Functional Reactivity: Eschewing the Imperative

An Overview of Functional Reactive Programming in the Context of Yampa

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Reactive programming

*Reactive systems*:  
- input arrives incrementally while system is running  
- output is generated in response to input in an interleaved fashion

Contrast *transformational systems*.

The notions of  
- time  
- time-varying entities, *signals*  
are inherent.
Functional Reactive Programming (FRP):

- Paradigm for reactive programming in a functional setting.
- Originated from Functional Reactive Animation (Fran) (Elliott & Hudak).
- Has evolved in a number of directions and into different concrete implementations.
Related languages

FRP related to:

- Synchronous languages, like Esterel, Lucid Synchrone.
- Modeling languages, like Simulink, Modelica.

Distinct features of FRP:

- First class reactive components.
- Allows highly dynamic system structure.
FRP applications

Some domains where FRP has been used:

- Graphical Animation (Fran: Elliott, Hudak)
- Robotics (Frob: Peterson, Hager, Hudak, Elliott, Pembeci, Nilsson)
- Vision (FVision: Peterson, Hudak, Reid, Hager)
- GUIs (Fruit: Courtney)
- Hybrid modeling (Nilsson, Hudak, Peterson)
Example: Robotics (1)

[PPDP’02, with Izzet Pembeci and Greg Hager, Johns Hopkins University]

Hardware setup:
Example: Robotics (2)

Software architecture:

```
  Application
  |      |
  Frob  FVision
  |      |
  FRP (Yampa)
  |      |
  Pioneer drivers  XVision2
```

Haskell

C/C++
Example: Robotics (3)
The most recent Yale FRP implementation is called **Yampa**:

- Embedding in Haskell; i.e. a Haskell library.
- Arrows used as the basic structuring framework.
- Advanced switching constructs allows for highly dynamic system structure.
Yampa?
Yampa?

Yet
Another
Mostly
Pointless
Acronym
Yampa?

Yet
Another
Mostly
Pointless
Acronym

???
Yampa?

Yet
Another
Mostly
Pointless
Acronym

???

No …
Yampa?

Yampa is a river . . .
Yampa?

...with long calmly flowing sections...
... and abrupt whitewater transitions in between.

A good metaphor for hybrid systems!
Key concept: **functions on signals**.

Intuition:

\[
\text{Signal } \alpha \approx \text{Time } \rightarrow \alpha \\
x :: \text{Signal T1} \\
y :: \text{Signal T2} \\
f :: \text{Signal T1} \rightarrow \text{Signal T2}
\]

Additionally: **causality** requirement.
Signal functions and state

Alternative view:

Functions on signals can encapsulate state.

\[ x(t) \rightarrow \begin{array}{c} f \\ \text{[state}(t)\text{]} \end{array} \rightarrow y(t) \]

\(state(t)\) summarizes input history \(x(t'), t' \in [0, t]\).

Functions on signals are either:

- **Stateful**: \(y(t)\) depends on \(x(t)\) and \(state(t)\)
- **Stateless**: \(y(t)\) depends only on \(x(t)\)
Signal functions are *first class entities*. Intuition: $\text{SF } \alpha \beta \approx \text{Signal } \alpha \to \text{Signal } \beta$
Signal functions in Yampa

- Signal functions are \textbf{first class entities}. Intuition: $\text{SF } \alpha \beta \approx \text{Signal } \alpha \rightarrow \text{Signal } \beta$

- Signals are \textbf{not} first class entities: they only exist indirectly through signal functions.
Signal functions in Yampa

- Signal functions are **first class entities**. Intuition: $\text{SF } \alpha \beta \equiv \text{Signal } \alpha \rightarrow \text{Signal } \beta$
- Signals are **not** first class entities: they only exist indirectly through signal functions.
- The strict separation between signals and signal functions distinguishes Yampa from earlier FRP implementations.
Describing systems

Systems are described by combining signal functions into larger signal functions:
Yampa and arrows

Yampa uses John Hughes’ *arrow* framework:
Signal functions are arrows.

