Implementing and Optimising Functional Reactive Programming Big-O Meetup, 14 Dec. 2016

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- This talk considers Yampa: an arrows-based FRP system embedded in Haskell.

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- In particular: many algebraic laws hold.
- These guide the implementation and optimisations: a theme of this talk.

FRP Applications (1)

Some domains where FRP or FRP-inspired approaches have been used:

- Robotics
- Vision
- Sound synthesis
- GUIs
- Virtual Reality Environments
- Games
- Distributed Event-based Systems

FRP Applications (2)

Example: Breakout in Yampa (and SDL) on a tablet:



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Take-home Game!

Or download one for free to your Android device!



Play Store: Pang-a-lambda (Keera Studios)

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 - Any function of the form $a \rightarrow M b$ where M is a monad (the "Kleisli construction").
- A number of *algebraic laws* must be satisfied: we will come back to those.
- Arrows due to Prof. John Hughes.

The Arrow framework (1)



Types signatures for some arrow F:

arr :: (a -> b) -> F a b
(>>>) :: F a b -> F b c -> F a c
first :: F a b -> F (a,c) (b,c)
loop :: F (a, c) (b, c) -> F a b

The Arrow framework (2)

Some derived combinators:





f ** * g f && & g

(***) :: F a b -> F c d -> F (a,c) (b,d) (&&&) :: F a b -> F a c -> F a (b,c)







Tedious way to program?



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Yes, can be. But syntactic support can be provided.

Key FRP Features

Combines conceptual simplicity of the *synchronous data flow* approach with the flexibility and abstraction power of higher-order functional programming:

Synchronous

- First class temporal abstractions
- Hybrid: mixed continuous and discrete time
- Dynamic system structure

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- First class temporal abstractions
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(But not everything labelled "FRP" supports them all.)

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FRP implemenation embedded in Haskell

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 - Signals: time-varying values
 - Signal Functions: functions on signals
 - Switching between signal functions

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Programming model:



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- Signals are a secondary notion: only exist indirectly.
- This is a key aspect allowing for a fundamentally simple, pure, implementation.

 Of course, FRP does not have to be implemented purely, and many FRP implementations are indeed not pure. But keeping it pure makes it easier to get correct. Good for reference if nothing else.
Yampa?

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Yampa?

Yet Another Mostly Pointless Acronym?

Yampa?

Yet Another Mostly Pointless Acronym?

Yampa is a river with long calmly flowing sections and abrupt whitewater transitions in between.



A good metaphor for hybrid systems!





Intuition:

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Intuition:

 $\texttt{Time}\approx\mathbb{R}$

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Intuition:

Time $\approx \mathbb{R}$ Signal a \approx Time -> a x :: Signal T1 y :: Signal T2



Intuition:

Time $\approx \mathbb{R}$ Signal a \approx Time -> a x :: Signal T1 y :: Signal T2 SF a b \approx Signal a -> Signal b f :: SF T1 T2

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Intuition:

Time ≈ ℝ Signal a ≈ Time -> a x :: Signal T1 y :: Signal T2 SF a b ≈ Signal a -> Signal b f :: SF T1 T2

Additionally, *causality* required: output at time t must be determined by input on interval [0, t].

Alternative view:

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Signal functions can encapsulate *state*.



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From this perspective, signal functions are:
stateful if y(t) depends on x(t) and state(t)
stateless if y(t) depends only on x(t)

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Signal functions can encapsulate *state*.

$$\begin{array}{c|c} x(t) & f & y(t) \\ \hline [state(t)] & \end{array}$$

state(t) summarizes input history x(t'), $t' \in [0, t]$. From this perspective, signal functions are: • *stateful* if y(t) depends on x(t) and state(t)• *stateless* if y(t) depends only on x(t)Signal functions form an arrow.

identity :: SF a a

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- identity :: SF a a
- constant :: b -> SF a b

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- integral :: VectorSpace a s=>SF a a $y(t) = \int_{0}^{t} x(\tau) \, \mathrm{d}\tau$

A basic implementation: SF (1)

Each signal function is essentially represented by a *transition function*. Arguments:

- Time passed since the previous time step.
- The current input value.

