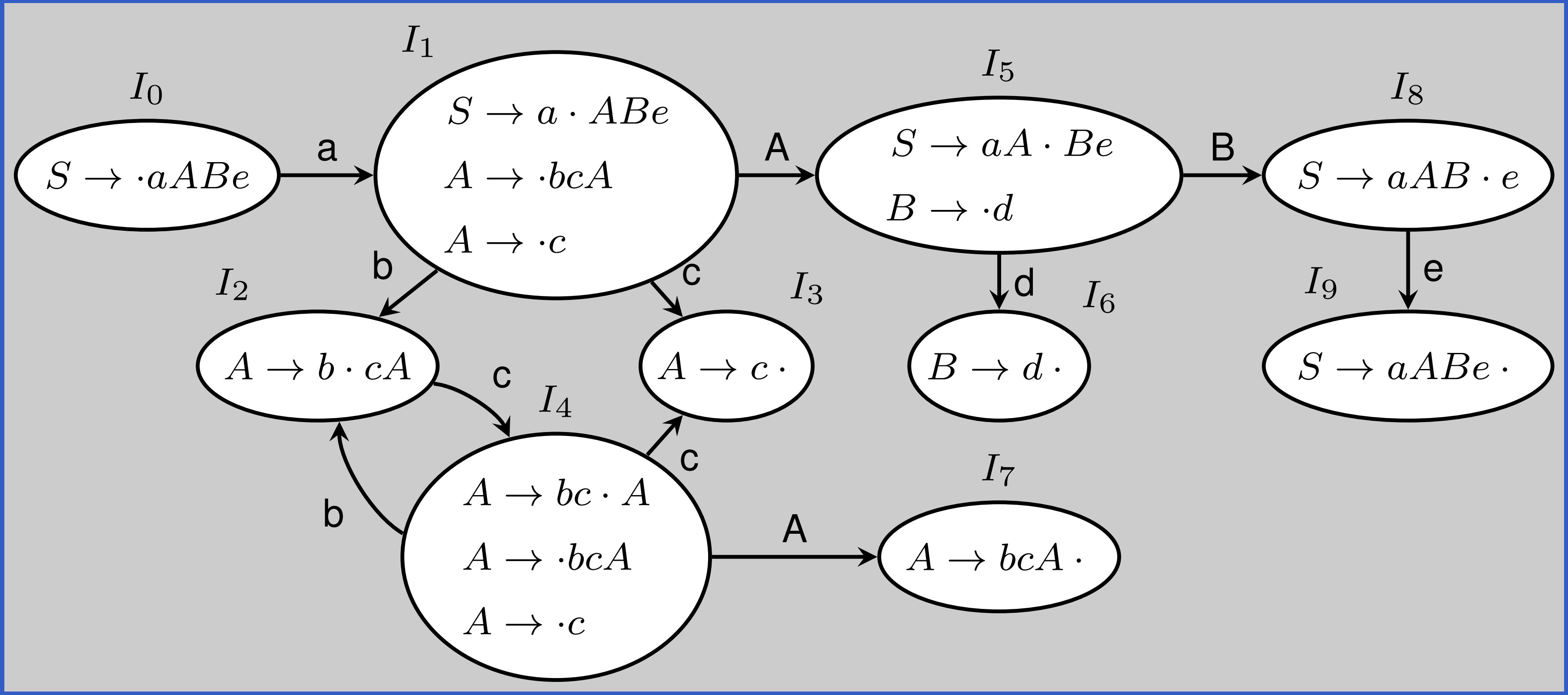
State	Stack (γ)	Input (w)	Move
I_2	ab	$ccde$	Shift
I_4	abc	cde	

LR(0) Parsing (10)



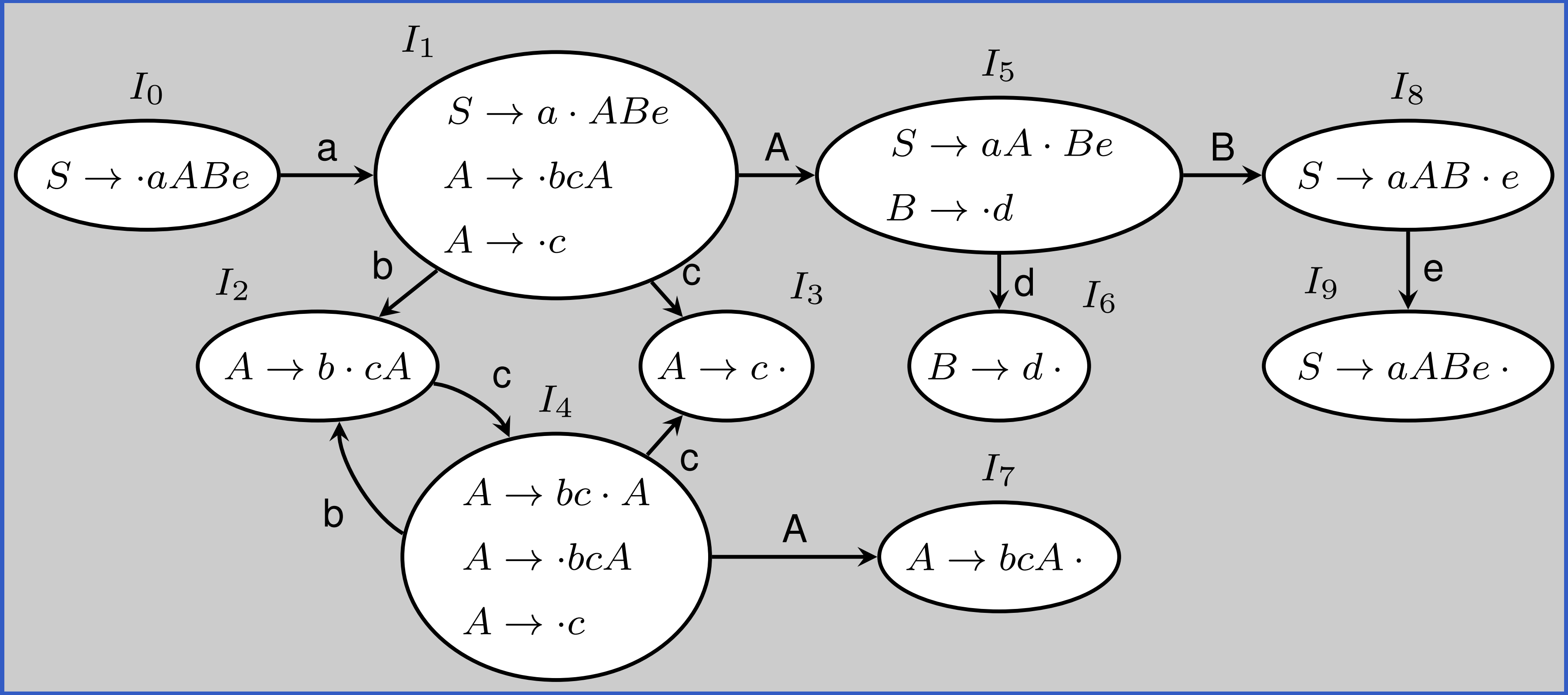
State	Stack (γ)	Input (w)	Move
I_2	ab	$ccde$	Shift
I_4	abc	cde	Shift

