All You Need Are Functions

A Brief Introduction to Functional Programming in Haskell

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Outline

- Why programming language research?
- What is functional programming and how is it different?
- A Taste of Haskell: A Pure, Lazy, Functional Language
- Some real-world examples (games!)

The Functional Programming Lab (1)

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These inform one another.

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- More reusable.
- More maintainable.

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The cost of bugs that make it into “the wild”?

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  Reason? Someone forgot to convert from imperial to metric units.

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Many and diverse reasons for failures: no one solution. But better programming language technology could have prevented some; e.g. the Mars orbiter crash.

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To put this differently: more ***what*** (logic), less ***how*** (control).

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Functional Programming is a type of declarative programming where programs are built ***exclusively*** from functions and function application.

In particular, functions in the basic mathematical sense: ***equational reasoning*** is applicable.

List of Squares: Python (1)

```
def squares(m,n):
    ss = []
    for i in range(m, n + 1):
        ss.append(i * i)
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[1, 4, 9, 16, 25]
```

List of Squares: Python (2)

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def squares(m,n):
    ss = []
    for i in range(m, n + 1):
        ss.append(i * i)
    return ss
```

Note:

- Step-by-step description of the algorithm: explicit ***control flow***; "***how***".
- The result list is constructed one element at a time.

List of Squares: Haskell

```
squares m n
    | m > n     = []
    | otherwise = m*m : squares (m+1) n

> squares 1 5
[1, 4, 9, 16, 25]
```

Note:

- Direct statement of ***what*** the list of squares is.
- Recursion.
- The result list is expressed as a whole.

Other differences: Function Types

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Haskell:

```
> :type squares
squares :: (Num a,Ord a) => a -> a -> [a]
```

For any numeric type `a`, `squares` is a function from two numbers of type `a` returning a list of numbers of the ***same*** type `a`.

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The Haskell version of `squares` is ***polymorphic***, or “of many shapes”: in this case, works for ***any*** numeric type as all we assumed was multiplication and addition.

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def foo():
    return squares([2,3,5,7])
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def foo():
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 >>>
```

The definition of `foo` is accepted!

```
>>> foo()
 TypeError: squares() takes exactly
2 arguments (1 given)
```

The error only caught when we attempt to run `foo`.

Dynamic vs. Static Typing (2)

Haskell:

```
> foo () = squares [(2::Int),3,5,7]
No instance for (Num [Int])
```

The error caught immediately: essentially we are told that a list of integers is not a number.

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Static typing certainly not unique to functional languages. But some of the most sophisticated type systems have been developed for functional languages.

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def fie(n):
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> let a = 10
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> let a = 20
> fie 2
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`fie 2` = `20` always! We can replace `fie 2` by `20` or vice versa anywhere without changing the meaning of a program. This is what is meant by ***equational reasoning***.

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Why is it (arguably) a practical advantage to program with pure functions?

A pure function has a simple, well-defined ***interface***: its meaning is independent of context and calling it does not cause any ***side effects***. As a consequence, much easier to:

- Understand large programs
- Reuse code
- Reason about code

Try Haskell (1)

Point your browser to `http://tryhaskell.org`.

- A string in Haskell is the same as a list of characters. I.e.

  ```
  ['a', 'b', 'c'] = "abc"
  ```

  Try it: type in `['a', 'b', 'c']` to verify.
- Try functions `head`, `tail`, `reverse`, `sort` on your name. E.g. `head "Henrik"`. What do they do?
- Write an expression that extracts:
  - The second letter of your name
  - The last letter of your name

Try Haskell (2)

- What is `[1..10]`?
- Write an expression for the list of all integers from 50 to 100.
- Do `head`, `tail`, `reverse` work on lists of numbers?
- What is the type of `head`, `tail`, `reverse`?
  Hint: just type in e.g. `head` and hit return.
  What do the types mean?
- What does the function `sum` do to a list of numbers?
- Write an expression to sum all integers from 1 to 1000.

Try Haskell (3)

- `(*2)` is a function that multiplies a number by 2; `(^2)` is a function that squares a number. Try!
- `map` is a ***higher order*** function: it takes a function as an argument and applies it to every element in a list. Explain the result of:
  - map (*2) [1..10]
  - map (^2) [1..10]
- Sum the squares from 1 to 1000.
- What does `words` do to your full name?
- Extract the initials from your full name.

Infinite Data Structures (1)

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This makes it possible to program with (conceptually) ***infinite*** data structures, such as lists.

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More generally, laziness promotes declarative programming. It allows us to focus more on “what”, less on “how”, as there is less need to worry about exactly when things get computed: they get computed automatically as and when needed.

Infinite Data Structures (2)

Given:

```
ones = 1 : ones

from n = n : from (n + 1)

nats = from 0
```

we have

```
> take 10 ones
[1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
> take 10 nats
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

The Sieve of Eratosthene

The following defines `primes` to be the list of ***all*** prime numbers!