Returns:

- A (possibly) updated representation of the signal function, the *continuation*.
- The current value of the output signal.

A basic implementation: SF (2)

type DTime = Double

data SF a b =
 SF {sfTF :: DTime -> a
 -> Transition a b}

type Transition a b = (SF a b, b)

The continuation encapsulates any internal state of the signal function. The type synonym DTime is the type used for the time deltas, > 0.

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writes the resulting output sample to the environment (typically I/O action).

 The loop then repeats, but uses the continuation returned from the transition function on the next iteration, thus ensuring any internal state is maintained.

A basic implementation: arr

arr :: (a -> b) -> SF a b
arr f = sf
where
sf = SF {sfTF = _ a -> (sf, f a)}

Note: It is obvious that arr constructs a **stateless** signal function since the returned continuation is exactly the signal function being defined, i.e. it never changes.

A basic implementation: >>>

For >>>, we have to combine their continuations into updated continuation for the composed arrow:

 $(>>>) \overrightarrow{\text{I: SF a b}} \rightarrow \overrightarrow{\text{SF b c}} \rightarrow \overrightarrow{\text{SF a c}}$ $(SF \{sfTF = tf1\}) >>> (SF \{sfTF=tf2\}) = SF \{sfTF = tf\}$

where

tf dt a =
$$(sf1' >>> sf2', c)$$

where

Note how **same** time delta is fed to both subordinate signal functions, thus ensuring **synchrony**.

A basic impl.: How to get started? (1)

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Instead:

- Initial SF representation makes a first transition given just an input sample.
- Makes that transition into a representation that expects time deltas from then on.

A basic impl.: How to get started? (2)

data SF a b =
 SF {sfTF :: a -> Transition a b}

data SF' a b =
 SF' {sfTF' :: DTime -> a
 -> Transition a b}

type Transition a b = (SF' a b, b)
SF' is internal, can be thought of as representing
a "running" signal function.

The arrow identity law:

arr id >>> a = a = a >>> arr id

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How can this be exploited?

1. Introduce a constructor representing arr id
 data SF a b = ...

SFId

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The arrow identity law:

arr id >>> a = a = a >>> arr id

How can this be exploited?

2. Make SF abstract by hiding all its constructors.
Optmimizing >>>: First Attempt (2)

3. Ensure SFId only gets used at intended type: identity :: SF a a identity = SFId

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4. Define optimizing version of >>>:
 (>>>) :: SF a b -> SF b c -> SF a c

SFId >>> sf = sf

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Optmimizing >>>: First Attempt (2)

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No optimization possible?

The type system does not get in the way of all optimizations. For example, for:

constant :: b -> SF a b constant b = arr (const b)

the following laws can readily be exploited:

sf >>> constant c = constant c
constant c >>> arr f = constant (f c)
But to do better, we need GADTs.

Generalized Algebraic Data Types

GADTs allow

- individual specification of return type of constructors
- the more precise type information to be taken into account during case analysis.

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Define optimizing version of >>> *exactly* as before:

- (>>>) :: SF a b -> SF b c -> SF a c
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(>>>) :: SF a b -> SF b c -> SF a c
...
SFId >>> sf = sf

Define optimizing version of >>> *exactly* as before:

(>>>) :: SF a b \rightarrow SF b c \rightarrow SF a c



Define optimizing version of >>> exactly as before:

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Other Ways? Statically?

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Other Ways? Statically?