Core signal function combinators:

- `arr :: (a -> b) -> SF a b`
- `>>> :: SF a b -> SF b c -> SF a c`
- `first :: SF a b -> SF (a,c) (b,c)`
- `loop :: SF (a,c) (b,c) -> SF a b`

Enough to express any conceivable “wiring”.
The arrow syntactic sugar

Using the basic combinators directly is often very cumbersome. Ross Paterson’s syntactic sugar for arrows provides a convenient alternative:

```plaintext
proc pat -> do [ rec ]
    pat_1 <- sfexp_1 <- exp_1
    pat_2 <- sfexp_2 <- exp_2
    ...
    pat_n <- sfexp_n <- exp_n
returnA <- exp
```

Also: `let pat = exp` $\equiv$ `pat <- arr id <- exp`
Some basic signal functions

- **identity** :: SF a a
  identity = arr id -- *semantics*

- **constant** :: b -> SF a b
  constant b = arr (const b) -- *semantics*

- **integral** :: VectorSpace a s->SF a a

- **time** :: SF a Time
  time = constant 1.0 >>> integral

- **(ˆ<<)** :: (b->c) -> SF a b -> SF a c
  f (ˆ<<) sf = sf >>> arr f
A bouncing ball

\[ y = y_0 + \int \dot{y} \, dt \]

\[ \dot{y} = \int -9.81 \]

On impact:

\[ \dot{y} = -\dot{y}(t-) \]

(fully elastic collision)
Modelling the bouncing ball: part 1

Free-falling ball:

type Pos = Double

\(\text{fallingBall} :: \) Pos -> Vel -> SF () (Pos, Vel)

\(\text{fallingBall} \ p0 \ v0 = \text{proc} () \rightarrow \text{do} \)
\(v \leftarrow (v0 +) \hat{\times} \text{integral} \leftarrow -9.81\)
\(p \leftarrow (p0 +) \hat{\times} \text{integral} \leftarrow v\)
\(\text{returnA} \leftarrow (p, v)\)
Conceptually, *discrete-time* signals are only defined at discrete points in time, often associated with the occurrence of some `event`.

Yampa models discrete-time signals by lifting the `range` of continuous-time signals:

```
data Event a = NoEvent | Event a
```

*Discrete-time signal* $= \text{Signal}(\text{Event } \alpha)$.

We often want to associate information with an event occurrence:

```
tag :: Event a -> b -> Event b
```
Some basic event sources

- never :: SF a (Event b)
- now :: b -> SF a (Event b)
- after :: Time -> b -> SF a (Event b)
- repeatedly ::
  Time -> b -> SF a (Event b)
- edge :: SF Bool (Event ())
Stateful event suppression

- `notYet :: SF (Event a) (Event a)`
- `once :: SF (Event a) (Event a)`
Modelling the bouncing ball: part 2

Detecting when the ball goes through the floor:

\[ \text{fallingBall': } :: \]
\[ \text{Pos } \to \text{Vel} \]
\[ \to \text{SF () ((Pos,Vel), Event (Pos,Vel))} \]

\[ \text{fallingBall'} \ p0 \ v0 = \text{proc () } \to \text{do} \]
\[ \text{pv@(p, _)} \gets \text{fallingBall} \ p0 \ v0 \gets () \]
\[ \text{hit } \gets \text{edge} \]
\[ \text{returnA } \gets (\text{pv, hit 'tag' pv}) \]
Switching

Q: How and when do signal functions “start”? 
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A: • *Switchers* “apply” a signal functions to its input signal at some point in time.
  
  • This creates a “running” signal function *instance*, which often replaces the previously running instance.
Q: How and when do signal functions “start”?  

A:  

- **Switchers** “apply” a signal function to its input signal at some point in time.  
- This creates a “running” signal function instance, which often replaces the previously running instance.

Switchers thus allow systems with **varying structure** to be described.
The basic switch

Idea:

- Allows one signal function to be replaced by another.
- Switching occurs on the first occurrence of the switching event source.

\[
\text{switch} :: \\
\text{SF a (b, Event c)} \\
\rightarrow (c \rightarrow \text{SF a b}) \\
\rightarrow \text{SF a b}
\]
The basic switch

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- Allows one signal function to be replaced by another.
- Switching occurs on the first occurrence of the switching event source.

\[
\text{switch ::} \\
\text{SF } a \ (b, \ Event \ c) \\
\rightarrow (c \rightarrow \text{SF } a \ b) \\
\rightarrow \text{SF } a \ b
\]
The basic switch

Idea:

- Allows one signal function to be replaced by another.
- Switching occurs on the first occurrence of the switching event source.

```
switch ::
  Function yielding SF to switch into SF a (b, Event c) -> (c -> SF a b) -> SF a b
```
Modelling the bouncing ball: part 3

Making the ball bounce:

```haskell
bouncingBall :: Pos -> SF () (Pos, Vel)
bouncingBall p0 = bbRec p0 0.0

where
    bbRec p0 v0 =
        switch (fallingBall' p0 v0) \(p,v) ->
            bbRec p (-v)
```

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Simulation of bouncing ball
Highly dynamic system structure?

Basic switch allows one signal function to be replaced by another.

- What about more general structural changes?