```
sieve (p : xs) =
  p : sieve [ x | x <- xs, x `mod` p /= 0 ]

primes = sieve (from 2)
```

The 10 first and the 10000th prime number:

```
> take 10 primes
[2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
> primes !! 9999
104729
```

So, What About *Real* Programs . . .

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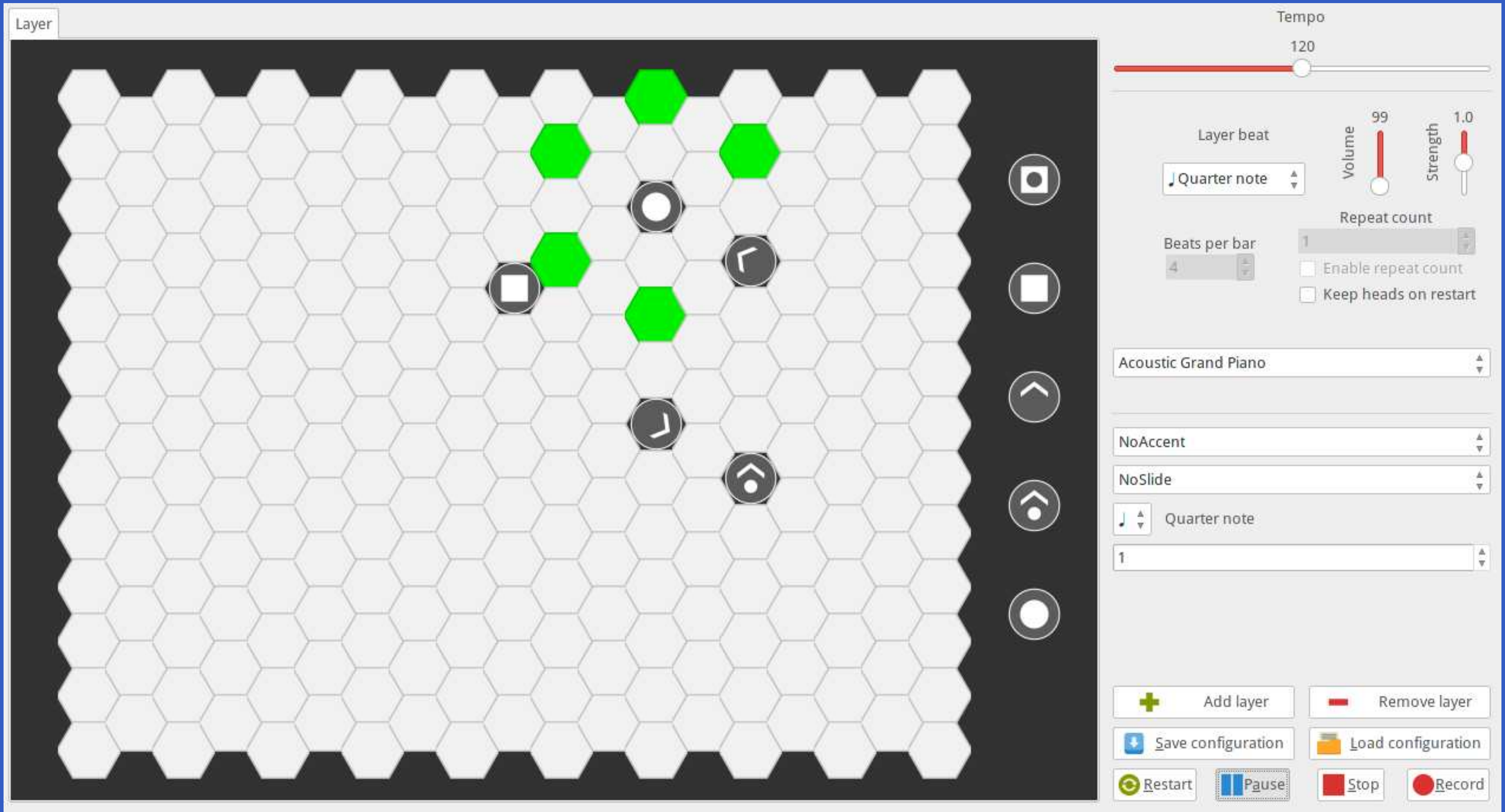
. . . like ***games***?

So, What About *Real* Programs ...

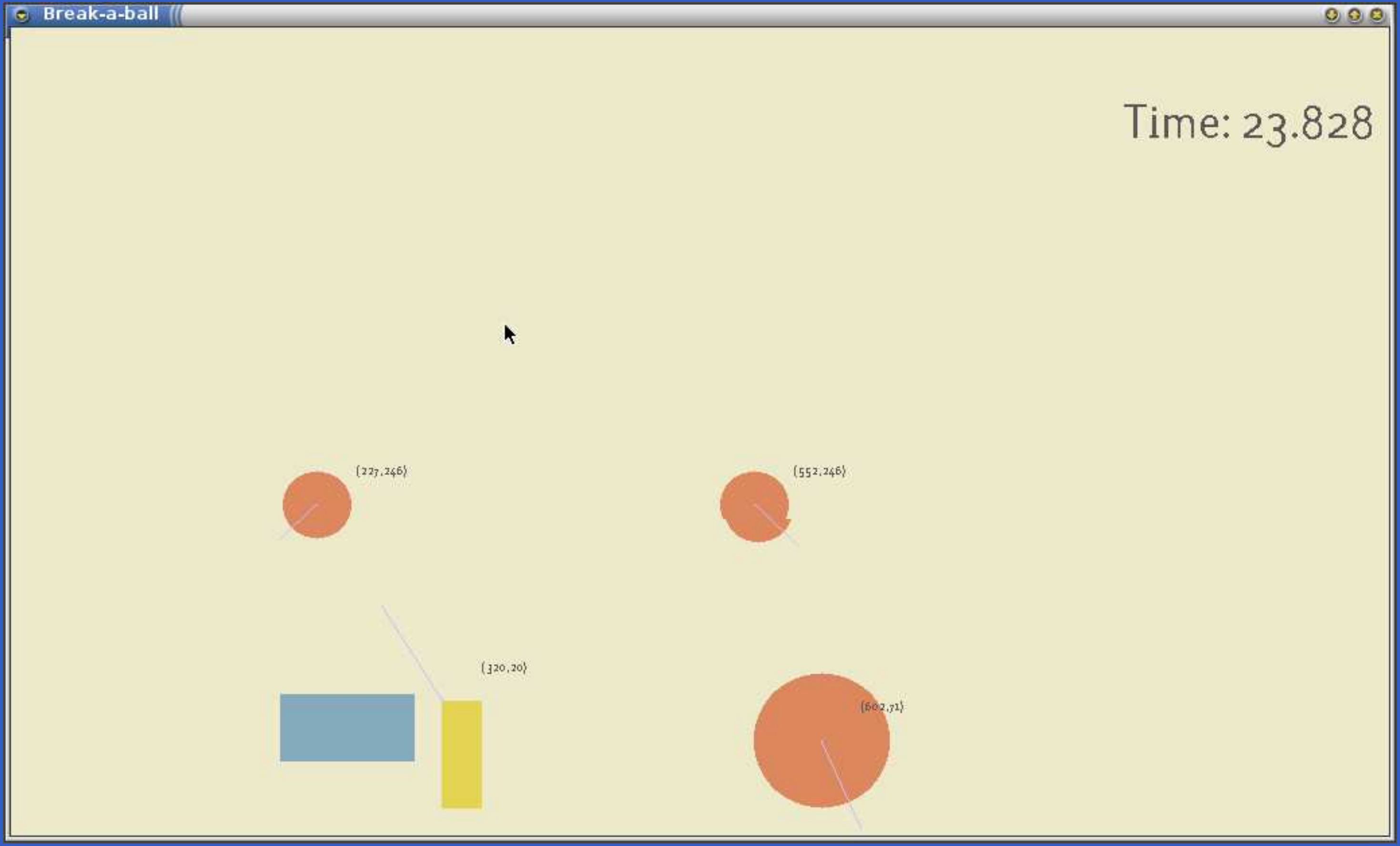
... like ***games***?

Or Musical Applications?



Take-home Game!

Download for free to your Android device!



Play Store: Pang-a-lambda (Keera Studios)

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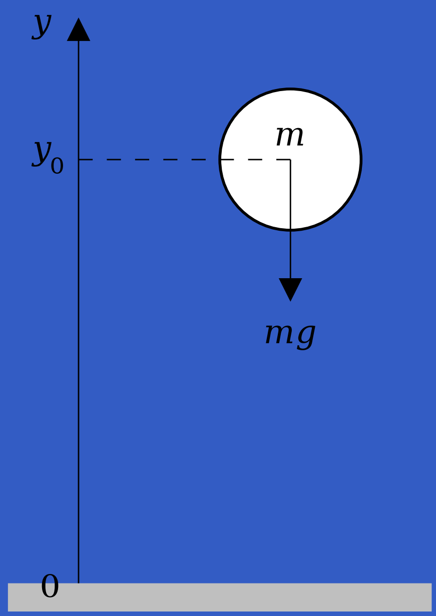
One possibility: pure functions on ***signals*** or ***time-varying values***:

- Player input
- Video output
- Input from a musical keyboard
- Notes to be played on a synthesizer
- Audio output

A Bouncing Ball

Lots of bouncing balls in Pang-a-lambda!



$$\begin{aligned} y &= y_0 + \int v \,\mathrm{d}t \\ v &= v_0 + \int -9.81 \end{aligned}$$

On impact:

$$v = -v(t-)$$

(fully elastic collision)

Mathematical equations that describe a falling ball: a simple ***physical model***.

Modelling a Free-falling Ball

```
type Pos = Double
type Vel = Double

fallingBall :: Pos -> Vel -> SF () (Pos, Vel)
fallingBall y0 v0 = proc () -> do
    v  <-  (v0 +) ^<< integral  -<  -9.81
    y  <-  (y0 +) ^<< integral  -<  v
    returnA -< (y, v)
```

Some different and extra symbols, but just superficial syntactic details: the ***structure*** remains the same. We have turned the mathematical model into a declarative program!

More information

- `http://www.haskell.org`
- John Hughes, recent retrospective: Why Functional Programming Matters `https://www.youtube.com/watch?v=FGQAP0Gx1W8`