



G53CMP: Handout Lecture 9

Contextual Analysis: Types and Type Systems I

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Values

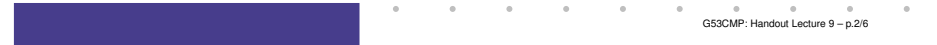
$v \rightarrow$	true	values:	true value
	false		false value
	nv		numeric value
$nv \rightarrow$		numeric values:	
	0		zero value
	succ nv		successor value



Example Language: Abstract Syntax

Example language. (Will be extended later.)

$t \rightarrow$	true	terms:	constant true
	false		constant false
	if t then t else t		conditional
	0		constant zero
	succ t		successor
	pred t		predecessor
	iszero t		zero test



One Step Evaluation Relation (1)

$t \rightarrow t'$ is an **evaluation relation** on terms. Read:
 t evaluates to t' in one step.

The evaluation relation constitutes an **operational semantics** for the example language.

$$\mathbf{if\ true\ then\ } t_2 \mathbf{\ else\ } t_3 \rightarrow t_2 \quad (\text{E-IFTRUE})$$

$$\mathbf{if\ false\ then\ } t_2 \mathbf{\ else\ } t_3 \rightarrow t_3 \quad (\text{E-IFFALSE})$$

$$\frac{t_1 \rightarrow t'_1}{\mathbf{if\ } t_1 \mathbf{\ then\ } t_2 \mathbf{\ else\ } t_3 \rightarrow \mathbf{if\ } t'_1 \mathbf{\ then\ } t_2 \mathbf{\ else\ } t_3} \quad (\text{E-IF})$$



One Step Evaluation Relation (2)

$$\frac{t_1 \longrightarrow t'_1}{\text{succ } t_1 \longrightarrow \text{succ } t'_1} \quad (\text{E-SUCC})$$

$$\text{pred } 0 \longrightarrow 0 \quad (\text{E-PREDZERO})$$

$$\text{pred } (\text{succ } nv_1) \longrightarrow nv_1 \quad (\text{E-PREDSUCC})$$

$$\frac{t_1 \longrightarrow t'_1}{\text{pred } t_1 \longrightarrow \text{pred } t'_1} \quad (\text{E-PRED})$$

One Step Evaluation Relation (3)

$$\text{iszero } 0 \longrightarrow \text{true} \quad (\text{E-ISZEROZERO})$$

$$\text{iszero } (\text{succ } nv_1) \longrightarrow \text{false} \quad (\text{E-ISZEROSUCC})$$

$$\frac{t_1 \longrightarrow t'_1}{\text{iszero } t_1 \longrightarrow \text{iszero } t'_1} \quad (\text{E-ISZERO})$$