LR(0) Parsing (10)



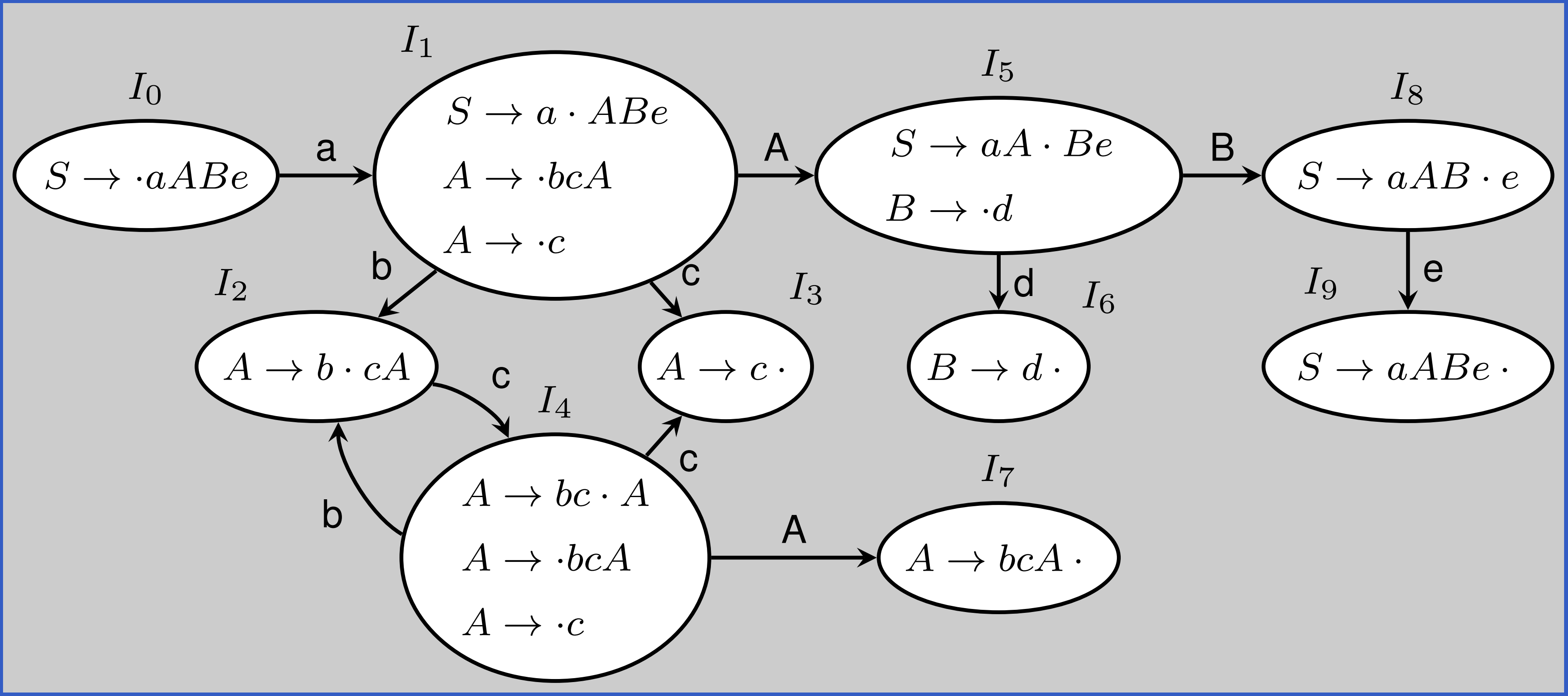
State	Stack (γ)	Input (w)	Move
I_2	ab	$ccde$	Shift
I_4	abc	cde	Shift
I_3	$abcc$	de	

LR(0) Parsing (10)