- What about state?
The challenge

George Russel said on the Haskell GUI list:

“I have to say I’m very sceptical about things like Fruit which rely on reactive animation, ever since I set our students an exercise implementing a simple space-invaders game in such a system, and had no end of a job producing an example solution. . . .
The challenge

George Russel said on the Haskell GUI list:

... Things like getting an alien spaceship to move slowly downward, moving randomly to the left and right, and bouncing off the walls, turned out to be a major headache. Also I think I had to use ‘error’ to get the message out to the outside world that the aliens had won. ...
George Russel said on the Haskell GUI list:

My suspicion is that reactive animation works very nicely for the examples constructed by reactive animation folk, but not for my examples.”
The game
Describing the alien behavior (1)

type Object = SF ObjInput ObjOutput

alien :: RandomGen g =>
    g -> Position2 -> Velocity -> Object
alien g p0 vyd = proc oi -> do
    rec
        -- Pick a desired horizontal position
        rx <- noiseR (xMin, xMax) g <- ()
        smpl <- occasionally g 5 () <- ()
        xd <- hold (point2X p0) <- smpl `tag` rx ...
...
Describing the alien behavior (2)

...  

```plaintext
-- Controller
let axd = 5 * (xd - point2X p) 
    - 3 * (vector2X v) 
ayd = 20 * (vyd - (vector2Y v)) 
ad  = vector2 axd ayd 
h   = vector2Theta ad 
...  
```
Describing the alien behavior (3)

...  

-- *Physics*

let a = vector2Polar

(min alienAccMax

(vector2Rho ad))

h

vp <- iPre v0 <- v

ffi <- forceField <- (p, vp)

v <- (v0 ^+^) ^<< impulseIntegral

<- (gravity ^+^ a, ffi)

p <- (p0 .+^) ^<< integral <- v

...
Describing the alien behavior (4)

...  

-- Shields

sl  <- shield  <- oiHit  oi
die <- edge    <- sl <= 0

returnA  <- ObjOutput

ooObsObjState  = oosAlien p h v,
ooKillReq      = die,
ooSpawnReq     = noEvent

where

v0 = zeroVector
Overall game structure

dpSwitch

route

ObjInput

ObjOutput

alien

gun

alien

bullet

killOrSpawn

[ObjectOutput]
Idea:

- Switch over *collections* of signal functions.
- On event, “freeze” running signal functions into collection of signal function *continuations*, preserving encapsulated *state*.
- Modify collection as needed and switch back in.
Need ability to express:

- How input routed to each signal function.
- When collection changes shape.
- How collection changes shape.

dpSwitch :: Functor col =>
    (forall sf . (a -> col sf -> col (b,sf)))
    -> col (SF b c)
    -> SF (a, col c) (Event d)
    -> (col (SF b c) -> d -> SF a (col c))
    -> SF a (col c)
dpSwitch

Need ability to express:

- How input routed to each signal function.
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- How collection changes shape.

\[ \text{dpSwitch} :: \text{Functor col} \Rightarrow \]
\[ (\forall sf . \, (a \rightarrow \text{col } sf \rightarrow \text{col } (b, sf))) \rightarrow \text{col } (\text{SF } b \, c) \]
\[ \rightarrow \text{SF } (a, \text{col } c) \, (\text{Event } d) \]
\[ \rightarrow (\text{col } (\text{SF } b \, c) \rightarrow d \rightarrow \text{SF } a \, (\text{col } c)) \]
\[ \rightarrow \text{SF } a \, (\text{col } c) \]
dpSwitch

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\[
dpSwitch :: \text{Functor } \text{col} \Rightarrow
(\forall \text{sf . } (a \rightarrow \text{col sf } \rightarrow \text{col (b, sf)}))
\rightarrow \text{col (SF b c)}
\rightarrow \text{SF } (a, \text{col c}) \ (\text{Event } d)
\rightarrow (\text{col (SF b c) } \rightarrow d \rightarrow \text{SF } a \ (\text{col c}))
\rightarrow \text{SF } a \ (\text{col c})
\]
dpSwitch

Need ability to express:

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\[
\text{dpSwitch} :: \text{ Functor } \text{ col} \Rightarrow \\
(\text{forall } \text{ sf} . \ (\text{a} \rightarrow \text{col} \ \text{sf} \rightarrow \text{col} \ (\text{b}, \text{sf}))) \\
\rightarrow \text{col} \ (\text{SF} \ \text{b} \ \text{c}) \\
\rightarrow \text{SF} \ (\text{a}, \ \text{col} \ \text{c}) \ (\text{Event} \ \text{d}) \\
\rightarrow \ (\text{col} \ (\text{SF} \ \text{b} \ \text{c}) \rightarrow \text{d} \rightarrow \text{SF} \ \text{a} \ (\text{col} \ \text{c})) \\
\rightarrow \text{SF} \ \text{a} \ (\text{col} \ \text{c})
\]
**dpSwitch**

