Overview

- Lectures and practical exercises
- Course web page:
  http://www.cs.nott.ac.uk/~nhn/ITU-FRP2010
- Outline is tentative:
  - Hard to know how long the the practical bits will take: should not rush unduly.
  - Happy to adapt.

This Lecture

- Brief introduction to FRP:
  - Central ideas
  - Key notions
  - Applications
  - FRP variants
- Classical FRP
  - Basic combinators
  - Semantics

FRP Applications (1)

Some domains where FRP or FRP-inspired approaches have been used:

- Graphical Animation (Fran: Elliott, Hudak)
- Robotics (Frob: Peterson, Hager, Hudak, Elliott, Pembeci, Nilsson)
- Vision (FVision: Peterson, Hudak, Reid, Hager)
- GUIs (Fruit: Courtney; Grapefruit: Jeltsch)
- Games (Courtney, Nilsson, Peterson, Cheong, ...)

FRP Applications (2)

- Virtual Reality Environments (Blom)
- Sound synthesis (Giorgidze, Nilsson)
- (Non-causal) modeling and simulation (Nilsson, Hudak, Peterson, Giorgidze)
- Experiment descriptions (Nielsen, Matheson, Nilsson)

Key FRP Features

- First class reactive entities.
- Synchronous: all system parts operate in synchrony.
- Support for hybrid (mixed continuous and discrete time) systems.
- Allows dynamic system structure.
Central Notions (1)

- Time-varying value or Signal. Intuition:
  \[ \text{Signal } \alpha \approx \text{Time} \rightarrow \alpha \]
- Signal Generator: maps a start time to a signal. Intuition:
  \[ \text{SG } \alpha \approx \text{Time} \rightarrow \text{Signal } \alpha \]
- Signal Function: maps a signal to a signal. Intuition:
  \[ \text{SF } \alpha \beta \approx \text{Signal } \alpha \rightarrow \text{Signal } \beta \]

Central Notions (2)

Additionally, general causality requirement:
output at time \( t \) must be determined by input on interval \([0, t]\).

Signal functions are said to be:
- pure or stateless if output at time \( t \) only depends on input at time \( t \)
- impure or stateful if output at time \( t \) depends on input over the interval \([0, t]\).

Generally also a notion of discrete time.

FRP Variants

A number of FRP variants have emerged. Key differences include what the central abstractions are. Some examples:
- Classic FRP: First class signal generators.
- Extended Classic FRP: First class signal generators and signals.
- Yampa: First class signal functions, signals a secondary notion.
- Elerea: First class signals and signal generators.

Example: Video Tracker

Video trackers are typically stateful signal functions:

Example: Robotics (1)

[PPDP'02, with Izzet Pembeci and Greg Hager, Johns Hopkins University]
Hardware setup:

Example: Robotics (2)

Software architecture:

Example: Robotics (3)

Example: Neuroscience Experiments

[TFP'09, Tom Nielsen, Tom Matheson, Henrik Nilsson]

Signal Functions and State

Alternative view:
Signal functions can encapsulate state.

\[ f : [\text{state}(t)] \rightarrow \text{output} \]

\( \text{state}(t) \) summarizes input history \( x(t'), t' \in [0, t] \).
Thus, really a kind of process.

From this perspective, signal functions are:
- stateful if \( y(t) \) depends on \( x(t) \) and \( \text{state}(t) \)
- stateless if \( y(t) \) depends only on \( x(t) \)
Classic FRP (1)

Classic FRP (CFRP): Fran derivative. Central abstractions:

- **Behavior**:
  - Polymorphic, (conceptually) continuous-time, signal generator.
  - Type constructor: $B \alpha$

- **Event**:
  - Polymorphic, discrete-time, signal generator.
  - Type constructor: $E \alpha$

Examples:

$7 :: B \text{Real}$

$time :: B \text{Time}$

$\star :: B \text{Real} \rightarrow B \text{Real}$

$\star :: (\alpha \rightarrow \beta) \rightarrow (B \alpha \rightarrow B \beta)$

$\text{integral} :: B \text{Real} \rightarrow B \text{Real}$

Classic FRP (2)

Some more examples:

$\text{never} :: E \alpha$

$\text{now} :: E ()$

$\text{after} :: \text{Time} \rightarrow E ()$

$\text{repeatedly} :: \text{Time} \rightarrow E ()$

$\text{edge} :: B \text{Bool} \rightarrow E ()$

$\text{hold} :: \alpha \rightarrow E \alpha \rightarrow B \alpha$

$lbp :: E ()$

$key :: E \text{Char}$

Classic FRP (3)

Switching and event mapping:

$\text{until} :: B \alpha \rightarrow E (B \alpha) \rightarrow B \alpha$

$\Rightarrow :: E \alpha \rightarrow (\alpha \rightarrow \beta) \rightarrow E \beta$

$\Rightarrow :: E \alpha \rightarrow \beta \rightarrow E \beta$

Typical CFRP Snippets (1)

$\text{color} :: B \text{Color}$

$\text{color} = \text{red} \ '\text{until}' \ lbp \rightarrow \text{blue}$

$\text{ball} :: B \text{Picture}$

$\text{ball} = \text{paint color circ}$

$\text{circ} :: B \text{Region}$

$\text{circ} = \text{translate} \ (\cos \text{time}, \sin \text{time})$

$\text{(circle 1)}$

Typical CFRP Snippets (2)

$\text{color2} = \text{red} \ '\text{until}'$

$\text{(lbp} \rightarrow \text{blue})$

$\text{key} \rightarrow \text{yellow}$

$\text{color3} = \text{red} \ '\text{until}'$

$\text{(edge} \ (\text{time} \geq 5) \rightarrow \text{blue})$

Classic FRP (4)

Semantic Functions (1)

$\text{at} : (B\alpha) \rightarrow \text{Time} \rightarrow \text{Time} \rightarrow \alpha$

$\text{occ} : (E\alpha) \rightarrow \text{Time} \rightarrow \text{Time} \rightarrow [\text{Time} \times \alpha]$

Intuitively, $\text{at}$ maps a behavior to a function from a start time and a time of interest to a value at that time.

Note that the type of $\text{at}$ can be parenthesized:

$\langle B\alpha \rangle \rightarrow (\text{Time} \rightarrow (\text{Time} \rightarrow \alpha))$

Thus, $\text{at}$ maps a behavior to a signal generator.

Semantic Functions (2)

$\text{at} : (B\alpha) \rightarrow \text{Time} \rightarrow \text{Time} \rightarrow \alpha$

$\text{occ} : (E\alpha) \rightarrow \text{Time} \rightarrow \text{Time} \rightarrow [\text{Time} \times \alpha]$

The function $\text{occ}$ gives meaning to events in a similar way, but the result is a finite list of time-ascending event occurrences from the start time to the time of interest.

Semantics (1)

Time, liftings, integration:

$\text{at}[\text{time}] T t = t$

$\text{at}[\text{lift0} c] T t = c$

$\text{at}[\text{lift1} f b] T t = [f] (\text{at}[b] T t)$

$\text{at}[\text{lift2} f b d] T t = [f] (\text{at}[b] T t) (\text{at}[d] T t)$

$\text{at}[\text{integral} b] T t = \int_t^\tau (\text{at}[b] T \tau) \text{d}r$
Semantics (2)

Basic events:

\[
\text{occ[never]} \quad T \ t = \ [] \\
\text{occ[now]} \quad T \ t = \ [(T, ())] \\
\text{occ[after } \tau \text{]} \quad T \ t = \ \begin{cases} 
\[] & T + \tau < t \\
(T + \tau, ()) & \text{otherwise}
\end{cases}
\]

Semantics (3)

\[
\text{occ[repeatedly } \tau \text{]} \quad T \ t = 
\begin{cases} 
\[] & n = 0 \\
((T + \tau, ()), (T + 2\tau, ()), \ldots, (T + n\tau, ())) & \text{otherwise}
\end{cases}
\]

where \( n \in \mathbb{N} \) is the largest number such that \( T + n\tau \leq t \).

Semantics (4)

Intuitively, the predicate event:

\[
\text{edge} :: \text{B} \text{ Bool} \rightarrow \text{E} ()
\]

occurs whenever the argument behavior changes from False to True.

However, surprisingly hard to characterize exactly (and, of course, not computable).

Implementation

Using infinite lists as streams, stream-based versions of the central CFRP abstractions can be realised as follows:

\[
\text{B} \ a = \text{[Time]} \rightarrow \text{[a]} \\
\text{E} \ a = \text{[Time]} \rightarrow \text{[Maybe a]}
\]

Note that this corresponds to signal generators: A prefix of \([\text{Time}]\) is a discretized approximation of an interval from the start time to the current time.

Faithfulness (1)

Of course, we can only hope to approximate the ideal, continuous semantics.

But, then, what is a faithful implementation?

- Wan and Hudak (2000) adapts the notion of uniform convergence to the setting of CFRP.
- They then show that the stream-based semantics of the CFRP converges to the ideal semantics in the limit as the maximal sampling interval tends to 0, establishing necessary side conditions where needed.

Faithfulness (2)

- Wan and Hudak still assume real reals and exact functions on the reals. Floating point arithmetic adds another level of difficulty.

Reading