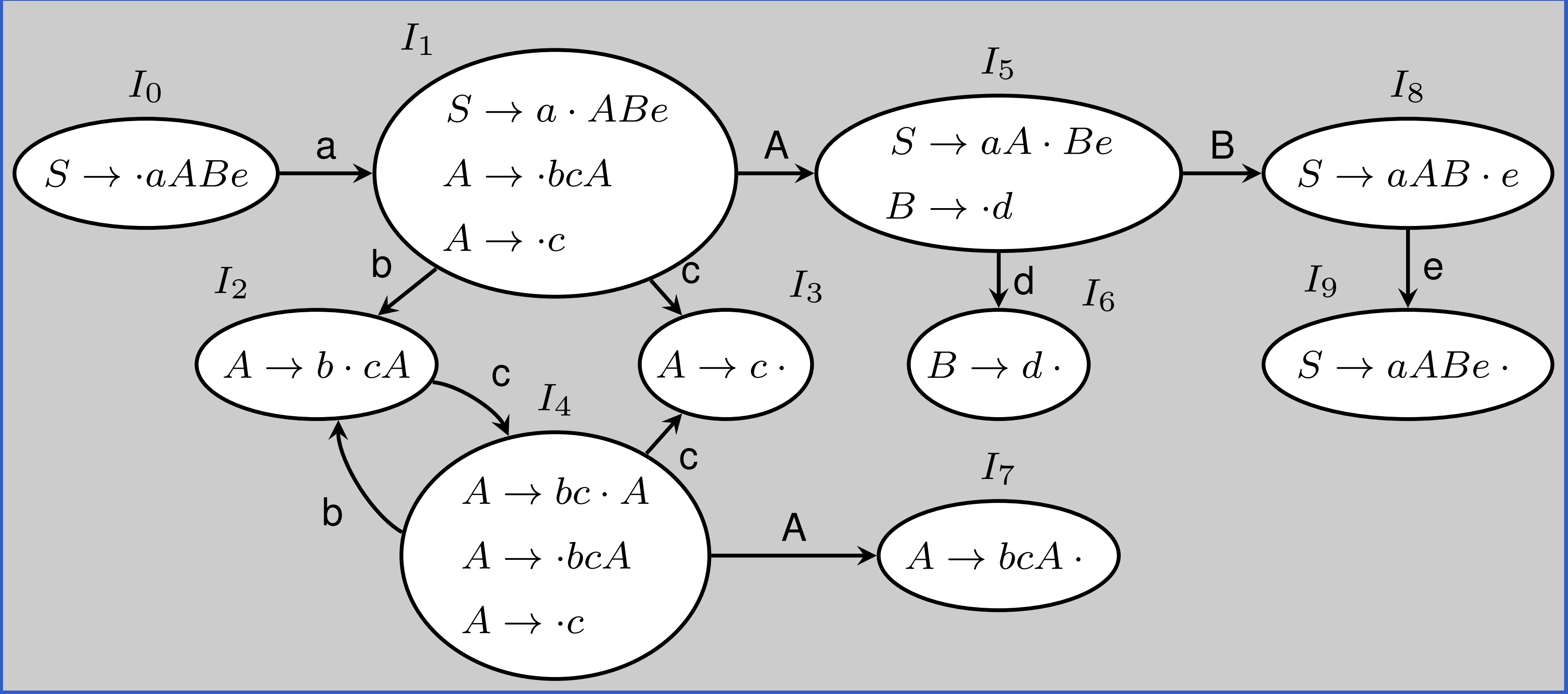
State	Stack (γ)	Input (w)	Move
I_2	ab	$ccde$	Shift
I_4	abc	cde	Shift
I_3	$abcc$	de	Reduce by $A \to c$

LR(0) Parsing (11)



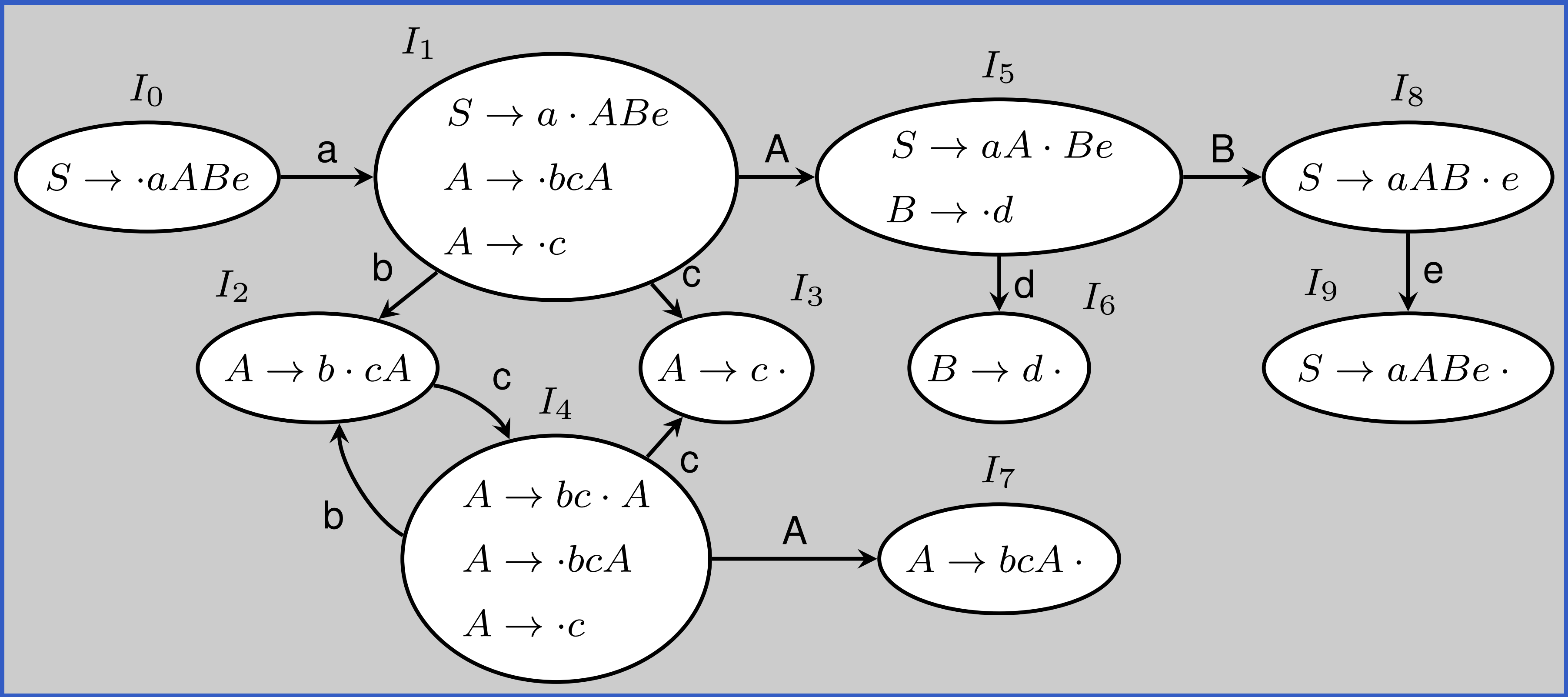
State	Stack (γ)	Input (w)	Move
I_7	$abcA$	de	

LR(0) Parsing (11)