- Other (typed) approaches include keeping coercion functions around as "evidence" for use at runtime (Hughes 2004). But imposes an overhead.
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arr g >>> switch (...) (_ -> arr f)

$$\xrightarrow{switch}$$
 arr g >>> arr f = arr (f . g)

Laws Exploited for Optimizations

General arrow laws:

(f >>> g) >>> h = f >>> (g >>> h)
 arr (g . f) = arr f >>> arr g
 arr id >>> f = f
 f = f >>> arr id

Laws involving const (the first is Yampa-specific):

sf >>> arr (const k) = arr (const k)
arr (const k)>>>arr f = arr (const(f k))

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- This can be exploited (Liu, Cheng, Hudak 2009) to define a *Causal Commutative Normal Form (CCNF)* for *switch-free* networks.
- Essentially CCNF is a Mealy Machine.
- Not exploited in Yampa, but this optimization has been used to obtain performance gains of two orders of magnitude (over Yampa-like performance).

Implementation (1)



Implementation (2)

data FunDesc a b where FDI :: FunDesc a a FDC :: b -> FunDesc a b FDG :: (a -> b) -> FunDesc a b



Implementation (2)



Recovering the function from a FunDesc:

fdFun :: FunDesc a b -> (a -> b)
fdFun FDI = id
fdFun (FDC b) = const b
fdFun (FDG f) = f

Implementation (2)





Implementation (3)

fdComp :: FunDesc a b -> FunDesc b c -> FunDesc a c fdComp FDI fd2 = fd2fdComp fd1 FDI = fd1fdComp (FDC b) fd2 =FDC ((fdFun fd2) b) fdComp (FDC c) = FDC c fdComp (FDG f1) fd2 =FDG (fdFun fd2 . f1)

Events

Yampa models *discrete-time* signals by lifting the *range* of continuous-time signals: data Event a = NoEvent | Event a *Discrete-time signal* = Signal (Event α).

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Yampa models *discrete-time* signals by lifting the *range* of continuous-time signals: data Event a = NoEvent | Event a*Discrete-time signal* = Signal (Event α). Consider composition of pure event processing:

- f :: Event a -> Event b
- g :: Event b -> Event c

arr f >>> arr g

Optimizing Event Processing (1)

Additional function descriptor: data FunDesc a b where ... FDE :: (Event a -> b) -> b -> FunDesc (Event a) b

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Optimizing Event Processing (1)

Additional function descriptor: data FunDesc a b where ... FDE :: (Event a -> b) -> b -> FunDesc (Event a) b

Extend the composition function: fdComp (FDE f1 f1ne) fd2 = FDE (f2 . f1) (f2 f1ne) where f2 = fdFun fd2

Optimizing Event Processing (2)

Extend the composition function: fdComp (FDG f1) (FDE f2 f2ne) = FDG f where f a = case f1 a of NoEvent -> f2ne f1a -> f2 f1a

Optimizing Event Processing (2)

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Optimizing Stateful Event Processing

A general stateful event processor:

ep :: (c -> a -> (c,b,b)) -> c -> b -> SF (Event a) b

Optimizing Stateful Event Processing

A general stateful event processor:

Composes nicely with stateful and stateless event processors!

Optimizing Stateful Event Processing

A general stateful event processor:

Composes nicely with stateful and stateless event processors! Introduce explicit representation:

data SF a b where

SFEP :: ...
-> (c -> a -> (c, b, b)) -> c -> b
-> SF (Event a) b
Cause for Concern

Code with GADT-based optimizations is getting large and complicated:

- Many more cases to consider.
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- Many more cases to consider.
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Is the result really a performance improvement? A number of Micro Benchmarks were carried out to verify that individual optimizations worked as intended, including:

- Space Invaders
- MIDI Event Processor

Benchmark 1: Space Invaders



Benchmark 2: MIDI Event Processor

High-level model of a MIDI event processor programmed to perform typical duties:



The MEP4



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Results

Benchmark	T_{U} [S]	$T_{\rm O}$ [S]	$T_{\rm O}/T_{\rm U}$
Space Inv.	0.95	0.86	0.93
MEP	19.39	10.31	0.48

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Most important gains:

- Insensitive to bracketing.
- A number of "pre-composed" combinators no longer needed, thus simplifying the Yampa API (and implementation).
- Much better event processing.