Need ability to express:

- How input routed to each signal function.
- When collection changes shape.
- How collection changes shape.

\[
\text{dpSwitch} :: \text{Functor col} \Rightarrow \\
(\forall \text{sf}. \; (a \to \text{col sf} \to \text{col (b,sf)})) \Rightarrow \\
\text{col (SF b c)} \Rightarrow \\
\text{SF (a, col c) (Event d)} \Rightarrow \\
(\text{col (SF b c) \to d \to SF a (col c)}) \Rightarrow \\
\text{SF a (col c)}
\]
Routing

Idea:

- The routing function decides which parts of the input to pass to each running signal function instance.
Routing

Idea:

- The routing function decides which parts of the input to pass to each running signal function instance.

- It achieves this by pairing a projection of the input with each running instance:
The game core

gameCore :: IL Object
    -> SF (GameInput, IL ObjOutput)
        (IL ObjOutput)

gameCore objs =
    dpSwitch route
    objs
    (arr killOrSpawn >>>> notYet)
    (\sfs’ f -> gameCore (f sfs’))
Closing the feedback loop (1)

```haskell
game :: RandomGen g =>
  g -> Int -> Velocity -> Score ->
  SF GameInput ((Int, [ObsObjState]),
                Event (Either Score Score))

game g nAliens vydAlien score0 = proc gi -> do
  rec
    oos <- gameCore objs0 <- (gi, oos)
    score <- accumHold score0
    <- aliensDied oos
    gameOver <- edge <- alienLanded oos
    newRound <- edge <- noAliensLeft oos
    ...
```
Closing the feedback loop (2)

... returnA <- ((score, 
    map ooObsObjState 
    (elemsIL oos)), 
    (newRound 'tag' (Left score)) 
    'lMerge' (gameOver 
    'tag' (Right score)))

where

objs0 =
    listToIL 
    (gun (Point2 0 50) 
     : mkAliens g (xMin+d) 900 nAliens)
Other functional approaches?

Transition function operating on world model with explicit state (e.g. Asteroids by Lüth):

- Model snapshot of world with *all* state components.
- Transition function takes input and current world snapshot to output and the next world snapshot.

One could also use this technique *within* Yampa to avoid switching over dynamic collections.
Why use Yampa, then?

• Yampa provides a lot of functionality for programming with time-varying values:
  - captures common patterns
  - packaged in a way that makes reuse very easy

• Yampa allows state to be nicely encapsulated by signal functions:
  - avoids keeping track of all state globally
  - adding more state is easy and usually does not imply any major changes to type or code structure
State in alien

Each of the following signal functions used in alien encapsulate state:

- noiseR
- occasionally
- hold
- iPre
- forceField
- impulseIntegral
- integral
- shield
- edge
Why not imperative, then?

If state is so important, why not stick to imperative/object-oriented programming where we have “state for free”? Yampa retains all advantages of declarative programming:
- High abstraction level.
- Referential transparency facilitates formal reasoning.
- Synchronous approach avoids “event-call-back soup”, meaning robust, easy-to-understand semantics.
Why not imperative, then?

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If state is so important, why not stick to imperative/object-oriented programming where we have “state for free”?

- Yampa retains all advantages of declarative programming:
  - High abstraction level.
  - Referential transparency facilitates formal reasoning.
- Synchronous approach avoids “event-call-back soup”, meaning robust, easy-to-understand semantics.
Drawbacks of Yampa?

- Choosing the right switch can be tricky.
- Subtle issues concerning when to use e.g. iPre, notYet.
- Syntax could be improved (with specialized pre-processor).
Related work (1)

- First-Order Systems: no dynamic collections
  - Esterel [Berry 92], Lustre [Caspi 87], Lucid Synchrone [Caspi 00], SimuLink, RT-FRP [Wan, Taha, Hudak 01]
- Fudgets [Carlsson and Hallgren 93, 98]
  - Continuation capture with extractSP
  - Dynamic Collections with dynListF
  - No synchronous bulk update
Related work (2)

- Fran [Elliott and Hudak 97, Elliott 99]
  - First class *signals*.
  - But dynamic collections?

- FranTk [Sage 99]
  - Dynamic collections, but only via \( \text{IO} \) monad.
Obtaining Yampa

Yampa 0.9 is available from

http://www.haskell.org/yampa