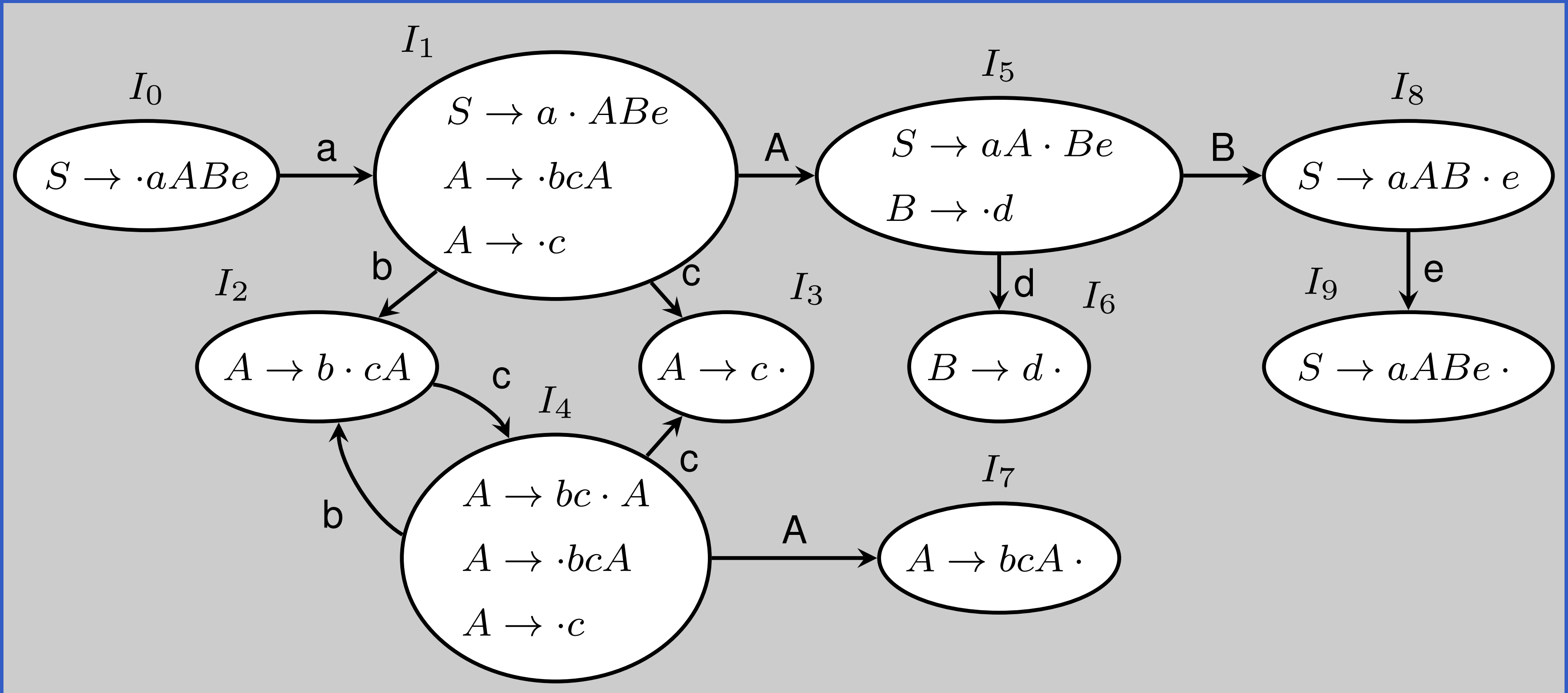
State	Stack (γ)	Input (w)	Move
I_7	$abcA$	de	Reduce by $A \to bcA$

LR(0) Parsing (11)



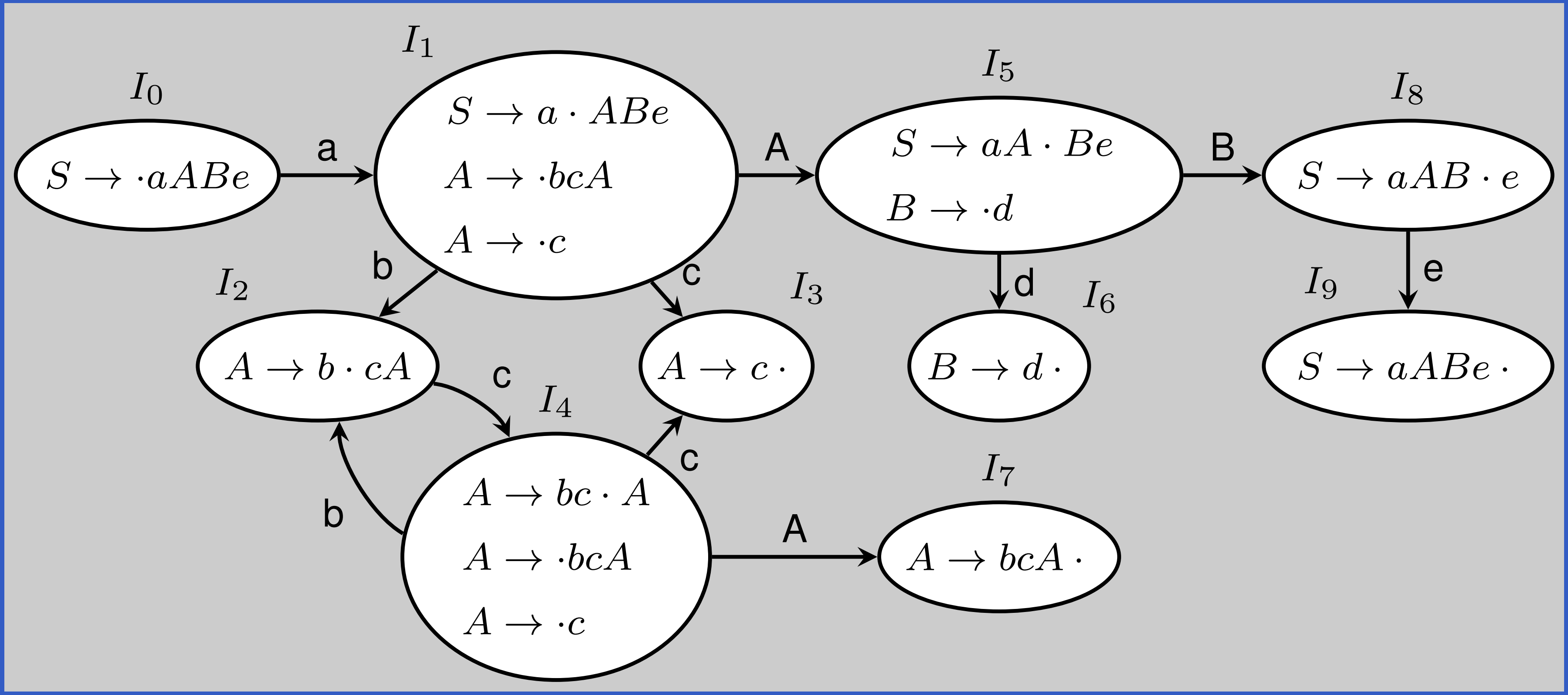
State	Stack (γ)	Input (w)	Move
I_7	$abcA$	de	Reduce by $A \to bcA$
I_5	aA	de	

LR(0) Parsing (11)



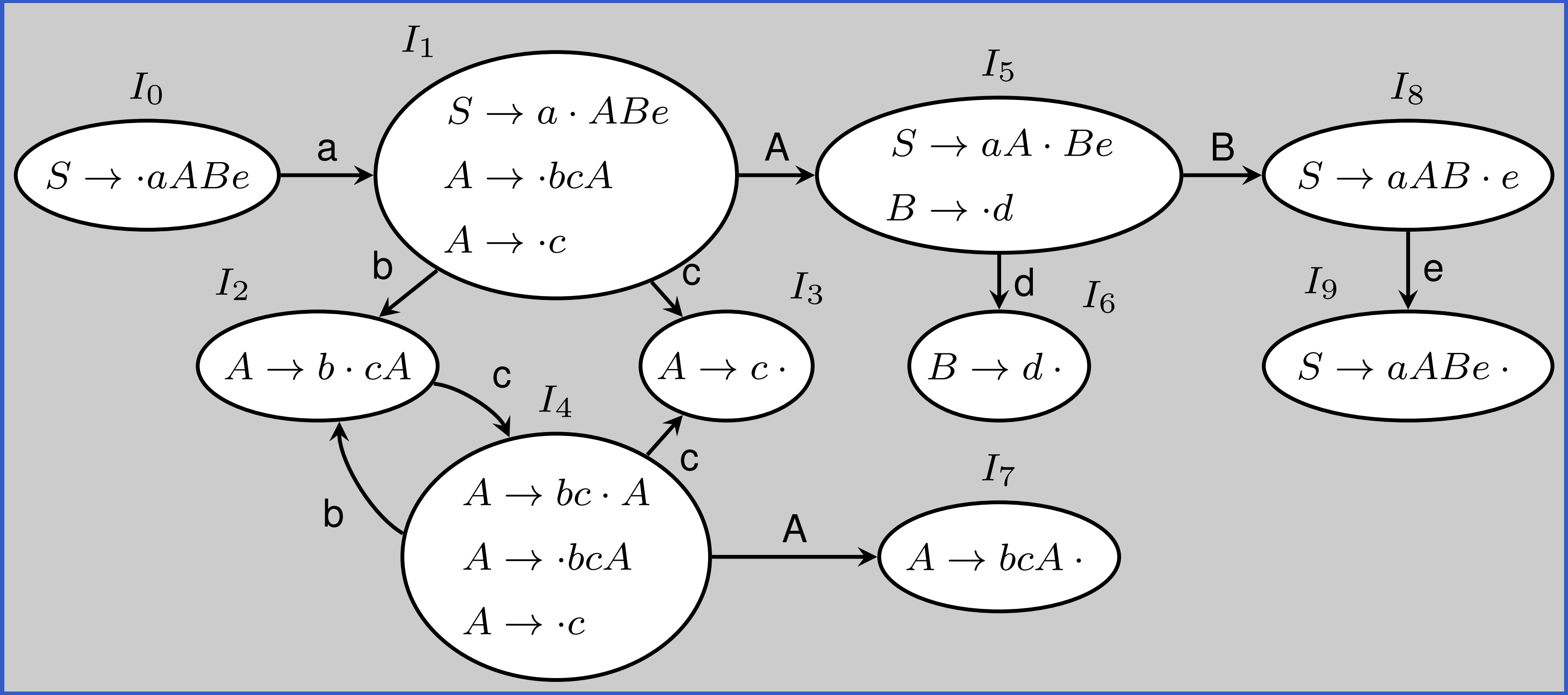
State	Stack (γ)	Input (w)	Move
I_7	$abcA$	de	Reduce by $A \to bcA$
I_5	aA	de	Shift

LR(0) Parsing (11)



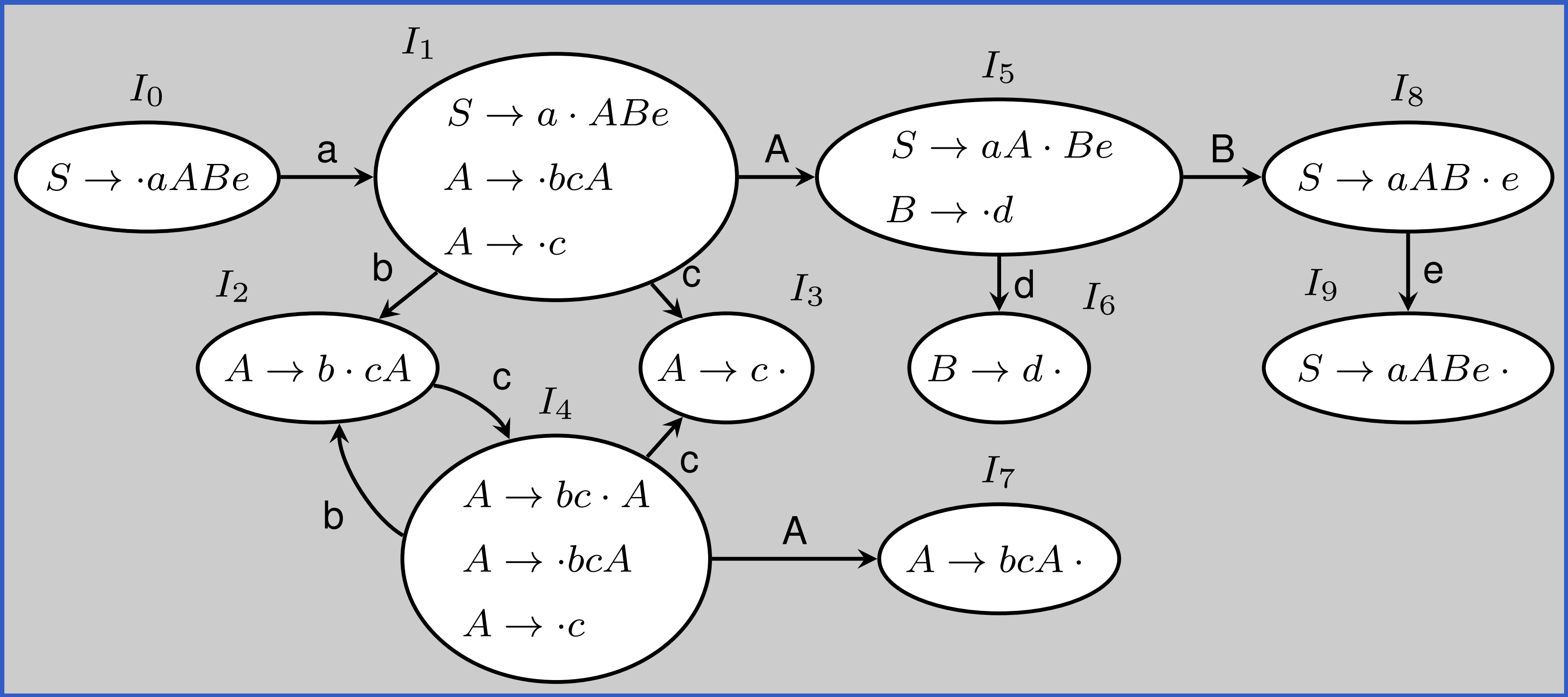
State	Stack (γ)	Input (w)	Move
I_7	$abcA$	de	Reduce by $A \to bcA$
I_5	aA	de	Shift
I_6	aAd	e	

LR(0) Parsing (11)



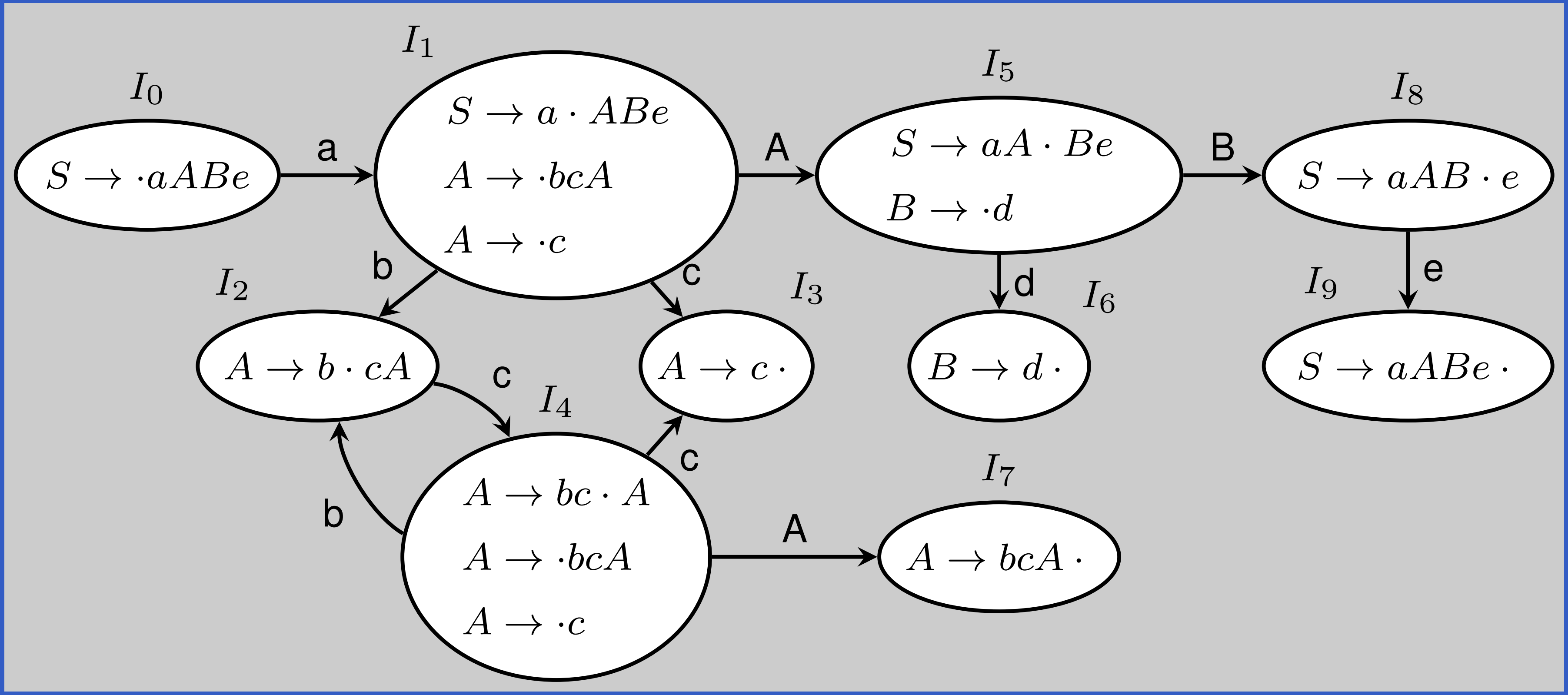
State	Stack (γ)	Input (w)	Move
I_7	$abcA$	de	Reduce by $A \to bcA$
I_5	aA	de	Shift
I_6	aAd	e	Reduce by $B \to d$

LR(0) Parsing (12)



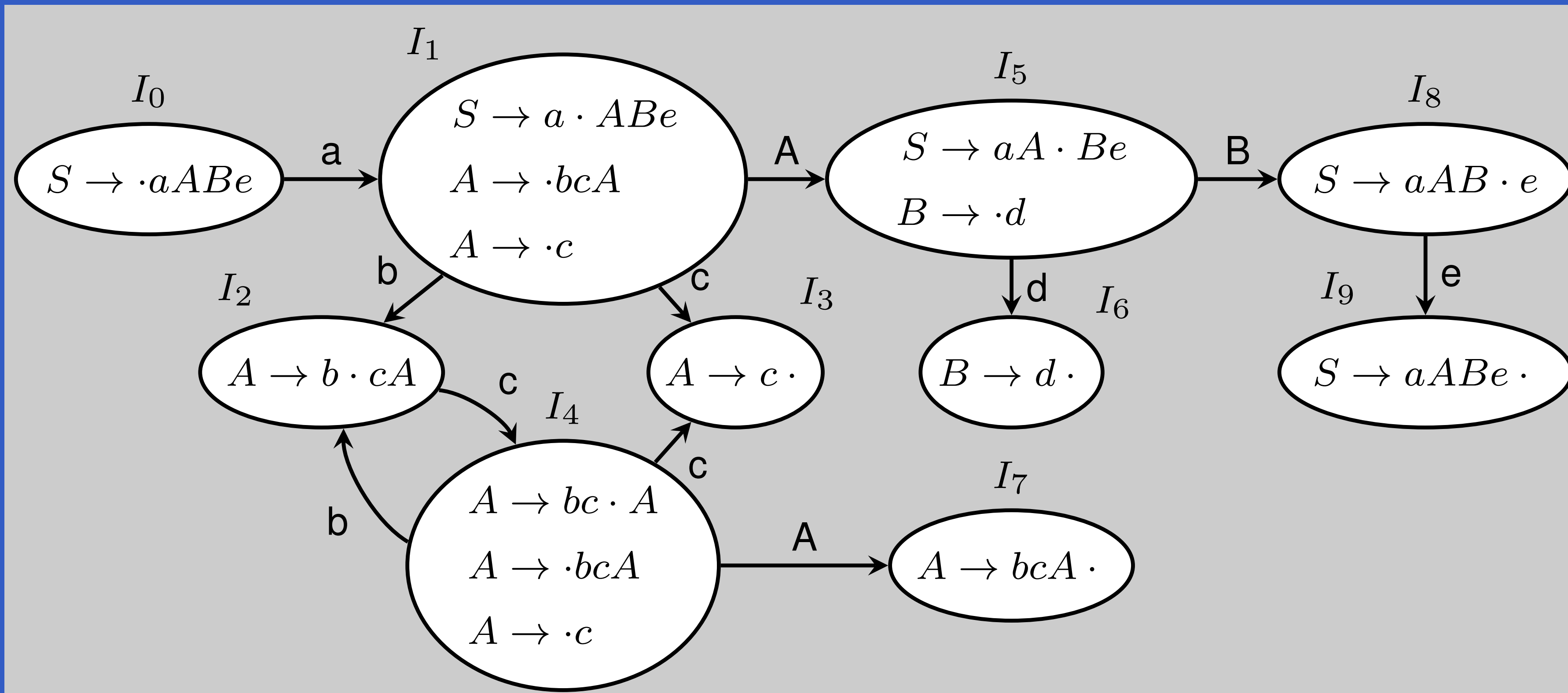
State	Stack (γ)	Input (w)	Move
I_8	aAB	e	

LR(0) Parsing (12)



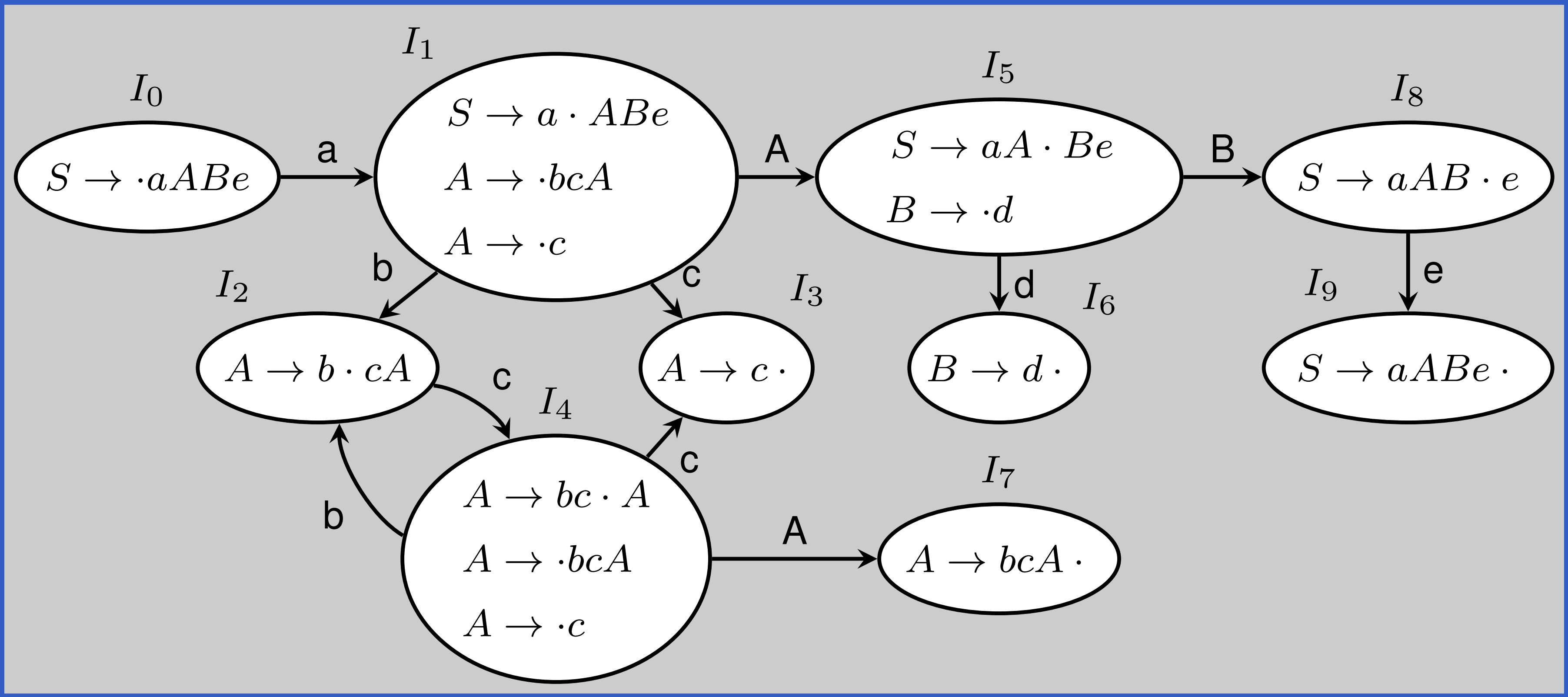
State	Stack (γ)	Input (w)	Move
I_8	aAB	e	Shift

LR(0) Parsing (12)



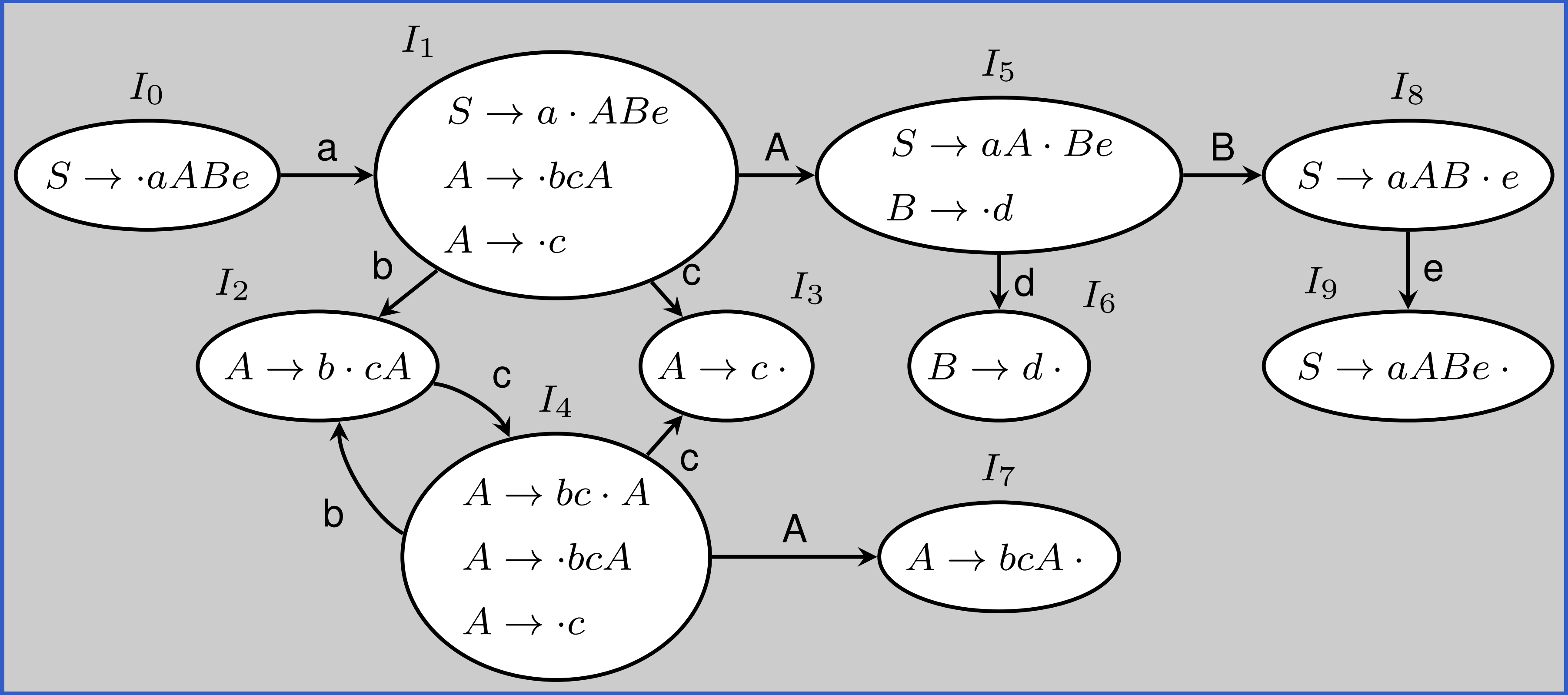
State	Stack (γ)	Input (w)	Move
I_8	aAB	e	Shift
I_9	$aABe$	ϵ	

LR(0) Parsing (12)



State	Stack (γ)	Input (w)	Move
I_8	aAB	e	Shift
I_9	$aABe$	ϵ	Reduce by $S \to aABe$

LR(0) Parsing (12)



State	Stack (γ)	Input (w)	Move
I_8	aAB	e	Shift
I_9	$aABe$	ϵ	Reduce by $S \to aABe$
	S	ϵ	Done

LR(0) Parsing (13)

Complete sequence (γw is right-sentential form):

State	Stack (γ)	Input (w)	Move
I_0	ϵ	$abccde$	Shift
I_1	a	$bccde$	Shift
I_2	ab	$ccde$	Shift
I_4	abc	cde	Shift
I_3	$abcc$	de	Reduce by $A \to c$
I_7	$abcA$	de	Reduce by $A \to bcA$
I_5	aA	de	Shift
I_6	aAd	e	Reduce by $B \to d$
I_8	aAB	e	Shift
I_9	$aABe$	ϵ	Reduce by $S \to aABe$
	S	ϵ	Done

Cf: $S \underset{rm}{\Rightarrow} aABe \underset{rm}{\Rightarrow} aAde \underset{rm}{\Rightarrow} abcAde \underset{rm}{\Rightarrow} abccde$

LR(0) Parsing (14)

Even more clear that the parser carries out the rightmost derivation in reverse if we look at the right-sentential forms γw of the reduction steps only:

$$
\begin{aligned}
abccde &\underset{rm}{\Leftarrow} \\
abcAde &\underset{rm}{\Leftarrow} \\
aAde &\underset{rm}{\Leftarrow} \\
aABe &\underset{rm}{\Leftarrow} \\
S &
\end{aligned}
$$

LR Parsing & Left/Right Recursion (1)

Remark: Note how the ***right-recursive*** production

$$A \to bcA$$

causes symbols bc to pile up on the parse stack until a reduction by

$$A \to c$$

can occur, in turn allowing the stacked symbols to be reduced away.

LR Parsing & Left/Right Recursion (2)

Even clearer if considering parsing of a string like

$$abcbcbccde \quad \text{or} \quad abcbcbcbcbccde$$

Exercise: Try parsing these!

Left-recursion allows reduction to happen sooner, thus keeping the size of the parse stack down. This is why left-recursive grammars often are preferred for LR parsing.

LR(1) Grammars (1)

- In practice, LR(0) tends to be a bit too restrictive.

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LR(1) Grammars (1)

- In practice, LR(0) tends to be a bit too restrictive.
- If we add one symbol of “lookahead” by determining the set of ***terminals that possibly could follow a handle*** being reduced by a production $A \to \beta$, then a wider class of grammars can be handled.
- Such grammars are called ***LR(1)***.

LR(1) Grammars (2)

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- Associate a ***lookahead set*** with items:

$$A \to \alpha \cdot \beta, \{a_1, a_2, \ldots, a_n\}$$

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LR(1) Grammars (2)

Idea:

- Associate a ***lookahead set*** with items:

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- On reduction, a complete item is ***only valid*** if the next input symbol belongs to its lookahead set.
- Thus it is OK to have two or more simultaneously valid complete items, as long as their lookahead sets are ***disjoint***.

(Similar to ***predictive*** recursive-descent parsing.)